Day 4: Probability of Mutually Inclusive and Exclusive Events

Warm-Up:

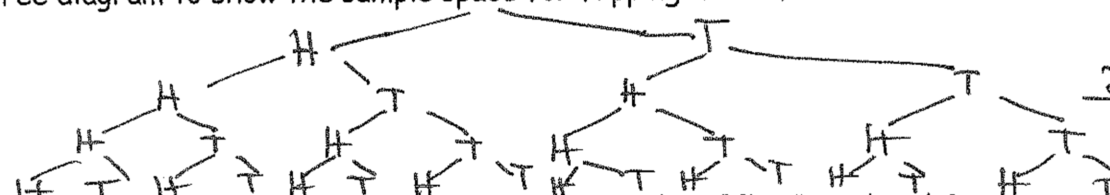
1. Your I-tunes card has enough for 3 of the 7 songs you want. In how many ways could you pick the songs?
   $_7C_3 = \boxed{35}$

2. We use 10 digits in our number system. How many 4-digit "numbers" can be formed if no digits are repeated ? (Zero is allowed in any position)
   ↳ permutation is ok
   $_{10}P_4 = \boxed{5040}$ OR $10 \cdot 9 \cdot 8 \cdot 7$

3. Confirm your answer to #2 using the Fundamental Counting Principle.
   $\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = \boxed{5040}$

4. Bad Frog Yogurt lets you pick 4 or fewer toppings from 40 choices and save 50 cents off of your order. How many ways can you get the savings?
   $_{40}C_4 + {_{40}C_3} + {_{40}C_2} + {_{40}C_1} + {_{40}C_0}$
   $91390 + 9880 + 780 + 40 + 1$
   $\boxed{102091}$

5. Create a tree diagram to show the sample space for flipping a coin four times.



check
$\underset{\text{1st flip}}{2} \cdot \underset{\text{2nd flip}}{2} \cdot \underset{\text{3rd flip}}{2} \cdot \underset{\text{4th flip}}{2} = 16$ items ✓

6. Using your answer to #5, what is the probability that all four "flips" are heads?
   $\boxed{1/16}$

Notes Day 4: Probability of Mutually Inclusive and Exclusive Events

Probability of an event NOT occurring

The probability that an event E will not occur is equal to one minus the probability that it will occur.

$$P(\text{not } E) = 1 - P(E)$$

Ex 1: Find the probability that you choose a number from 1 to ten that is not 6.

$P(\text{not } 6) = 1 - P(6)$
$= 1 - 1/10 \quad \boxed{\frac{9}{10}}$

Ex 2: Find the probability that you deal a card that is not a diamond.

$P(\text{not diamond}) = 1 - P(\text{diamond})$
$= 1 - \frac{13}{52} = 1 - \frac{1}{4} = \boxed{3/4}$

Ex 3: You draw a card that is not a red face card (Jack, Queen, King)

$P(\text{not red face}) = 1 - P(\text{red face})$
$= 1 - \frac{6}{52} = \frac{46}{52} = \boxed{\frac{23}{26}}$

Ex 4: You select someone in the class who is not wearing jeans.

$P(\text{not jeans}) = 1 - P(\text{jeans})$

Ex: In the classic lottery game, each player chooses 6 different numbers from 1 to 48. If all of the numbers match the 6 picked, they win. What is the probability of not winning?

$_{48}C_6 = 12,271,512$ ways to pick numbers

$P(\text{win}) = 1 - \frac{1}{12,271,512} = 1 - P(\text{win})$

$= \frac{12,271,511}{12,271,512}$

$= \boxed{.99999992}$

Mutually Exclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or you roll a 2?

- Can these both occur at the same time? Why or why not? NO. 2 is not an odd number so you can't have an odd # and a 2 at same time.

Mutually Exclusive Events: events that cannot occur at the same time

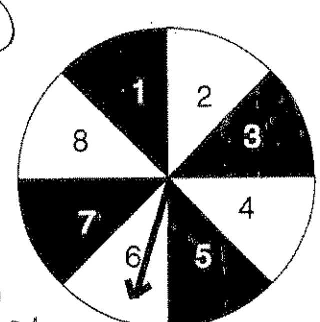
+ The probability of two mutually exclusive events occurring at the same time, P(A and B), is 0 (impossible → so prob = 0)

Ex/ Are the events mutually exclusive? Explain. → venn Diagram can help

4 | 6

1) Spinning a 4 or a 6 at the same time on a single spin.

yes - can't occur at same time.



2) Spinning an even number or a multiple of 3 at the same time on a single spin.

Even: 2, 4, 8 | 6 | Mult of 3: 3

2, 4, 6, 8   3, 6

NO. 6 is even and a multiple of 3 so these can occur at once.

3) Spinning an even number or a prime number on a single spin.

Even: 4, 6, 8 | 2 | Prime: 3, 5, 7

2, 4, 6, 8   2, 3, 5, 7

NO. 2 is even and prime so these can occur at once.

4) Spinning an even number or a number less than 2 on a single spin.

Even: 2, 4, 6, 8 | #<2: 1

2, 4, 6, 8   1

yes. These can't occur at the same time.

To find the probability of one of two **mutually exclusive** events occurring, use the following formula:

$$P(A \text{ or } B) = P(A) + P(B)$$

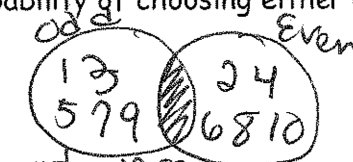
or

$$P(A \cup B) = P(A) + P(B)$$

* If A and B are two mutually exclusive events (no overlap) then $P(A \text{ and } B) = 0$

Examples: Extra Example: Prop brown hair or green eyes in class
P(A or B) = P(A) + P(B) − P(A and B)

1. If you randomly chose one of the integers 1 – 10, what is the probability of choosing either an odd number or an even number?



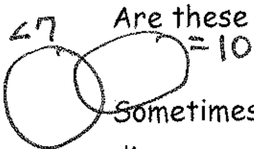
Are these mutually exclusive events? Why or why not?
Yes. A number can't be even and odd at once.

Complete the following statement: P(odd or even) = P(odd) + P(even)

Now fill in with numbers: P(odd or even) = $\frac{5}{10}$ + $\frac{5}{10}$ = 1

Does this answer make sense?
Yes. A number must be even or odd, so since these are the only 2 possibilities, their sum should be 1 for 100%.

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10?
Are these events mutually exclusive? Yes. Sum can't be < 7 and = 10 because 10 is greater than 7



Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice:

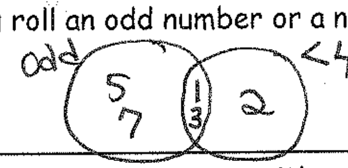
	1	2	3	4	5	6
1	2	3	4	5	6	
2	3	4	5	6	7	
3	4	5	6	7	8	
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P(getting a sum less than 7 OR sum of 10) = P(sum < 7) + P(sum = 10) = $\frac{15}{36} + \frac{3}{36} = \frac{18}{36} = \boxed{\frac{1}{2}}$

This means half of the time we roll two dice, their sum should be < 7 or = 10.

Mutually Inclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or a number less than 4?



Can these both occur at the same time? If so, when?
Yes. 1 and 3 are both odd and less than 4.

Mutually Inclusive Events: two events that can occur at the same time

Probability of the Union of Two Events: The Addition Rule:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \text{ and } B)$

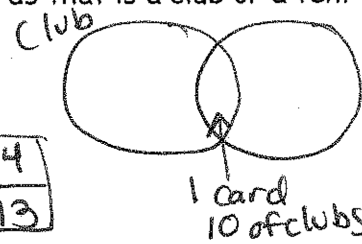
*** Use this for Both mutually exclusive and inclusive ***

Examples:

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?
   P(choosing a club or a ten) =

   $$P(\text{club}) + P(10) - P(\underbrace{\text{club} + 10}_{\text{overlap}})$$

   $$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$$

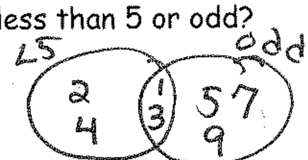
   

   ✱ Mut. Inclusive

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

   $$P(<5) + P(\text{odd}) - P(<5 \text{ and odd})$$

   $$\frac{4}{10} + \frac{5}{10} - \frac{2}{10} = \boxed{\frac{7}{10}}$$

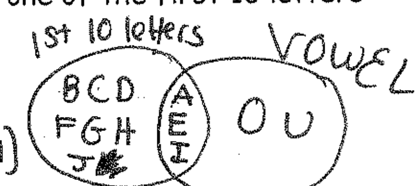
   

   ✱ Mut. Inclusive

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

   $$P(\text{1st 10 letters or vowel}) = P(\text{1st 10 letters}) + P(\text{vowel}) - P(\text{1st 10 and vowel})$$

   $$= \frac{10}{26} + \frac{5}{26} - \frac{3}{26} = \frac{12}{26} = \boxed{\frac{6}{13}}$$

   ✱ Mut. Inclusive

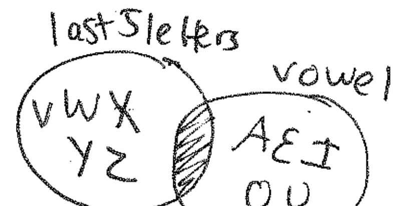
4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

   $$P(\text{last 5 letters or vowel}) - 0$$

   no overlap to subtract away

   $$= P(\text{last 5 letters}) + P(\text{vowel}) - 0$$

   $$= \frac{5}{26} + \frac{5}{26} = \frac{10}{26} = \boxed{\frac{5}{13}}$$

   ✱ mut. exclusive

   no need to subtract out the overlap