Unit 6 Day 2
Basic Probability

$${}_nC_r = \frac{n!}{(n-r)!\bullet r!}$$

Warm Up

$${}_nP_r = \frac{n!}{(n-r)!}$$

1. Suppose you are asked to list, in order of preference, the three best movies you have seen. If you saw 20 movies, in how many ways can the 3 best be chosen and ranked? Show work by hand here. ☺

2. There are 6 women and 5 men interviewing for 4 cashier positions at Walmart.

   a) In how many ways can the 4 positions be filled?

   b) In how many ways can the positions be filled if all women are hired?

   c) In how many ways can the positions be filled if 2 women and 2 men are hired?

3. How many distinguishable permutations are possible using the letters of the following words:

   a) ATHENS

   ★b) BASKETBALL

   ★c) SUBSTITUTE

   ★d) ICICLE

★**Hint: how did we take care of "repeats" with the combination formula?**

$${}_nC_r = \frac{n!}{(n-r)!\bullet r!}$$

Warm Up

$${}_nP_r = \frac{n!}{(n-r)!}$$

1. Suppose you are asked to list, in order of preference, the three best movies you have seen. If you saw 20 movies, in how many ways can the 3 best be chosen and ranked?

   ${}_{20}P_3 = 6840$ Permutation because ranking -> order matters

2. There are 6 women and 5 men interviewing for 4 cashier positions at Walmart. **Combination because all the same position -> no order**

   a) In how many ways can the 4 positions be filled?

   ${}_{11}C_4 = 330$ 11 total people to choose from, pick 4 of them

   b) In how many ways can the positions be filled if all women are hired?

   ${}_6C_4 = 15$ 6 total women to choose from, pick 4 of them

   c) In how many ways can the positions be filled if 2 women and 2 men are hired?

   $\underset{\text{Women}}{{}_6C_2} \bullet \underset{\text{Men}}{{}_5C_2} = 150$ Use counting principle because choose women, then choose men.

$$_nC_r = \frac{n!}{(n-r)!\bullet r!}$$

Warm Up

$$_nP_r = \frac{n!}{(n-r)!}$$

3. How many distinguishable permutations are possible using the letters of the following words:

a) ATHENS

a) $_6P_6$ or 6! = 720

b) BASKETBALL

b) $\frac{10!}{(2!\ 2!\ 2!)}$ = 453600

<- B, A, and L are repeated twice, so divide out the repeats

c) SUBSTITUTE

c) $\frac{10!}{(2!\ 2!\ 3!)}$ = 151200

<- S and U are repeated twice, & T is repeated 3 times, so divide out repeats

d) ICICLE

d) $\frac{6!}{(2!\ 2!)}$ = 180

<- I and C are repeated twice, so divide out the repeats

Probability HW Answers

1. 210
2. 210
3. $3 \cdot 5 = 15$
4. $3 \cdot 7 = 21$
5. $4^8 = 65536$
6. ${}_{20}P_3 = 6840$
7. ${}_{14}C_6 = 3003$
8. ${}_{8}P_3 = 336$
9. ${}_{9}P_5 = 15120$
10. ${}_{17}C_8 = 24310$
11. ${}_{100}C_{18} = 3.066 \times 10^{19}$
12. ${}_{5}C_3 = 10$
13. ${}_{2}C_1 \cdot {}_{5}C_2 \cdot {}_{4}C_2 = 120$
14. ${}_{30}P_3 = 24360$
15. ${}_{12}P_5 - {}_{15}C_6 = 90035$

HW Answers: Cumulative Rev. After Unit 5

16) a) GHE; ASA b) GHI; SSS c) JIL; SAS

17) a) all real #s (any x-value can go into equation)

b) $x \geq 0$ (because no negative prices)

c) \$277.50 d) \$0.55 or 55 cents

e) \$300 per day

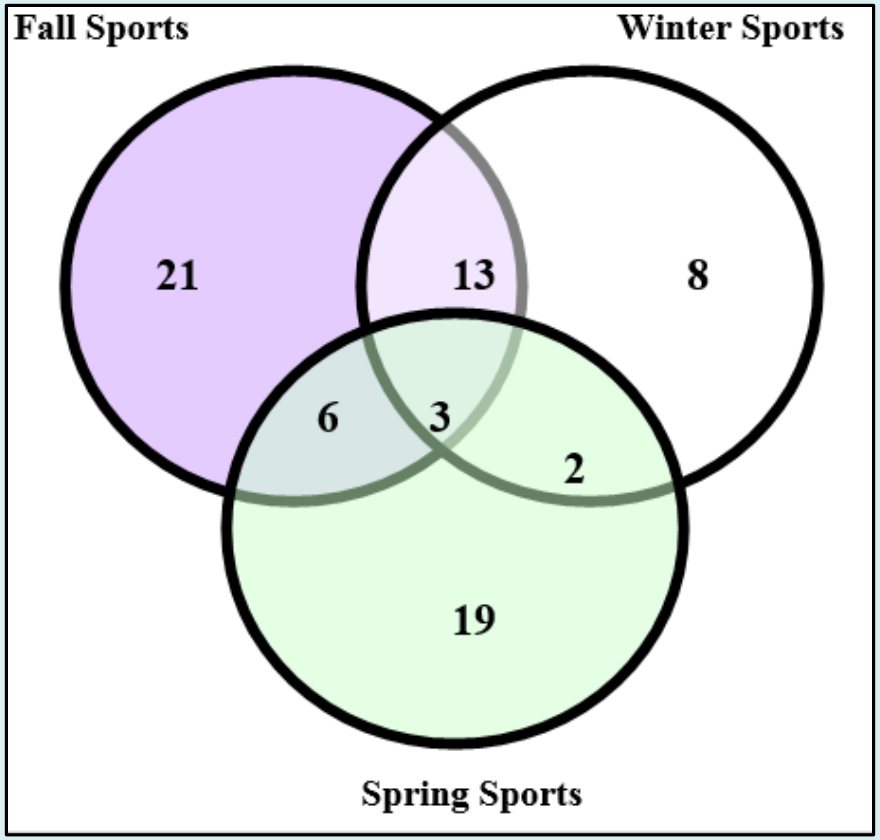
18) 6 19) $18 \leq x < 19.5$ 20) 3

21) a) $3x^2 + 6x - 4$ c) $y = x - 4$

b) $3x^2 + 3x - 16$ d) $3x^2 + 35x + 92$

Tonight's Homework

- Packet p. 3-5



Study your formulas!

- Remember you'll need to know these formulas for the quiz AND test!
- ALSO make sure you know how to show your work by hand…including showing that factorial means multiplying down to 1 & how some of the numbers divide away!

$$ {}_nC_r = \frac{n!}{(n-r)!\bullet r!} $$

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Notes Day 2

Basic Probability & Odds, Sample Spaces

Basic Probability

- **Sample Space:** A list of all possible outcomes of a given experiment.

a. Tossing a coin	b. Rolling a six sided die	c. Drawing a marble from a bag containing two red, three blue, and one white marble
Heads, Tails	1, 2, 3, 4, 5, 6	R, R, B, B, B, W

Intersection of two sets **(A ∩ B):**

all the elements that appear in both sets

* Elements in A **AND** B

Example: Given set A: {3,4,5,6,7}, and set B: {5,6,7,8,9,10}, find (A ∩ B).

(A ∩ B) = {5,6,7}

Union of two sets **(A ∪ B):**

Everything **in both sets**

*Elements in A **OR** B

Example: Given set A: {3,4,5} and set B: {5,6,7}, find (A ∪ B).

(A ∪ B) = {3,4,5,6,7}

You TRY

Example: Given the following sets, find

$A \cap B$ and $A \cup B$.

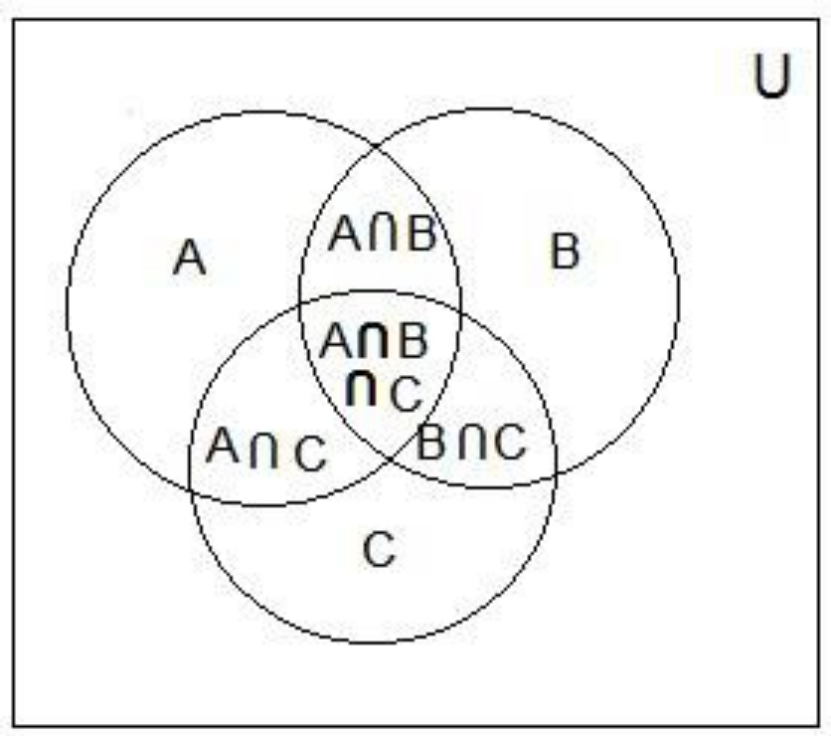
$A = \{1, 3, 5, 7, 9, 11, 13, 15\}$

$B = \{0,3,6,9,12,15\}$

$A \cap B =$ **$\{3, 9, 15\}$**

$A \cup B =$ **$\{0,1,3,5,6,7,9,11,12,13,15\}$**

Venn Diagram: a diagram that shows all possible relationships between a collection of sets



Example: Use the Venn Diagram to answer the following questions. Let A = Factors of 12 and B = Factors of 16:

1.What are the elements of set A?

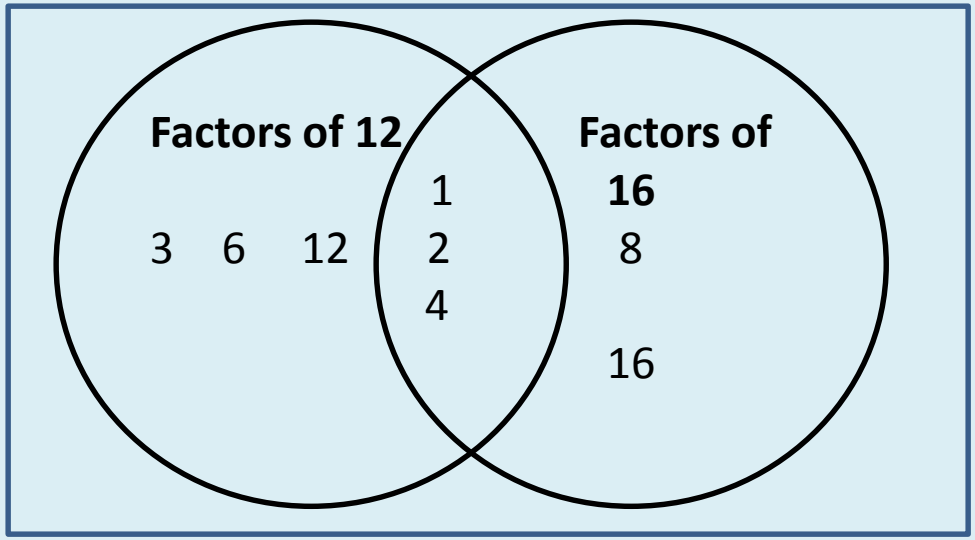
A = {1, 2, 4, 3, 6, 12}

2.What are the elements of set B?

B = {1, 2, 4, 8, 16}

3.Why are 1, 2, and 4 in both sets?

They are factors of 12 and 16.



4. What is A ∩ B?

A ∩ B= {1, 2, 4}

5. What is A ∪ B?

A ∪ B ={1, 2, 4, 3, 6, 8, 12, 16}

Example: In a class of 60 students, 21 sign up for chorus, 29 sign up for band, and 5 take both. 15 students in the class are not enrolled in either band or chorus.

If the sample space, S, is the set of all students in the class, let students in chorus be set A and students in band be set B.

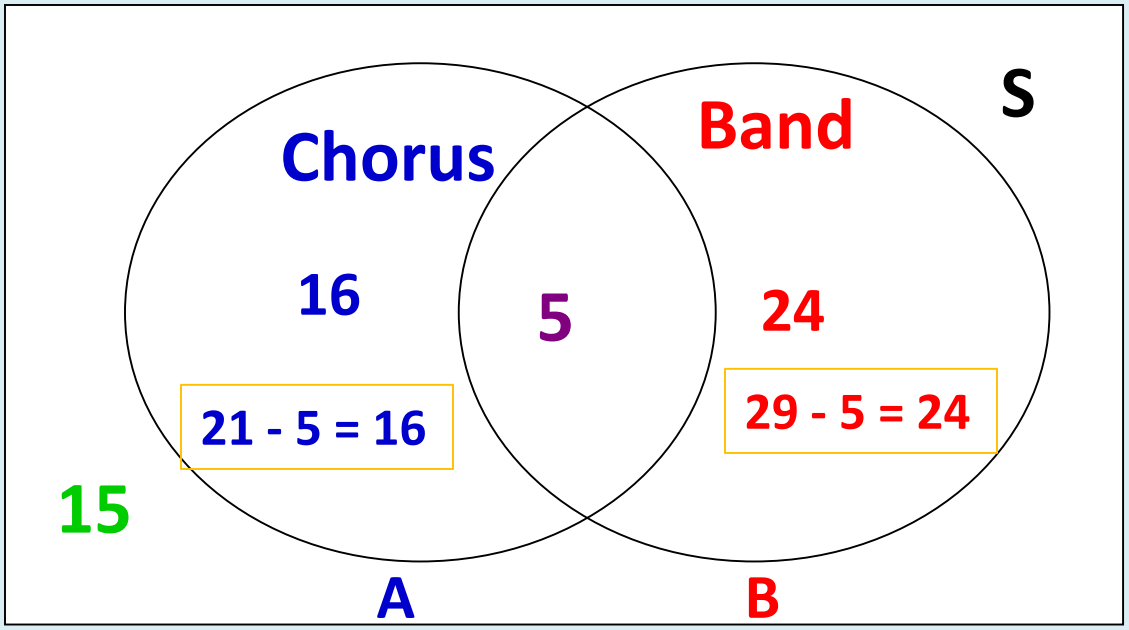
What is $A \cup B$?

$16 + 5 + 24 = 45$

$A \cup B = \{ 45 \}$

What is $A \cap B$?

$A \cap B = \{ 5 \}$



- **Compliment** of a set**:**

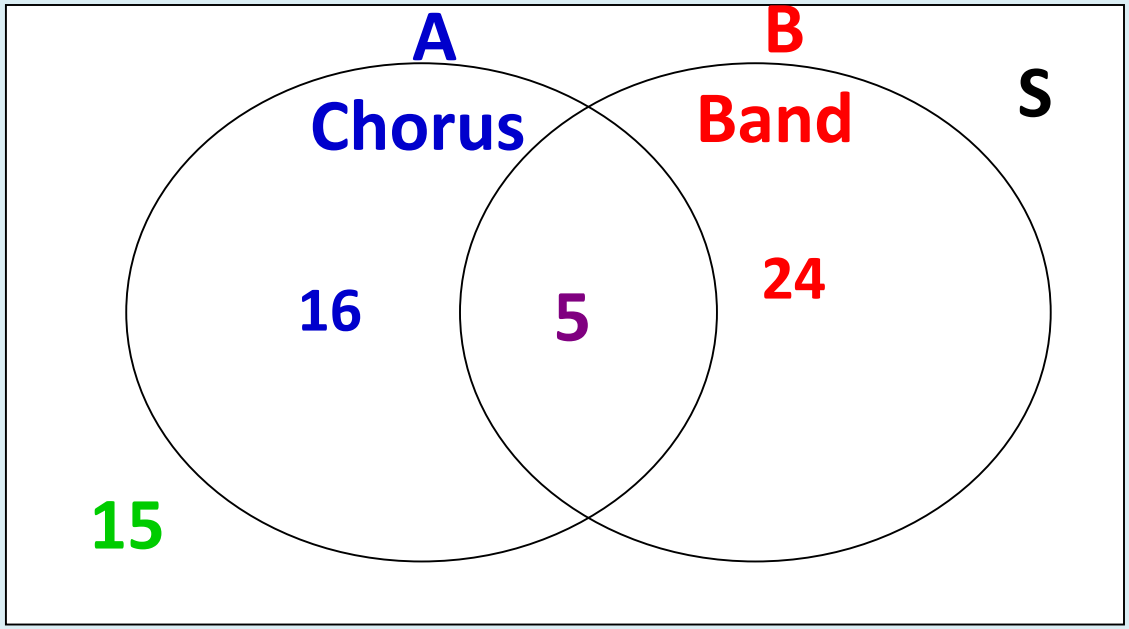
  all elements in the universal set that are not in the initial set

Ex: S = {…-3, -2, -1, 0, 1, 2, 3, 4,…}
and A = {…-2, 0, 2, 4,…}

If A is a subset of S, what is A^C?

A^C = {…-3, -1, 1, 3…}

You Try Example: Use the Venn Diagram to find the following:



What is A^C?

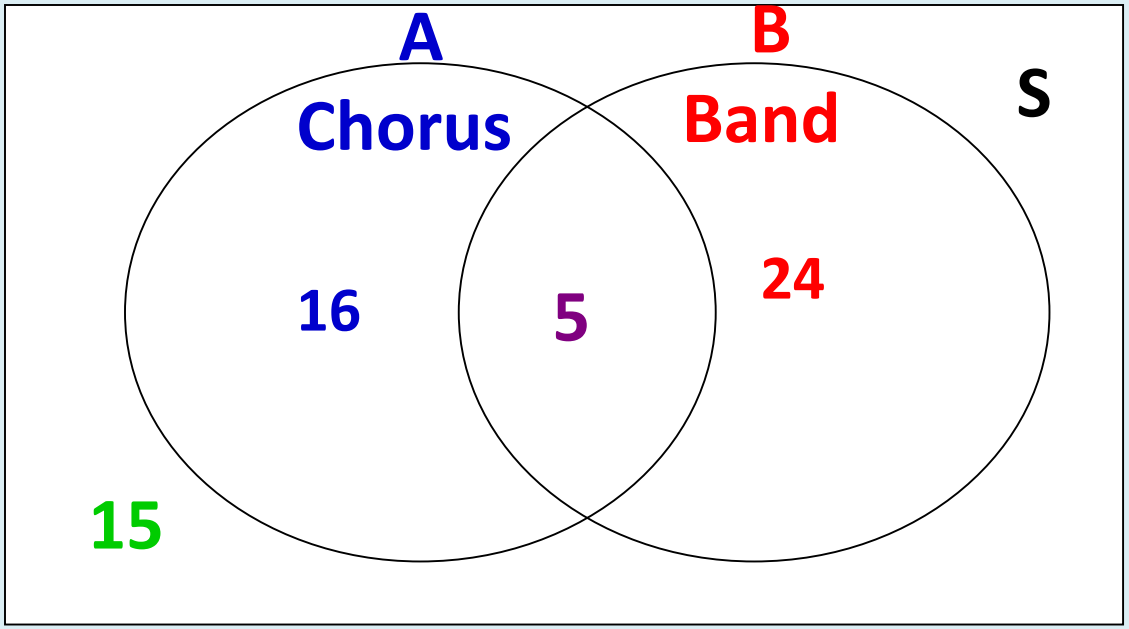
What is B^C?

What is $(A \cap B)^C$?

What is $(A \cup B)^C$?

Hint: Thinking about the "real life" meaning can help! ☺

Example: Use the Venn Diagram to find the following:



What is A^C?

$A^C = 39$

What is B^C?

$B^C = 31$

What is $(A \cap B)^C$?

$(A \cap B)^C = 55$

What is $(A \cup B)^C$?

$(A \cup B)^C = 15$

Basic Probability

Probability of an Event: P(E) =

$$\frac{\text{\# of ways an event can happen}}{\text{Total \# of possible outcomes}}$$

Example 1:

A spinner has 4 equal sectors colored yellow, blue, green and red. After spinning the spinner, what is the probability of landing on each color?

P(yellow) = 1/4 **P(green) = 1/4**

P(blue) = 1/4 **P(red) = 1/4**

You Try! Example 2:

A single 6-sided die is rolled. What is the probability of each outcome? What is the probability of rolling an even number? Of rolling an odd number?

P(1) = $\frac{1}{6}$

P(2) = $\frac{1}{6}$

P(3) = $\frac{1}{6}$

P(4) = $\frac{1}{6}$

P(5) = $\frac{1}{6}$

P(6) = $\frac{1}{6}$

P(even)= $\frac{3}{6} = \frac{1}{2}$

P(odd) = $\frac{3}{6} = \frac{1}{2}$

Note that **$P(A^C)$** is every outcome **except (or not)** A, so we can find $P(A^C)$ by finding:

$$\mathbf{P(A^C) = 1 - P(A)}$$

- Why do you think this works?

A and A^C are the only options, so the sum of their probabilities should be 1 for 100%.

$P(A^C) + P(A) = 1$
If you solve for $P(A^C)$, you get the above formula. ☺

Example 3:

A pair of dice is rolled. What is the probability of **NOT** rolling doubles?

For a complex problem like this we need a sample space. A table is good here since we have 2 dice. Let's create the table together!

1st Die \ 2nd Die	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Use the Counting Principle to check your sample space!

6 • 6 = 36 items

Checking for understanding…. Why do we need 1, 3 AND 3, 1?

Remember, $P(A^C) = 1 - P(A)$

P(doubles) = 6/36 = 1/6

P(not doubles) = 1 – P(doubles) = 1 - 1/6 = 5/6

Example 4:

A pair of dice are rolled. What is the probability of rolling a sum of 10 or less?

*What is the complement of rolling “10 or less”?

Rolling more than 10

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

P(10 or less) = 1 – P(11 or 12)

= 1 – [P(11) + P(12)]

= 1 – (2/36 + 1/36) = 33/36 = 11/12

Ex 5: An experiment consists of tossing three coins. List the sample space for the outcomes of the experiment.

Let's do a tree diagram together to get the sample space!

You Try!

- Find the following probabilities:
  - a. P(all heads) **1/8**
  - b. P(two tails) **3/8**
  - c. P(no heads) **1/8**
  - d. P(at least one tail) **7/8**

- How could you use compliments to find d?

P(at least 1 tail) = 1 – P(no tails)

= 1 – P(all heads) = 1 – 1/8 = 7/8

You Try!

- Complete examples 6 and 7 in notes
  - Answer questions # 14-19
  - On the next 2 slides if you don't have your notes….

You Try! **Example 6:** A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

List the sample space for this experiment.

Find the following probabilities:

a. P(red) _____

b. P(blue or white) _____

c. P(not yellow) _____

You Try! **Example 7:** A card is drawn at random from a standard deck of cards. Find each of the following:

P(heart) _____

P(black card) _____

P(2 or jack) _____

P(not a heart) _____

You Try Answers!

Example 6: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

List the sample space for this experiment.

R R R B B Y W W
R R R B B Y W

Find the following probabilities:

a. P(red) _____ **2/5 (simplify 6/15)**

b. P(blue or white) _____ **7/15 (count up ones that are blue or white)**

c. P(not yellow) _____ **13/15 (do $P(Y^c) = 1 - P(Y) = 1 - 2/15$)**

You Try Answers!

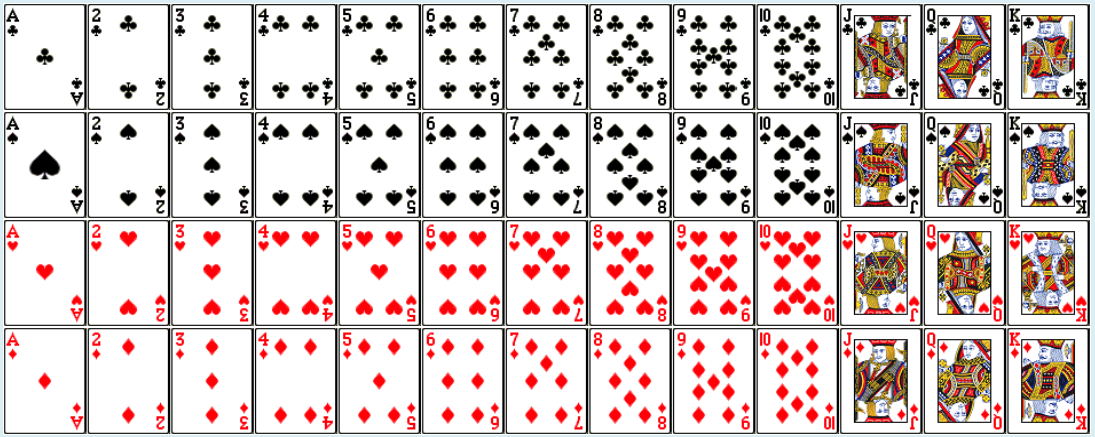
Example 7: A card is drawn at random from a standard deck of cards. Find each of the following:

P(heart) _____ **1/4 (simplify 13/52)**

P(black card) _____ **1/2 (simplify 26/52)**

P(2 or jack) _____ **2/13 (simplify 8/52)**

P(not a heart) _____ **3/4 (do $P(H^c) = 1 - P(H) = 1 - 13/52$)**



Odds: The **odds** of an event occurring are equal to the ratio of **favorable outcomes** to **unfavorable outcomes**.

Odds = ***Number of successes: Number of failures***

The weather forecast for Saturday says there is a 75% chance of rain. What are the odds that it will rain on Saturday?

What does the 75% in this problem mean?

The probability of it raining is 75%.

With these conditions, it will rain 75% of the time.

The favorable outcome in this problem is that it rains:

Odds(rain) = **75 : 25**

Rain Not Rain

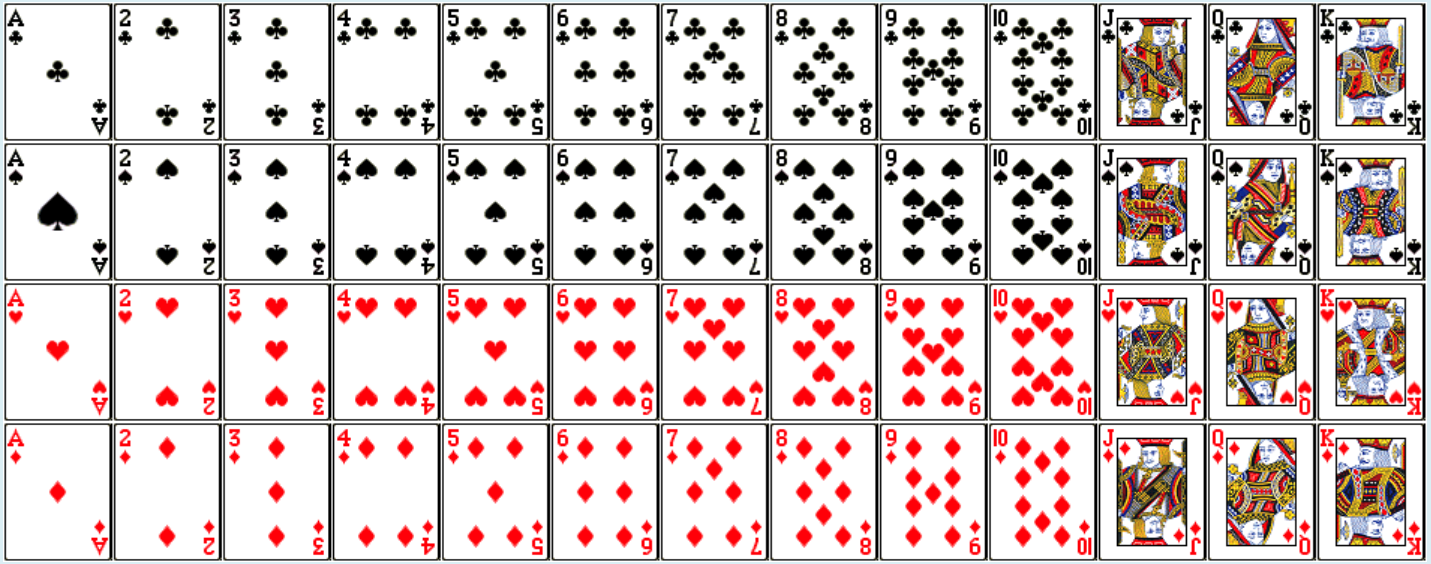
You Try!

Odds = ***Number of successes: Number of failures***

What are the odds of drawing an ace at random from a standard deck of cards?

4 : 48

Ace Not Ace



PRACTICE: Exit Ticket

A gumball machine contains gumballs of five different colors: **36 red**, 44 white, **15 blue**, **20 green**, and **5 orange**. The machine dispenser randomly selects one gumball. What is the probability that the gumball is:

a) Green?
b) Not green?
c) Not orange?
d) Orange?
e) Not a color in the flag of the USA?
f) Red, white or blue?

a) 20/120 = 1/6
b) 1 – 1/6 = 5/6
c) 1 – P(O) = 1 – 5/120 = 23/24
d) 1/24
e) 1 – 19/24 = 5/24
f) 95/120 = 19/24

Homework

- Packet p. 3-5