Day 6: Two Way Tables & Conditional Probability

Warm-Up:

For the following questions, use the fundamental counting principle, permutation or combination formulas.

1) As I'm choosing a stocking for my nephew, I'm given a choice of 3 colors for the stocking itself. I plan to have his name embroidered. I have 8 thread choices and 5 font choices. How many variations of stocking can be made for my nephew?

2) I have a candy jar filled with 250 different candies. How many ways can I grab a handful of 7 yummy confections to eat?

3) The VonTrapp family is taking pictures (they have 7 children). How many ways can we line the children up?

4) The VonTrapp children are being ornery. Kurt and Brigitta have to be on either side of the picture (away from one another). Now, how many ways can I line the children up?

5) I need to choose a password for my computer. It must be 5 characters long. I can choose any of the 26 letters of the alphabet (lowercase only) or any number as a character. How many possible passwords can I create?

Warm-Up:

For the following questions, use the fundamental counting principle, permutation or combination formulas.

1) As I'm choosing a stocking for my nephew, I'm given a choice of 3 colors for the stocking itself. I plan to have his name embroidered. I have 8 thread choices and 5 font choices. How many variations of stocking can be made for my nephew?

120

2) I have a candy jar filled with 250 different candies. How many ways can I grab a handful of 7 yummy confections to eat?

1.11×10^{13}

3) The VonTrapp family is taking pictures (they have 7 children). How many ways can we line the children up?

5040

4) The VonTrapp children are being ornery. Kurt and Brigitta have to be on either side of the picture (away from one another). Now, how many ways can I line the children up?

240

5) I need to choose a password for my computer. It must be 5 characters long. I can choose any of the 26 letters of the alphabet (lowercase only) or any number as a character. How many possible passwords can I create?

60,466,176

Warm-Up: Did you get #4?

The VonTrapp children are being ornery. Kurt and Brigitta have to be on either side of the picture (away from one another). Now, how many ways can I line the children up? (Remember from #3, they have 7 children)

2   *   5 * 4 * 3 * 2 * 1   * **1**   **= 240**

K or B   any of other 5 kids   **K or B**

(whichever 1 is left)

Probability Homework Answers
Packet Page 8 – 9 All

1. a) 5/36   **(Table for #1 on next slide)**
   b) 5/18
   c) 13/18
2. ½; Yes
3. ¼; Yes
4. 0
5. 1/9
   (mutually inclusive)
6. 5/9

7. a) 53/71
   b) 18/71
8. 11/26
9. a) 7/26
   b) 5/26
   c) 5/13

Table for #1

1. a) 5/36
   sum = 6

   b) 5/18
   sum < 6

   c) 13/18
   sum ≥ 6

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Tonight's Homework!

Packet Page 10 - 11 All

Remember to start studying for your
quiz on Wednesday after break! ☺

Also, make sure you've attended 2 tutorials –
they will be due SOON after the break!

Notes Day 6:
# Two Way frequency Tables and Conditional Probability

Conditional Probability

- Contains a **CONDITION** that may **LIMIT** the sample space for an event

- Can be written with the notation **P(B|A)**, which is read "the probability of event B, **GIVEN** event A"

- How likely is one event to happen, given that another event **HAS** happened?

- Percentages/probability based on the **ROW** or **COLUMN** total of the given event -> for two-way table problems

- More complex "given" problems may require use of this formula:

Probability of B given A = $P(B|A) = \dfrac{P(B \text{ and } A)}{P(A)}$

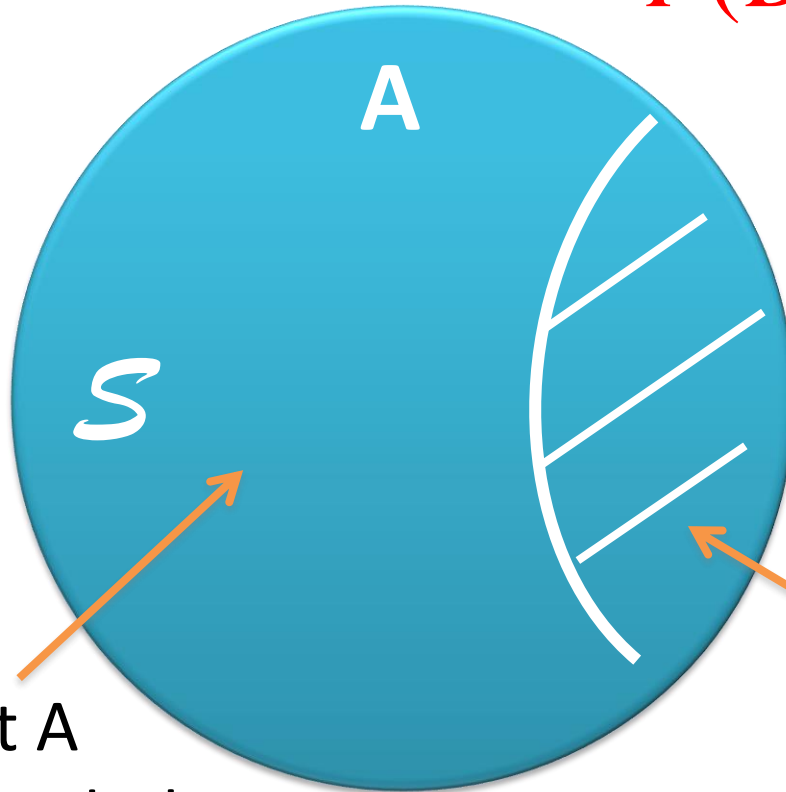# Why does this formula work?

Remember: Joint Probability   P(A and B)
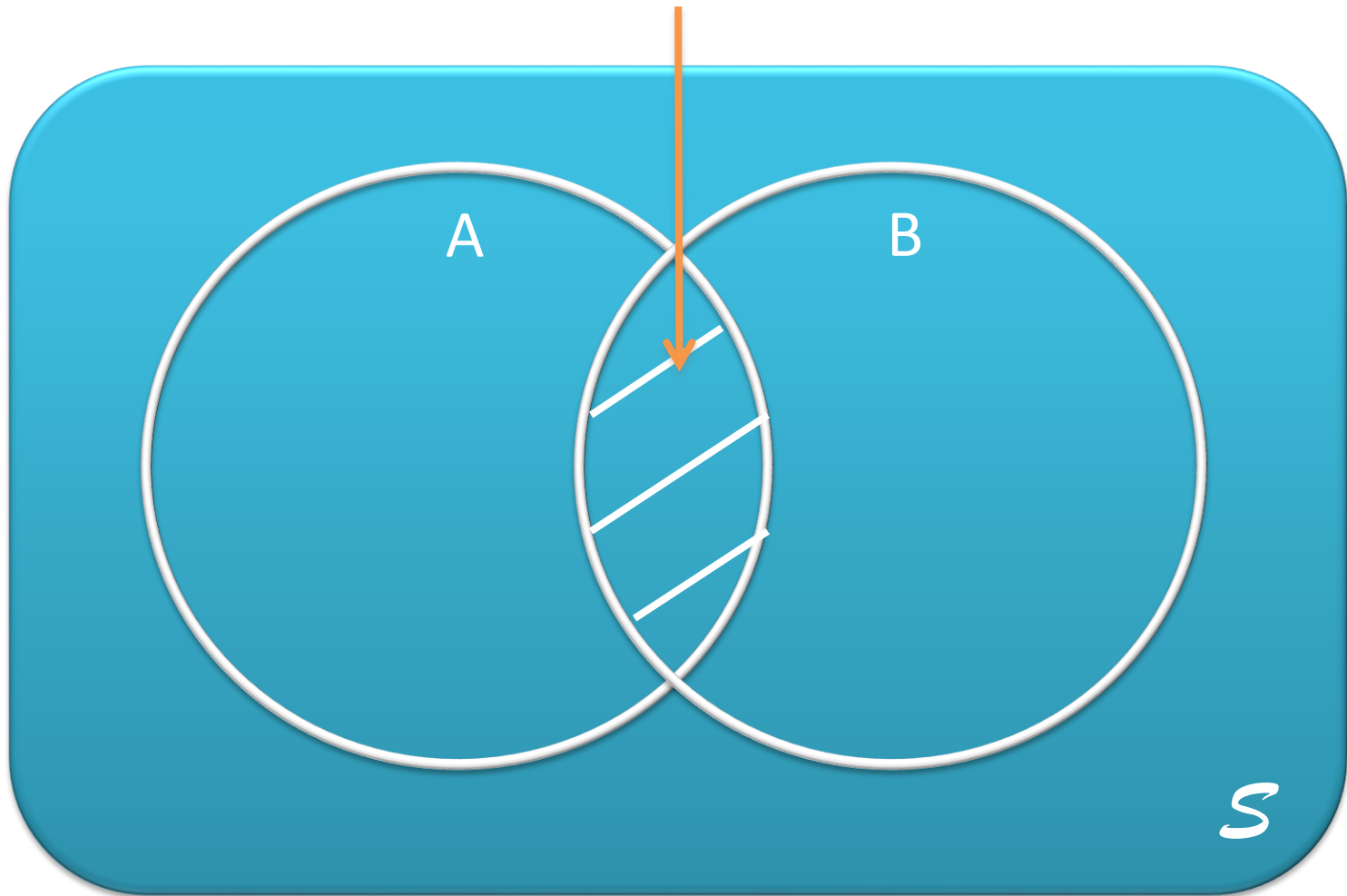
Conditional Probability

$$P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)}$$

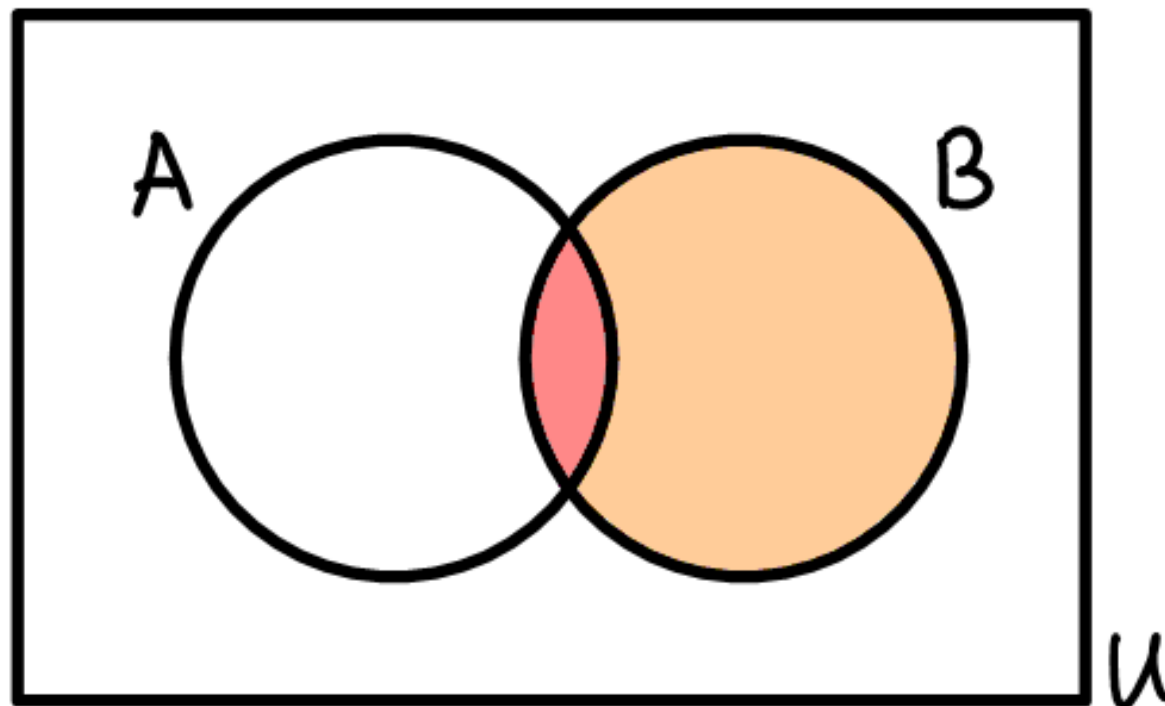Since Event A has happened, the sample space S is reduced to the outcomes in A

P(A and B) represents the outcomes from B that are included in A

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

P(2nd club | 1st club)

= P(2nd club and 1st club)/P(1st club)

= (13/52 x 12/51)/(13/52)

= 12/51 or 4/17

The probability of drawing a club on the second draw given the first card is a club is 4/17 or 23.5%

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

P(brown | blue)

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

= P(brown and blue)/P(blue)

= (6/8 x 2/7)/(6/8)

= 2/7

The probability of drawing a brown marble given the first marble was blue is 2/7 or 28.6%

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

P(brown eyes | brown hair)

= P(brown eyes and brown hair)/P(brown hair)

= .05/.7

= .071

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

The probability of a student having brown eyes given he or she has brown hair is 7.1%

4. In Mrs. Walden's class, 65% of the students have brown hair, 30% have green eyes, and 8% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes?

P(green eyes | brown hair)

= P(green eyes and brown hair)/P(brown hair)

= 0.08/0.65

= 0.123

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

The probability of a student having green eyes given he or she has brown hair is 12.3%

Using Two-Way Frequency Tables to Compute Conditional Probabilities

- In Math 1 you learned how to put data in a two-way frequency table (using counts) or a two-way relative frequency table (using percents), and use the tables to find joint and marginal frequencies and conditional probabilities.

- Let's look at some examples to review this.

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	
11th or 12th	41	58	184	7	
Total					

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

a. What is the probability that the student walked to school?

- 88/500 = 22/125

b. P(9th or 10th grader)

- 210/500 = 21/50

c. P(rode the bus OR 11th or 12th grader)

- 147/500 + 290/500 – 41/500 = 396/500 = 99/125

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

d. What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

P(11th or 12th | car)

* We only want to look at the car column for this probability!

= 11th or 12th graders in cars/total in cars

= 184/254 = 92/127   (or 72.4%)

The probability that a person is in 11th or 12th grade given that they rode in a car is 92/127 (or 72.4%).

	Bus	Walk	Car	Other	Total
9th or 10th	106	30	70	4	210
11th or 12th	41	58	184	7	290
Total	147	88	254	11	500

e. What is P(Walk|9th or 10th grade)?

= walkers who are 9th or 10th / all 9th or 10th

= 30/210

= 1/7 or 14.2%

The probability that a person walks to school given he or she is in 9th or 10th grade is 1/7 (or 14.2%).

2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult			
Child			
Total			

Let's discuss how to fill in the two-way table together!

You Try – Find the Probabilities!

|	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

a. Find P(vanilla)
   = 78/224
   = 39/112

b. Find P(child)
   = 131/224

c. Find P(vanilla | adult)
   = (52/224) / (93/224)
   = 52/93
(OR look @ adult row)

d. Find P(child | chocolate)
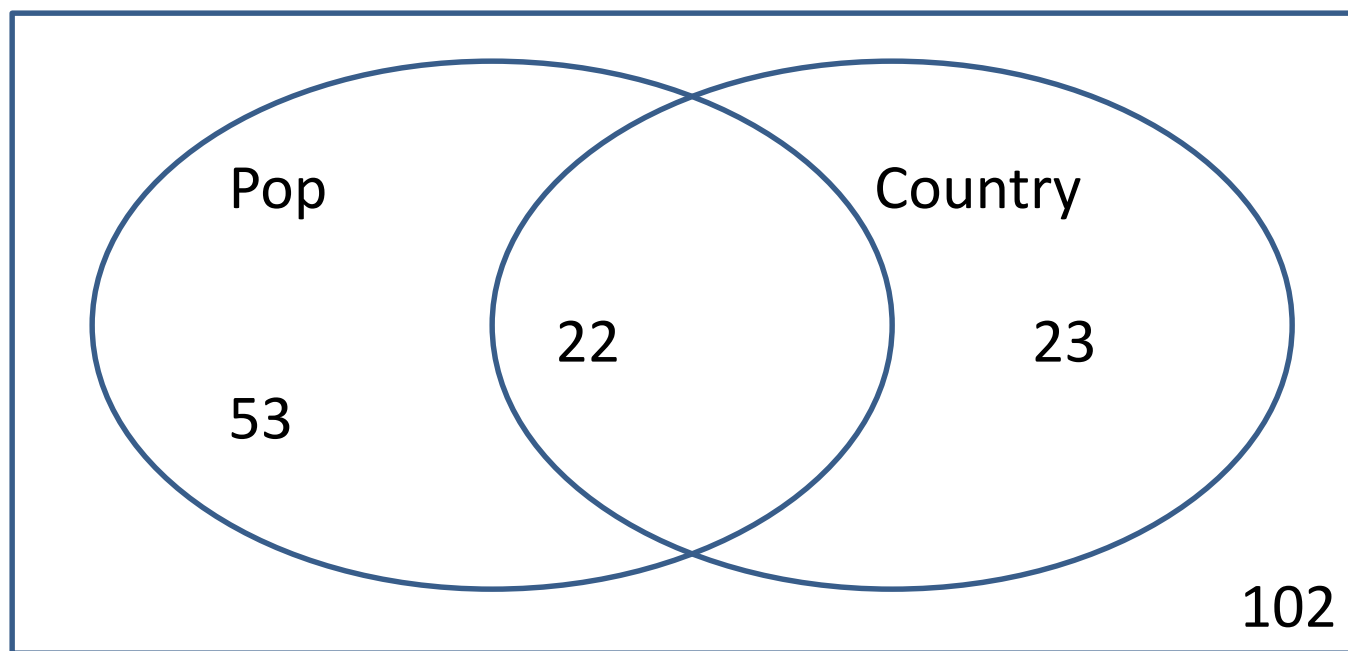   = (105/224) / (146/224)
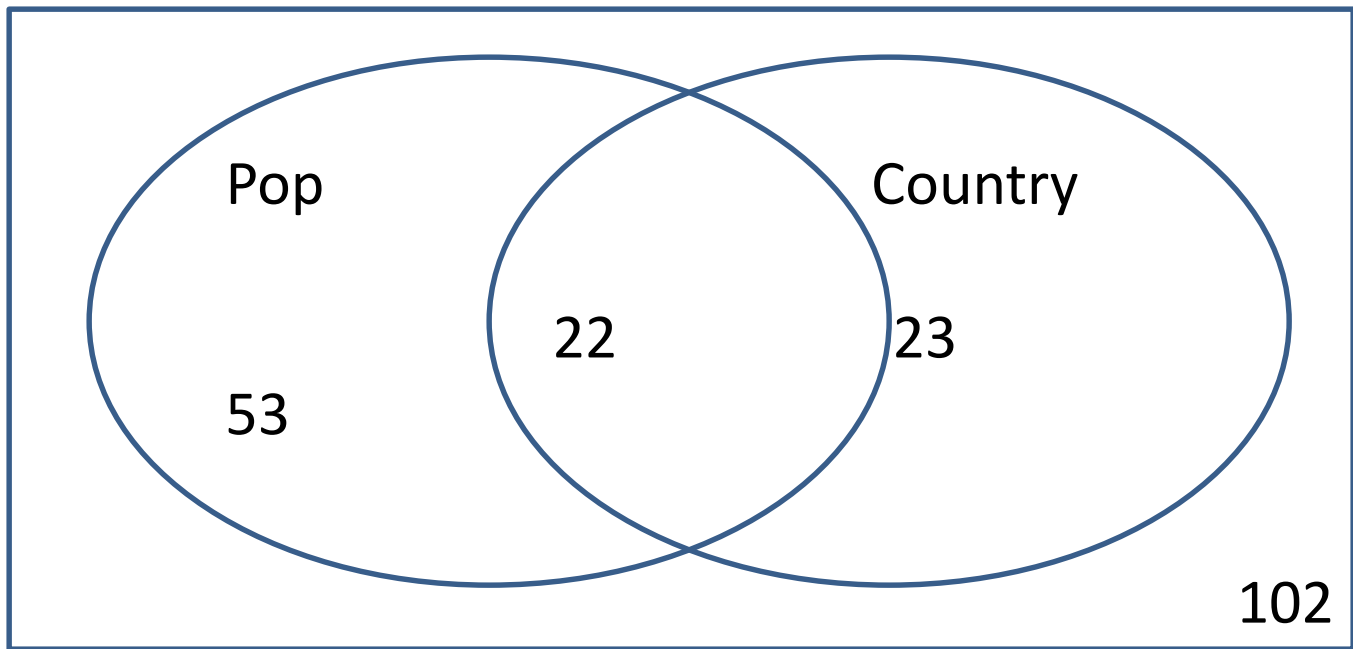   = 105/146
(OR look @ choc. column)

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

1ˢᵗ You Try Fill in Venn Diagram!
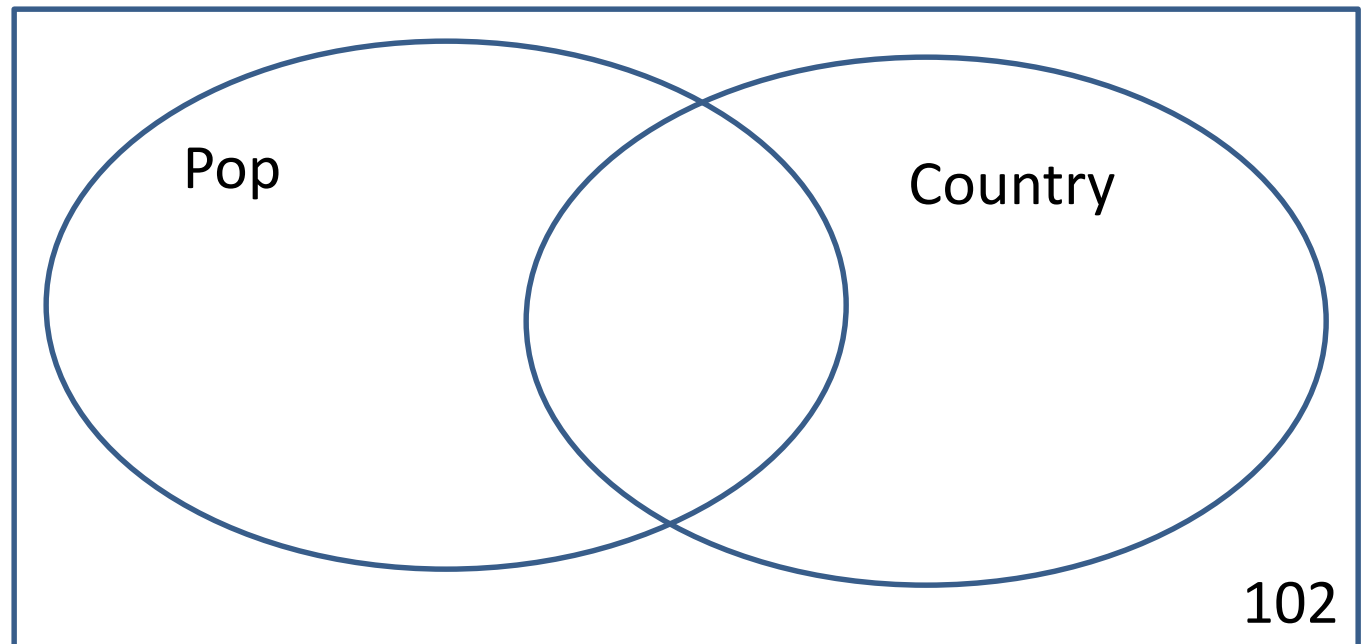
Pop

Country

102

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.
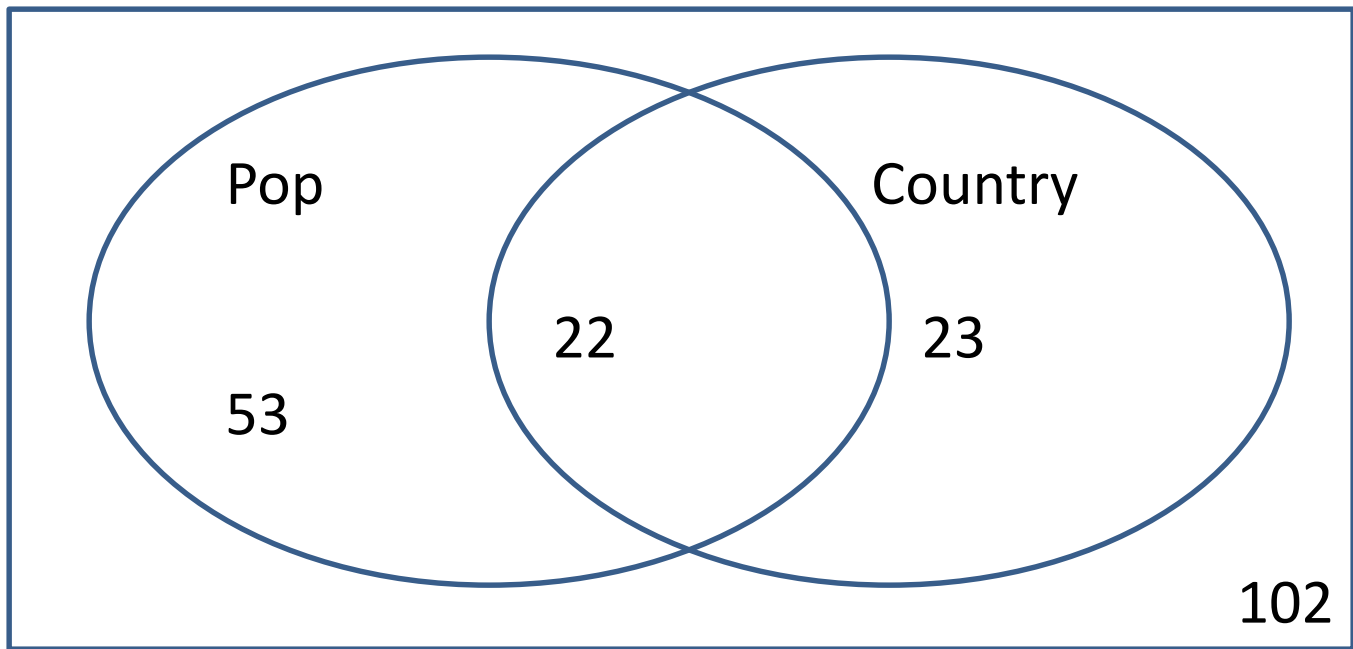
Check Your Venn Diagram THEN finish a-d!

Pop

Country

22

23

53

102

You Try!

Venn diagram:
- Pop
- Country
- 22 (intersection)
- 53 (Pop only)
- 23 (Country only)
- 102 (outside)

a) Probability that a randomly selected student listens to pop music given that they listen to country music **22/45**

b) P(pop U country) **49/100**

c) P(pop ∩ country) **11/100**

d) P(country ᶜ) **31/40**

Did you get part a?

a) Probability that a randomly selected student listens to pop music given that they listen to country music

P(Pop|Country)

= 22/(22+23)   look at "country" circle

    (or use formula   (22/200)/(45/200)   and get same answer)

= 22/45 or 48.9%

48.9% of students who listen to country also listen to pop.

Using Conditional Probability to Determine if Events are Independent

- If two events are statistically independent of each other, then:

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

- Let's revisit some previous examples and decide if the events are independent.

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. Each player draws two cards, without replacement. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent?

Let event A = draw a club and event B = draw a club.

$$P(A) = 13/52 \text{ or } 1/4 = .25$$

13 clubs out of 52 cards

$$P(B) = 13/52 \text{ or } 1/4 = .25$$

P(drawing a second club after drawing the first club) =

$$P(B|A) = 12/51 \text{ or } 4/17 \approx .235$$

Only 12 clubs left and only 51 cards left

So, $P(B) \neq P(B|A)$

Thus, the events of drawing a club followed by drawing another club without replacement are NOT independent events.

2. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. Each player draws a card, looks at it, then replaces the card randomly in the deck. Then they draw a second card. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent?

Let event A = draw a club and event B = draw a club

$$P(A) = 13/52 \text{ or } 1/4 = .25$$

13 clubs out of 52 cards

$$P(B) = 13/52 \text{ or } 1/4 = .25$$

P(drawing a second club after drawing the first club) =

$$P(B|A) = 13/52 \text{ or } 1/4 = .25$$

Still 13 clubs out of 52 cards

$$P(B) = P(B|A)$$

Similarly, we can show that $P(A) = P(A|B)$

Thus, the events of drawing a club with replacement followed by drawing another club are independent events.

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

P(brown eyes|brown hair)

= P(brown eyes and brown hair)/P(brown hair)

= .05/.7

= .071

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

P(brown eyes) = 0.25

P(brown eyes|brown hair) ≠ P(brown eyes)
  so these are dependent events

	Vanilla	Chocolate	Total
Adult	52	41	93
Child	26	105	131
Total	78	146	224

4. Determine whether age and choice of ice cream are independent events.

We could start by looking at the P(vanilla|adult) and P(vanilla). If they are the same, then the events are independent.

P(vanilla|adult) = 52/93 = 55.9%

P(vanilla) = 78/224 = 34.8%

P(vanilla|adult) ≠ P(vanilla), so the events are dependent!

Practice:
Complete Notes p. 24
"Given Discovery"

Ex: Suppose you manage a restaurant that serves chicken wings that are mild or hot, and boneless or regular. From your experience you know that of boneless wings bought, 75% of them are mild, and of the regular wings bought, 70% are hot. Only 4 out of 10 costumers buy boneless wings.

1) Create a tree diagram for the scenario displaying all the possibilities and probabilities.
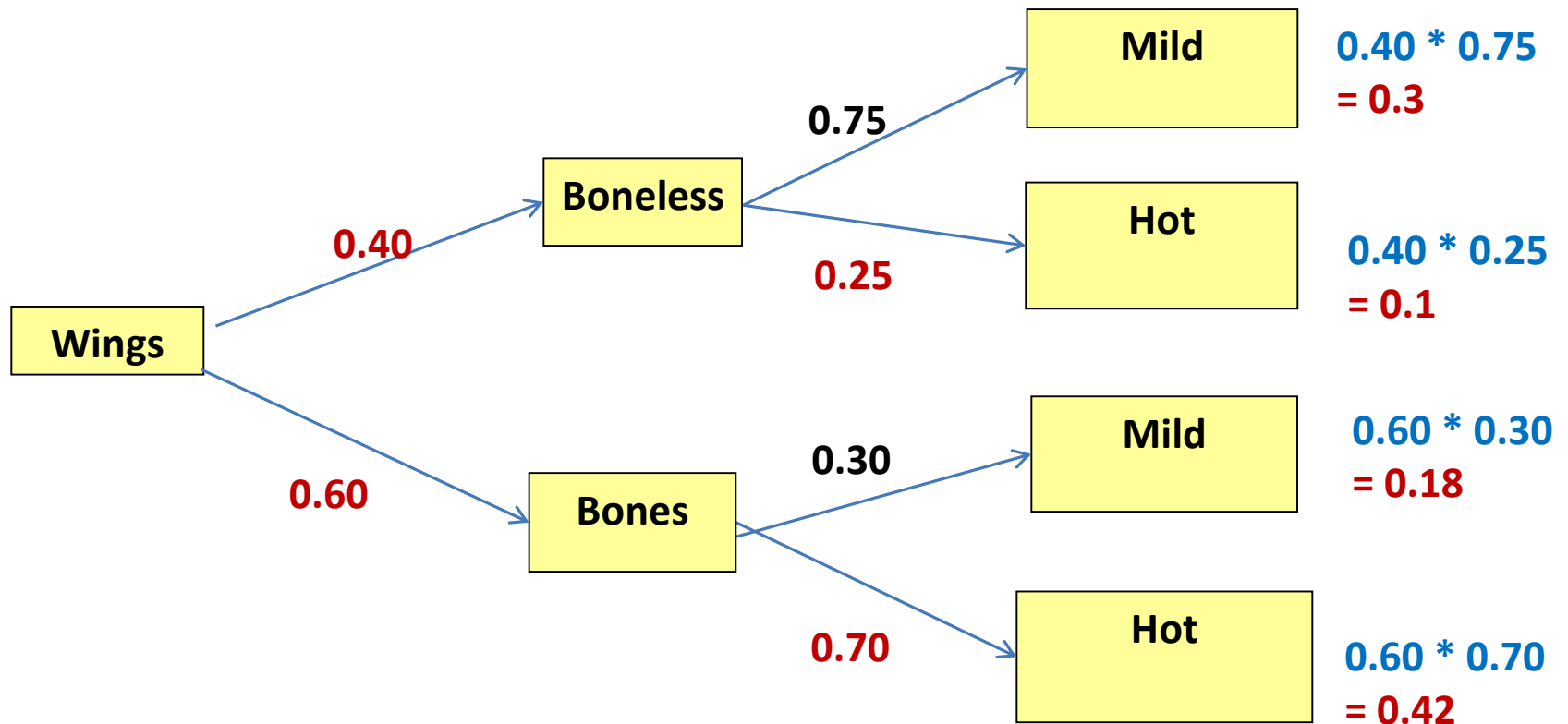
2) P(boneless and hot wings)

$$(.4)(.75) = .1$$

10%

3) P(hot | boneless)

$$\frac{(.4)(.25)}{(.4)} = .25 = 25\%$$

4) P(hot)

$$(.4)(.25) + (.6)(.7) = .52 = 52\%$$

5) P(boneless |hot)

$$\frac{(.4)(.25)}{(.4)(.25) + (.6)(.7)} = .192$$

$$= 19.2\%$$

6) P(mild wings)

$$(.4)(.75) + (.6)(.3) = .48$$

48%

7) If a person orders regular wings, what is the probability they choose mild?

30%

8) P(boneless |mild)

$$\frac{(.4)(.75)}{(.4)(.75) + (.6)(.3)} = .625 = 62.5\%$$

9) Of the boneless wings, what is the probability someone orders mild?

75%

Tonight's Homework!

Packet Page 10 - 11 All

Remember to start studying for your
quiz on Wednesday after break! ☺

Also, make sure you've attended 2 tutorials!