Day 2: Basic Probability

Warm-Up:

1. Suppose you are asked to list, in order of preference, the three best movies you have seen. If you saw 20 movies, in how many ways can the 3 best be chosen and ranked?

order matters - "ranked" $_{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot \cancel{17 \cdot 16 \cdot 15 \cdots 3 \cdot 2 \cdot 1}}{\cancel{17 \cdot 16 \cdot 15 \cdots 3 \cdot 2 \cdot 1}}$ **6840**

2. There are 6 women and 5 men interviewing for 4 cashier positions at Walmart.

a) In how many ways can the 4 positions be filled?

order not matter - all cashier $_{11}C_4 =$ **330**

b) In how many ways can the positions be filled if all women are hired?

6 women to choose from $_6C_4 =$ **15**

c) In how many ways can the positions be filled if 2 women and 2 men are hired?

$\underset{\text{women}}{_6C_2} \cdot \underset{\text{men}}{_5C_2} = 15 \cdot 10 =$ **150**

3. How many distinguishable permutations are possible using the letters of the following words:

a) ATHENS $6! \text{ OR } {}_6P_6 =$ **720**

b) BASKETBALL $= 10!$ (3W, 3W's, 2B's, 2A's, 2L's) $= \frac{10!}{(2!2!2!)} =$ **453600**

c) SUBSTITUTE (3U, 3W's, 1B) $\frac{10!}{(2!2!3!)} =$ **151200** ← 2S's, 2U's, 3T's

d) ICICLE (W's, 2's) $\frac{6!}{(2!2!)} =$ **180** ← 2I's, 2C's

Notes Day 2: Basic Probability

Sample Space: all the possible outcomes of a given experiment

List the sample space, S, for each of the following:

a. Tossing a coin	b. Rolling a six sided die	c. Drawing a marble from a bag containing two red, three blue, and one white marble
H T	1 2 3 4 5 6	R R B B B W

Intersection of two sets ($A \cap B$): all the elements that appear in Both sets * Elements in A AND B

Ex/ A: {3,4,5,6,7} and B: {5,6,7,8,9,10}, $A \cap B = \{5,6,7\}$

Union of two sets ($A \cup B$): all the elements in both sets * Elements in A OR B (all items in A, B, or both A+B)

Ex/ A: {3,4,5} and B: {5,6,7}

$A \cup B$: {3,4,5,6,7}

Example: Given the following sets, find $A \cap B$ and $A \cup B$

You Try!

$A = \{1,3,5,7,9,11,13,15\}$ $B = \{0,3,6,9,12,15\}$

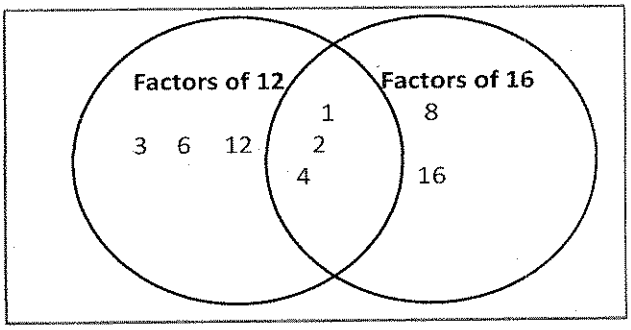
$A \cap B =$ $\{3, 9, 15\}$ intersection → what is in both A & B

$A \cup B =$ $\{0, 1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$ union → what's in either A or B

Venn Diagram: a diagram that shows all possible relationships between a collection of sets

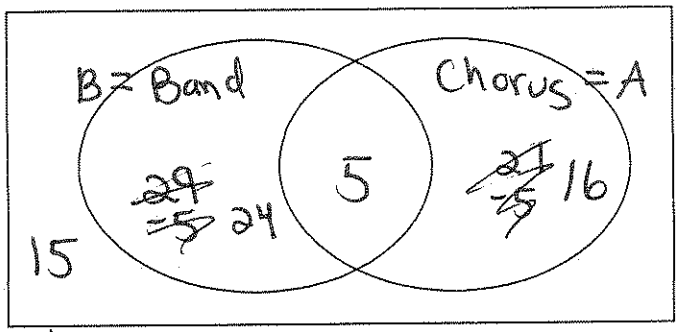
Picture:

Example: Use the Venn Diagram to answer the following questions:



1. What are the elements of set A?
   Factors of 12 → $\{1, 2, 3, 4, 6, 12\}$
2. What are the elements of set B?
   Factors of 16 → $\{1, 2, 4, 8, 16\}$
3. Why are 1, 2, and 4 in both sets?
   They are factors of both 12 and 16
4. What is $A \cap B$? intersection → the middle/overlap
   $\{1, 2, 4\}$
5. What is $A \cup B$? union → what's in either
   $\{1, 2, 3, 4, 6, 8, 12, 16\}$

Example: In a class of 60 students, 21 sign up for chorus, 29 sign up for band, and 5 take both. 15 students in the class are not enrolled in either band or chorus.



6. Put this information into a Venn Diagram. If the sample space, S, is the set of all students in the class, let students in chorus be set A and students in band be set B.
7. What is $A \cup B$? students in band or chorus (or both)
8. What is $A \cap B$? students in band and chorus

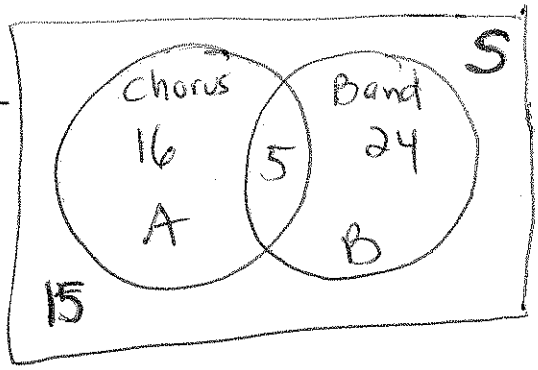
Compliment of a set: all elements in the universal set that are not in the initial set

- Ex: $S = \{\ldots -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ $A = \{\ldots -2, 0, 2, 4, \ldots\}$ → even #s

  If A is a subset of S, what is A^c? $A^c = \{-3, -1, 1, 3, 5\}$ → odd #s

  ↑ little "c" like exponent is notation for complement

Example: Use the Venn Diagram above to find the following:



9. What is A^c? $A^c = 39$ B^c? 31
   60 total − 21 in chorus; 60 total − 29 in band

10. What is $(A \cap B)^c$? 55
    60 − 5 = students NOT in band & chorus (both)

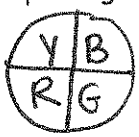
11. What is $(A \cup B)^c$? 15
    60 − (16 + 5 + 24); 60 − (45) = 15 students in neither band nor chorus

You Try!

Basic Probability

Probability of an Event: P(E) = # of ways an event can happen / total # of possible outcomes

Example 1: A spinner has 4 equal sectors colored yellow, blue, green and red. After spinning the spinner, what is the probability of landing on each color?



P(yellow) = 1/4 P(green) = 1/4

P(blue) = 1/4 P(red) = 1/4

Example 2: A single 6-sided die is rolled. What is the probability of each outcome? What is the probability of rolling an even number? Of rolling an odd number?

P(1) = 1/6 P(2) = 1/6 P(3) = 1/6 P(4) = 1/6 P(5) = 1/6 P(6) = 1/6

P(even) = 3/6 = 1/2 P(odd) = 3/6 = 1/2

Probability of A^c

Note that **$P(A^c)$** is every outcome **except (or not)** A, so we can find $P(A^c)$ by finding

$P(A^c) = 1 - P(A)$.

Why do you think this works? items are either in A or A^c so their sum $P(A) + P(A^c) = 1$ for 100% because all options are included

Example 3: (not in notes)

A pair of dice is rolled. What is the probability of **NOT** rolling doubles?

*There are 6 ways to roll doubles.
1,1 2,2 3,3 4,4 5,5 6,6

$\frac{6}{\text{1st die}} \cdot \frac{6}{\text{and die}} = 36$ total

Fix "dice"

P(doubles) = 6/36 = 1/6

P(NOT doubles) = $\frac{36}{36} - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$

OR $P(ND) = 1 - P(D) = 1 - 1/6$

Example 4: A pair of dice are rolled. What is the probability of rolling a sum of 10 or less?

*What is the complement of rolling "10 or less"?

↳ rolling > 10 → 11 or 12
5,6 6,6
6,5

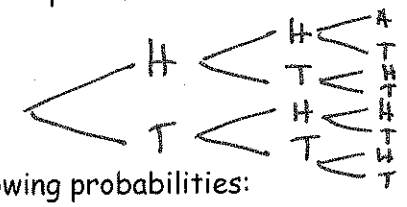
$P(\text{sum of 10 or less}) = 1 - P(\text{sum} > 10)$

$= 1 - 3/36 = \frac{33}{36}$

Practice Example: An experiment consists of tossing three coins. → need tree if >1 of something

$\frac{11}{12}$

12. List the sample space for the outcomes of the experiment.



HHH HTH
HHT HTT
THH TTH
THT TTT

$\frac{2}{1\text{st coin}} \cdot \frac{2}{2\text{nd coin}} \cdot \frac{2}{3\text{rd coin}} = 8$ in sample space

13. Find the following probabilities:

a. P(all heads) 1/8 HHH

b. P(two tails) 3/8 HHT, THH, HTH

c. P(no heads) 1/8 TTT

d. P(at least one tail) 7/8 $= 1 - P(\text{all heads}) = 1 - \frac{1}{8}$

HHT HTH
HTT
THH TTH
THT TTT

e. How could you use compliments to find d?

P(>1 T) = 1 − P(all heads) = $1 - \frac{1}{8}$

Example: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random. → 1 list is fine since just 1 thing drawn

14. List the sample space for this experiment.

R R R B B Y W W
R R R B B Y W

15 total marbles

15. Find the following probabilities:

a. P(red) 6/15 = 2/5

b. P(blue or white) 7/15

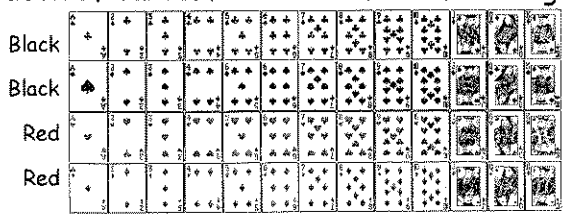
c. P(not yellow) 13/15

Note that we could either count all the outcomes that are not yellow or we could think of this as being 1 - P(yellow).

Why is this? a marble is either yellow or not yellow... those are the only options so

$P(\text{not yellow}) = 1 - P(\text{yellow})$ because $P(\text{yellow}) + P(\text{not yellow}) = 1$

Example: A card is drawn at random from a standard deck of cards. Find each of the following:



16. P(heart) $13/52 = \boxed{1/4}$

17. P(black card) $26/52 = \boxed{1/2}$

18. P(2 or jack) $8/52 \ \boxed{2/13}$ (4 2s, 4 Js)

19. P(not a heart) $\boxed{3/4}$

$= 1 - P(\heartsuit) = 1 - 13/52 = 39/52$

Odds: The **odds** of an event occurring are equal to the ratio of favorable outcomes to unfavorable outcomes

Odds = # of successes : # of failures

20. The weather forecast for Saturday says there is a 75% chance of rain. What are the odds that it will rain on Saturday?

- What does the 75% in this problem mean?

  75% of the time with these conditions it will rain
  25% of the time with these conditions it won't rain

- The favorable outcome in this problem is that it rains:

  this happens 75% of the time

- Odds(rain) = 75 : 25 (rain : not rain)

- Should you make outdoor plans for Saturday? Probably not... more likely to rain than to not rain

Exit Ticket

21. A gumball machine contains gumballs of five different colors: 36 red, 44 white, 15 blue, 20 green, and 5 orange. The machine dispenser randomly selects one gumball. What is the probability that the gumball selected is: 120 total gumballs

a) Green? $= 20/120 = \boxed{1/6}$

b) Not green? $= 1 - P(G) = 1 - 1/6 \ \boxed{5/6}$

c) Not orange? $= 1 - P(\text{Or}) = 1 - 5/120 = 115/120 = \boxed{23/24}$

d) Orange? $= 5/120 = \boxed{1/24}$

e) Not a color in the flag of the USA? $= 1 - P(\text{color of USA flag}) = 1 - \frac{36+44+15}{120} = 1 - \frac{95}{120}$

$= \frac{25}{120} = \boxed{\frac{5}{24}}$

f) Red, white or blue?

$\frac{36+44+15}{120} = \frac{95}{120} = \boxed{\frac{19}{24}}$

a) What are the odds of drawing an ace at random from a standard deck of cards? $\boxed{4:48}$