Day 5: Mutually Exclusive and Inclusive Events

Honors Math 2

Unit 6: Probability

Warm-up on Notebook paper (NOT in notes)

1. A local restaurant is offering taco specials. You can choose 1, 2 or 3 tacos from a total of 5 choices. If you must choose different tacos, how many possible taco combinations can you select?
2. You are creating a schedule for 1 day of your club's community service. You need a team for campus clean-up, 3 teams to serve at the soup kitchen, and two teams to collect can donations. In how many ways can you select the teams from a set of 3 campus clean-up groups, 6 soup kitchen groups, and 7 can collection groups?
3. In a group of 56 students, 32 take a CTE class, 20 take a Foreign Language, and 7 take both. Let **C** = CTE and **F** = Foreign Language.
   a. *What is* $C \cup F$ *?*
   b. *What is* $C \cap F$ *?*
   c. *What is* C^C *?*
4. How many distinguishable arrangements are possible using the letters of the word PARALLEL?
5. You have 6 different rings. How many ways can you wear them on 3 fingers?

Warm-up ANSWERS

1. A local restaurant is offering taco specials. You can choose 1, 2 or 3 tacos from a total of 5 choices. If you must choose different tacos, how many possible taco combinations can you select?

$$_5C_3 + {}_5C_2 + {}_5C_1 = 25$$

Combination because you are choosing a collection so order does not matter, and you can't repeat items.

2. You are creating a schedule for 1 day of your club's community service. You need a team for campus clean-up, 3 teams to serve at the soup kitchen, and two teams to collect can donations. In how many ways can you select the teams from a set of 3 campus clean-up groups, 6 soup kitchen groups, and 7 can collection groups?

$$\underset{\text{Clean-Up}}{{}_3C_1} \bullet \underset{\text{Soup Kitchen}}{{}_6C_3} \bullet \underset{\text{Can Collection}}{{}_7C_2} = 1{,}260$$

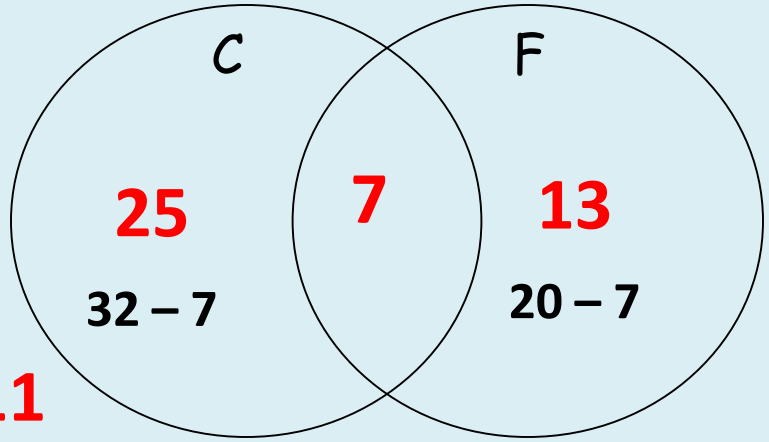
Warm-up ANSWERS

3. In a group of 56 students, 32 take a CTE class, 20 take a Foreign Language, and 7 take both. Let **C** = CTE and **F** = Foreign Language.

FIRST, compelte a Venn Diagram, like the one shown

a. What is $C \cup F$ *?*

Union means in C or F or both 25 + 7 + 13 = 45



b. What is $C \cap F$ *?*

Intersection means in C AND F = 7

c. What is C^C *?*

Complement means NOT in C

11 + 13 = 24

Warm-up ANSWERS

4. How many distinguishable arrangements are possible using the letters of the word PARALLEL?

$$\frac{8!}{2!\,3!} = 3360$$

5. You have 6 different rings. How many ways can you wear them on 3 fingers?

$${}_6P_3 = 120$$

TONIGHT'S HOMEWORK

Packet p. 8 and 9

Remember to start studying for your quiz on Wednesday after break! ☺

Also, make sure you've attended 2 tutorials – they will be due SOON after the break!

HW Answers: Cumulative Review

10) a) 270° rotation b) (x, y) -> (y, -x)

11) a) reflection over x-axis

b) (x, y) -> (x, -y) c) (x, y) -> (x+3, y+4)

12) a) next = now $\cdot$0.88; start = 28,500

b) $y = 28500(0.88)^x$

c) $15040.36

13) $y = 8(1.5)^x$

14) $Y = 0.06(x)^{2.0131}$; 69.37 minutes

15) a) Area = $x(120 - 2x)$

b) Area = $120x - 2x^2$

c) 1800 ft^2

Cumulative Review Homework Answers

#'s 22 - 26

22) $m\angle R = 70°$, $m\angle S = 70°$, $m\angle T = 40°$,
$SR = 3.6$, $TR = 6$

23) a) $y = |x - 4| + 2$ b) V (4, 2)
c) $(-\infty, \infty)$ d) $[2, \infty)$
e) right 4, up 2

24 – 25) On next slide

26) a) $y = \frac{5}{x} - 2$
b) $(-\infty, 2) \cup (2, \infty)$
c) $(-\infty, 0) \cup (0, \infty)$
d) left 2

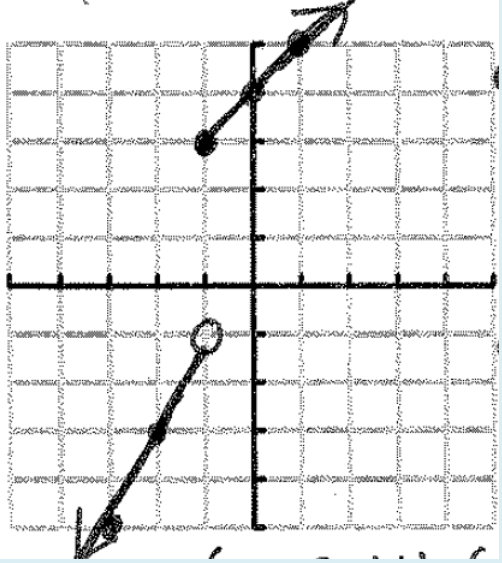
Cumulative Review Homework Answers
#24 AND 25

24) a) 6 b) –5 c) 3

d) $(-\infty, \infty)$

e) $(-\infty, -1) \cup [3, \infty)$

$$g(x) = \begin{cases} x + 4 & \text{if } x \geq -1 \\ 2x + 1 & \text{if } x < -1 \end{cases}$$



25) b) (–3.5, –2.25)

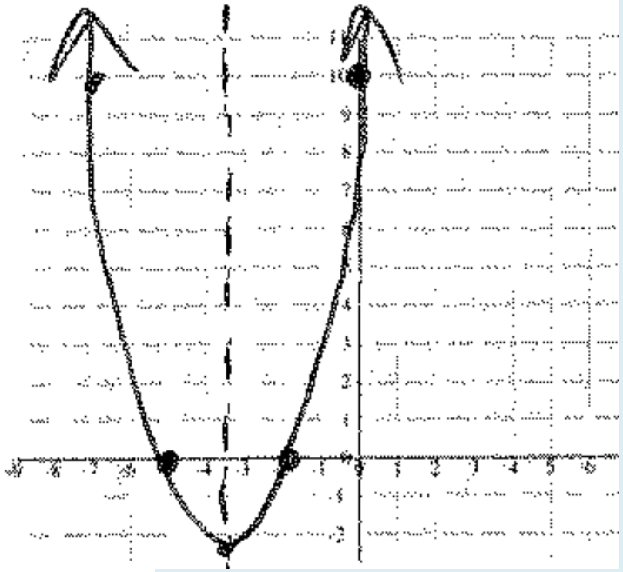
c & d) (–5, 0) (–2, 0)

e) x = –3.5

f) opens up

g) (0, 10)

25. Given $f(x) = x^2 + 7x + 10$.



Notes Day 5: Mutually Exclusive and Inclusive Events

Probability of event **NOT** occurring

The probability that an event E will not occur is equal to one **minus** the probability that it will occur

P(not E) = **1 - P(Event)**

Example:

If 3 prizes for every 1000 raffle tickets,

P(not win) = 1- P(win) = 1 – 3/1000 = 997/1000

P(not E) = 1 - P(Event)

Ex 1: Find the probability that you choose a number from 1 to ten that is not 6.

1 – (1/10) = 9/10

Ex 2: Find the probability that you deal a card that is not a diamond.

1 – (13/52) = 39/52 = 3/4

Ex 3: You draw a card that is not a red face card (Jack, Queen, King)

1 – (6/52) = 46/52 = 23/26

P(not E) = 1 - P(Event)

Ex 4: You select someone in the class who is not wearing jeans.

Ex 5: In the classic lottery game, each player chooses 6 different numbers from 1 to 48. If all of the numbers match the 6 picked, they win. What is the probability of not winning?

“OR” Probability Discovery

Notes p. 16 - 17

Checkpoints:

Check in after you complete
Part A #10 and #11

Part B #7 and 8

(Notes p.18) Summary:

Mutually Exclusive Events

- Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or you roll a 2?
    - Can these both occur at the same time? Why or why not?

      NO. 1, 3, 5, 2 are the numbers so there is not an overlap here.

      P(odd or 2) = 1, 3, 5, 2 = 4/6 = 2/3

- **Mutually Exclusive Events (or Disjoint Events):** Two or more events that cannot occur at the same time.
- The probability of two mutually exclusive events occurring at the same time , P(A and B), is 0.

Probability of Mutually Exclusive Events

- To find the probability of one of two **mutually exclusive** events occurring, use the following formula:

$$P(A \text{ or } B) = P(A) + P(B)$$

or

$$P(A \cup B) = P(A) + P(B)$$

*If A and B are mutually exclusive (no overlap), then P(A and B) = **0 .**

Mutually Inclusive Events

- Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or a number less than 4?
  - Can these both occur at the same time? If so, when?

    YES. 1, 3, 5, 1, 2, 3 are the numbers so there *IS* an overlap here.

    P(odd or less than 4) = 1, 3, 5, 2 = 4/6 = 2/3

- **Mutually Inclusive Events:** Two events that can occur at the same time.

Probability of the Union of Two Events:

The Addition Rule

- We just saw that the formula for finding the probability of two mutually inclusive events can also be used for mutually exclusive events, so let's think of it as the formula for finding the probability of the union of two events or the Addition Rule:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

P(odd or less than 4) = P(odd) + P(<4) – P(odd and < 4)

$$= \underset{1,3,5}{3/6} + \underset{1,2,3}{3/6} - \underset{1,3}{2/6} = 4/6 = 2/3$$

*****Use this for both Mutually Exclusive and Inclusive events*****

Another example of why we have to calculate differently for OR probability with overlap! (Not in Notes)

Ex/ Probability of selecting someone with green eyes or brown hair from the class.

Overlap or No overlap?

Use P(A or B) = P(A) + P(B) - P(A and B)

P(green eyes or brown hair)

= P(green eyes) + P(brown hair) – P(gr. eyes & brown hair)

= _____ + _____ - _____ = _____

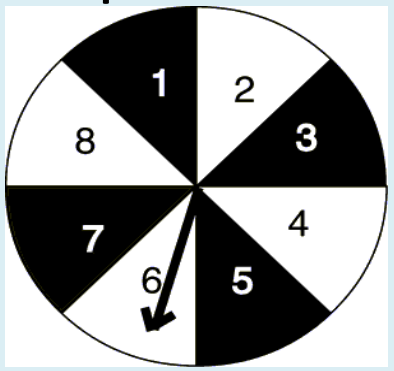
Mutually Inclusive Events

- **Mutually Inclusive Events:** Two events that can occur at the same time.

- Video on Mutually Inclusive Events

Remember:

Events that cannot happen at the **same time** are mutually exclusive events. There is "no overlap".



Examples TOGETHER: Are the events mutually exclusive? Explain.

1) Spinning a 4 or a 6 at the same time on a single spin.

They can't both happen at once, you can't end up with a 4 and a 6 in one spin – Mutually Exclusive

2) Spinning an even number or a multiple of 3 at the same time on a single spin.

They can both happen at once, if you spin a 6 it is a multiple of 3 (3*2) AND it is even – NOT Mutually Exclusive

You try ☺ Are the events mutually exclusive?

3) Spinning an even number or a prime number on a single spin.

Who can remind us what "prime" means?

A prime number is a number greater than 1 that can only be divided by 1 and itself

They can both happen at once, 2 is even and also prime – Not Mutually Exclusive

4) Spinning an even number or a number less than 2 on a single spin.

They cannot both happen at once, 1 is not even and it's the only number on the spinner less than 2 – Mutually Exclusive

Examples of "OR" probability

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?

Are they mutually exclusive or mutually inclusive?

Mutually inclusive (there's some overlap)

P(choosing a club or a ten)

= P(club) + P(ten) – P(10 of clubs)

= 13/52 + 4/52 – 1/52

= 16/52

= 4/13

The probability of choosing a club or a ten is 4/13

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

Are they mutually exclusive or mutually inclusive?

Mutually inclusive (there's some overlap)

P(<5 or odd)

= P(<5) + P(odd) – P(<5 and odd)

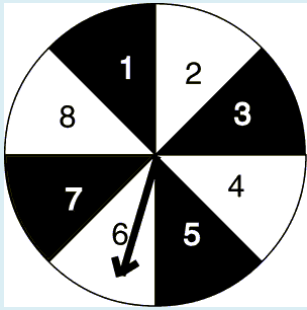
<5 = {1,2,3,4} odd = {1,3,5,7,9}

= 4/10 + 5/10 – 2/10

= 7/10

The probability of choosing a number less than 5 or an odd number is 7/10

(Beneath examples 3 and 4)



Ex: Spinning a 4 or 6 on a 1-8 spinner (Numbers 1, 2, 3, 4, 5, 6, 7 and 8 occur on the spinner).

Are they mutually exclusive or mutually inclusive?

Mutually Exclusive (there's NO overlap)

Probability of 4 or 6:

P(4) + P(6) – P(4 and 6)

1/8 + 1/8 – 0

= 2/8 = ¼

OR Probability of 4 or 6: 2/8 = ¼

You Try!

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

Are they mutually exclusive or mutually inclusive?

Mutually inclusive (there's some overlap)

P(one of the first 10 letters or vowel)

= P(one of the first 10 letters) + P(vowel) – P(first 10 and vowel)
= 10/26 + 5/26 – 3/26
= 12/26 or 6/13

The probability of choosing either one of the first 10 letters or a vowel is 6/13

4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

Are they mutually exclusive or mutually inclusive?

Mutually exclusive (there's NO overlap)

P(one of the last 5 letters or vowel)
= P(one of last 5 letters) + P(vowel) – P(last 5 & vowel)
= 5/26 + 5/26 – 0
= 10/26 or 5/13

The probability of choosing either one of the first 10 letters or a vowel is 5/13

Practice/Examples

Finish Notes p. 19

Practice/Examples

1. If you randomly chose one of the integers 1 – 10, what is the probability of choosing either an odd number or an even number?

Are these mutually exclusive events? Why or why not?

Yes. They are mutually exclusive because there is no overlap.

Complete the following statement:

P(odd or even) = P(**odd**) + P(**even**)

Now fill in with numbers:

P(odd or even) = **5/10** + **5/10**

P(odd or even) = **½ + ½** **= 1**

Does this answer make sense?

Yes. Numbers are either even or odd, so P(even or odd) should be 100% to include all numbers!

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10?

Are these events mutually exclusive or mutually inclusive? Why? **They are mutually exclusive because there is no overlap.**

Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice to help calculate the probability requested:

Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4				
3	4					
4						
5						
6						

Completed table and problem is on the next slide ->

Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P(getting a sum less than 7 OR sum of 10)

= P(sum less than 7) + P(sum of 10)

= 15/36 + 3/36

= 18/36 = ½

The probability of rolling a sum less than 7 or a sum of 10 is ½ or 50%.

Exit Ticket!

1. For a radio show, a DJ can play 4 songs. If there are 8 to select from, in how many ways can the program for this show be arranged?

2. An election ballot asks voters to select no more than three city commissioners but at least one from a group of six candidates. In how many ways can this be done?

3. Consider a set of cards labeled 1-10. Let set A = even numbers and set B = # greater than 8. Find the probability of A or B.

4. Using the situation from problem #3, what is the probability you select an even number given you selected a number greater than 8?

Exit Ticket Answers

1. For a radio show, a DJ can play 4 songs. If there are 8 to select from, in how many ways can the program for this show be arranged?

   ${}_8P_4 = 1680$

2. An election ballot asks voters to select no more than three city commissioners but at least one from a group of six candidates. In how many ways can this be done?

   ${}_6C_3 + {}_6C_2 + {}_6C_1 = 41$

3. Consider a set of cards labeled 1-10. Let set A = even numbers and set B = # greater than 8. Find the probability of A or B.

   $P(A \text{ or } B) = 5/10 + 2/10 - 1/10 = 6/10 = 3/5$

4. Using the situation from problem #3, what is the probability you select an even number given you selected a number greater than 8?

   $(1/10) / (2/10) = 5/10 = ½$

   (OR the numbers >8 are 9, 10 and 1 of the 2 is even ☺)

TONIGHT'S HOMEWORK

Packet p. 8 and 9