Day 5: Tree Diagrams, Conditional Probability, and Two Way Tables

Warm-Up:

Use the fundamental counting principle, permutation or combination formulas to answer the following.

1) As I'm choosing a stocking for my nephew, I'm given a choice of 3 colors for the stocking itself. I plan to have his name embroidered. I have 8 thread choices and 5 font choices. How many variations of stocking can be made for my nephew?

$\underset{\text{color stocking}}{\underline{3}} \cdot \underset{\text{thread choice}}{\underline{8}} \cdot \underset{\text{font choice}}{\underline{5}} = \boxed{120}$

2) I have a candy jar filled with 250 different candies. How many ways can I grab a handful of 7 yummy confections to eat?

① can't be repeated ② order doesn't matter = combination

${}_{250}C_7 = \boxed{1.1126 \cdot 10^{13}}$

3) The VonTrapp family is taking pictures (they have 7 children). How many ways can we line the children up?

${}_7P_7 = \boxed{5040}$

OR $\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

① can't be repeated (child in only 1 spot) ② order matters "line up"

4) The VonTrapp children are being ornery. Kurt and Brigitta have to be on either side of the picture (away from one another). Now, how many ways can I line the children up?

$\underset{\text{Kurt or Brigitta}}{\underline{2}} \cdot \overbrace{\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}}^{\text{1 of other 5 kids}} \cdot \underset{\text{Kurt or Brigitta}}{\underline{1}}$ $\boxed{240}$ OR $\underset{\text{line others}}{{}_5P_5} \cdot \underset{\text{line K+B}}{{}_2P_2}$

5) I need to choose a password for my computer. It must be 5 characters long. I can choose any of the 26 letters of the alphabet (lowercase only) or any number as a character. How many possible passwords can I create?

$\underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36} \cdot \underline{36}$ or $36^5 = \boxed{60,466,176}$

36 possibilities 26 letters, #s 0-9 for all

*Doesn't say can't repeat SO must use counting principle (not C or P)

Notes Day 5: Two Way Frequency Tables and Conditional Probability

Conditional Probability:

- contains a condition that may limit the sample space for an event
- can be written with the notation P(B|A), which is read "the probability of event B, given event A" *given event A means event A has happened
- how likely is one event to happen, given that another event has happened?
- percentages/probability based on the row or column total of the given event if looking at two-way table
- more complex "given" problems may require use of this formula:

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Examples of Conditional Probability:

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

$\frac{P(\text{club} \ \& \ \text{2nd club} \mid \text{1st club})}{P(\text{1st club})} = \frac{\frac{13}{52} \cdot \frac{12}{51} = \frac{12}{51}}{\frac{13}{52}} = \boxed{\frac{4}{17}}$

OR 13 total clubs, 1 drawn 1st so 12 left

$P(\text{club} \mid \text{club}) = \frac{12}{51} = \boxed{\frac{4}{17}}$

51 total left

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

not replaced

$\frac{P(\text{1st blue} \ \& \ \text{2nd brown})}{P(\text{1st blue})} = \frac{\frac{6}{8} \cdot \frac{2}{7}}{\frac{6}{8}} = \boxed{\frac{2}{7}}$

OR $P(\text{brown} \mid \text{blue}) = \frac{2}{7}$ ← 2 brown, ← 7 total left

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

↳ P (brown eyes given brown hair)

$= P(\text{brown eyes} \mid \text{brown hair}) = \frac{P(\text{brown eyes} \ \& \ \text{brown hair})}{P(\text{brown hair})} = \frac{.05}{.70}$

$= .0714$

$\boxed{7.14\%}$

Using Two-Way Frequency Tables to Compute Conditional Probabilities

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

[redacted]	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	210
11^{th} or 12^{th}	41	58	184	7	290
Total	147	88	254	11	500

Suppose we randomly select one student.

a) What is the probability that the student walked to school? $\frac{88}{500} = \boxed{\frac{22}{125}}$

look at 2nd column →

b) $P(9^{th}$ or 10^{th} grader)

$= \frac{210}{500} = \boxed{\frac{21}{50}}$

look at 1st row.

c) P(rode the bus OR 11^{th} or 12^{th} grader)

$P(\text{rode bus}) + P(11^{th} \text{ or } 12^{th}) - P(\text{rode bus} \ \& \ 11^{th} \text{ or } 12^{th})$

$= \frac{147}{500} + \frac{290}{500} - \frac{41}{500} = \frac{396}{500} = \boxed{\frac{99}{125}}$

d) What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

$P(11^{th} \text{ or } 12^{th} \mid \text{rode in car})$ $\boxed{\frac{92}{127}}$

$= \frac{P(11^{th} \text{ or } 12^{th} \ \& \ \text{rode car})}{P(\text{rode car})} = \frac{184/500}{254/500}$

e) What is P(Walk|9th or 10th grade)?

$\frac{P(\text{Walk} \ \& \ 9^{th} \text{ or } 10^{th} \text{ grade})}{P(9^{th} \text{ or } 10^{th} \text{ grade})}$

$= \frac{30/500}{210/500} = \frac{30}{210} = \boxed{\frac{1}{7}}$

2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

total

fill in together

	Vanilla	Chocolate	Total
Adult	52 (in problem)	41 (do 93-52)	93 (in problem)
Child	26 (do 78-52)	105 (do 146-41)	131 (do 26+105 or 224-93)
Total	78 (do 224-146)	146 (in problem)	224 (in problem)

You Try

a) Find P(vanilla) $= \frac{78}{224} = \boxed{\frac{39}{112}}$

b) Find P(child) $= \boxed{\frac{131}{224}}$

c) Find P(vanilla|adult)

$= \frac{P(\text{vanilla} + \text{adult})}{P(\text{adult})} = \frac{\frac{52}{224}}{\frac{93}{224}} = \boxed{\frac{52}{93}}$

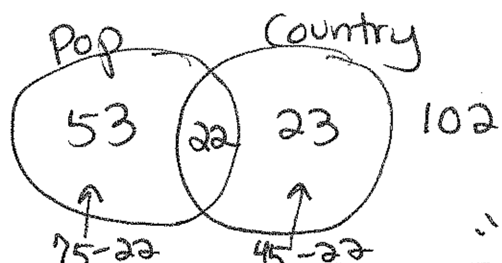
OR zoom in to adult row

d) Find P(child|chocolate)

$= \frac{P(\text{child} + \text{chocolate})}{P(\text{chocolate})} = \frac{105/224}{146/224} = \boxed{\frac{105}{146}}$

OR zoom in to chocolate column

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the following



a) probability that a randomly selected student listens to pop music given that they listen country music.

$P(\text{pop} \mid \text{country}) = \frac{P(\text{pop} + \text{country})}{P(\text{country})}$

OR zoom in to country bubble $= \frac{22/200}{45/200} = \boxed{\frac{22}{45}}$

b) P(pop ∪ country)

"union" OR → pop or country or both

$= P(\text{just pop}) + P(\text{just country}) + P(\text{pop} + \text{country})$

$= \frac{53}{200} + \frac{23}{200} + \frac{22}{200} = \frac{98}{200} = \boxed{\frac{49}{100}}$

c) P(pop ∩ country)

intersection "AND" → both

$\frac{22}{200} = \boxed{\frac{11}{100}}$

d) P(country ^{c})

$= 1 - P(\text{country}) = 1 - \frac{45}{200} = \frac{155}{200} = \boxed{\frac{31}{40}}$

OR

$P(\text{just pop}) + P(\text{not pop or country}) = \frac{53}{200} + \frac{102}{200}$

Using Conditional Probability to Determine if Events are Independent

If two events are statistically independent of each other, then:

$P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$

Let's revisit some previous examples and decide if the events are independent.

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit without replacement. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

   $P(\text{club}) = \frac{13}{52} = \frac{1}{4}$; $P(\text{club} \mid \text{club}) = 12/51$ from earlier today

   - Are the two events independent?
   - Let drawing the first club be event A and drawing the second club be event B.

   by formula ⟹ If independent, $P(\text{club} \mid \text{club}) = P(\text{club})$ ⟹ $\boxed{\text{Not indep}}$

   $12/51 \neq 1/4$

2. You are playing a game of cards where the winner is determined by drawing ~~tow~~ two cards of the same suit. Each player draws a card, looks at it, then replaces the card randomly in the deck. Then they draw a second card. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent? $\boxed{\text{Dep}}$

   Fix "two"

   $P(\text{club}) = \frac{13}{52} = 1/4$

   $P(\text{club} \mid \text{club}) = 13/52 = 1/4$ because "replaced"

   By formula, indep. if $P(\text{club} \mid \text{club}) = P(\text{club})$

   $1/4 = 1/4$ ⟹ $\boxed{\text{Indep}}$

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment.

   - If the student has brown hair, what is the probability that the student also has brown eyes?

     From earlier $P(\text{breyes} \mid \text{brhair}) = \frac{P(\text{br hair} + \text{breyes})}{P(\text{br hair})} = \frac{0.05}{0.70} = 0.071$; $P(\text{breyes}) = 0.25$

   - Are event A, having brown hair, and event B, having brown eyes, independent?

     $P(\text{breyes} \mid \text{br hair}) \neq P(\text{breyes})$

     $0.071 \neq 0.25$

     Not Indep $\boxed{\text{Dependent}}$

4. Using the table from the ice cream shop problem, determine whether age and choice of ice cream are independent events.

   $P(\text{vanilla} \mid \text{adult}) = \frac{P(\text{vanilla} + \text{adult})}{P(\text{adult})} = \frac{52/224}{93/224} = \boxed{\frac{52}{93}}$ OR look @ adult row

   $P(\text{vanilla}) = \boxed{39/112}$

   If indep, $P(\text{vanilla} \mid \text{adult}) = P(\text{vanilla})$

   $52/93 \neq 39/112$

   Not indep $\boxed{\text{Dependent}}$