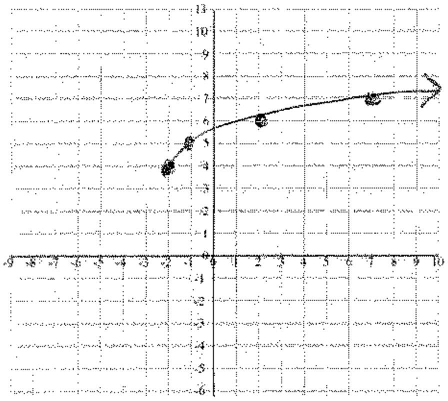
Day 1: Counting Methods, Permutations & Combinations

Warm-Up:

$= \sqrt{x+2} + 4$ ← up 4, left 2

1. Given the equation $y = 4 + \sqrt{x+2}$ draw the graph, being sure to indicate at least 3 points clearly.



Then determine the following:

a. Identify its vertex $(-2, 4)$

b. Identify its domain $x \geq -2$

c. Identify its range $y \geq 4$

d. How is this function translated from its parent graph?

left 2, up 4

e. If this graph was translated to the right 5 units, what would the new equation be? $y = 4 + \sqrt{x-3}$

Vertex $(-2, 4)$ to right 5 is $(-2+5, 4)$ new vertex $(3, 4)$

Solve

2. $2\sqrt[3]{(x-1)^4} + 4 = 36$

$-4 \quad -4$

• Isolate radical 1st

$\frac{2\sqrt[3]{(x-1)^4}}{2} = \frac{32}{2}$

$\sqrt[3]{(x-1)^4} = 16$ cube to do inverse of cube root

$(x-1)^4 = 4096$

$\sqrt[4]{(x-1)^4} = \sqrt[4]{4096}$ do 4th root

$x-1 = \pm 8$

$x-1 = 8$, $x-1 = -8$

$\boxed{x = 9, -7}$

3. $\sqrt{x+7} - x = 1$

$+x \quad +x$

$\sqrt{x+7} = x+1$

$(\sqrt{x+7})^2 = (x+1)^2$

$x+7 = (x+1)(x+1)$ use FOIL

$x+7 = x^2 + 2x + 1$

$-x \; -7 \qquad -x \; -7$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$3 \cdot -2 = -6$

$3 + -2 = 1$

$x = -3, 2$

$\sqrt{-3+7} - -3 = 1$

$5 \neq 1$ ← extraneous solution

$\sqrt{2+7} - 2 = 1$

$\sqrt{9} - 2 = 1$

$3 - 2 = 1$

$\boxed{x = 2}$

Notes Day 1: Counting Methods, Permutations & Combinations

I. Introduction

Probability Defined: Probability is the likelihood of something happening

Prob = $\frac{\#\text{ of desired}}{\#\text{ of total}}$

II. Basic Counting Methods for Determining the Number of Possible Outcomes

A. Fundamental Counting Principle:

a. **Tree Diagrams:**

Example #1: LG will manufacture 5 different cellular phones: Ally, Extravert, Intuition, Cosmos and Optimus. Each phone comes in two different colors: Black or Red. Make a tree diagram representing the different products. How many different products can the company display?

See next pages

b. **In general:** If there are ____ ways to make a first selection and ____ ways to make a second selection, then there are ____________ ways to make the two selections simultaneously. This is called the Fundamental Counting Principle.

Ex #1 above: 5 different types of phones in 2 different colors.

How many different products will LG display?

ii. **Basic Counting Methods for Determining the Number of Possible Outcomes**

a. Tree Diagrams:

Example #1: LG will manufacture 5 different cellular phones: Ally, Extravert, Intuition, Cosmos and Optimus. Each phone comes in two different colors: Black or Red. Make a tree diagram representing the different products.

How many different products can the company display?.

$\boxed{10}$

A E I C O

B R B R B R B R B R

b. In general:

- If there are *m* ways to make a first selection and *n* ways to make a second selection, then there are *m times n* ways to make the two selections simultaneously. This is called the **Fundamental Counting Principle**

$5 \cdot 2$

Ex #1 above:

5 different cell phones in 2 different colors. How many different products?

$5 \cdot 2 = \boxed{10}$

More Practice (White Boards)

Ex #2: Elizabeth is going to completely refurbish her car. She can choose from 4 exterior colors: white, red, blue and black. She can choose from two interior colors: black and tan. She can choose from two sets of rims: chrome and alloy. How many different ways can Elizabeth remake her car? Make a tree diagram and use the Counting Principle.

$\underset{\text{ext color}}{\underline{4}} \cdot \underset{\text{int color}}{\underline{2}} \cdot \underset{\text{rim}}{\underline{2}} = \boxed{16 \text{ ways}}$

Ex #3

Passwords for employees at a company in Raleigh NC are 8 digits long and must be numerical (numbers only). How many passwords are possible? (Passwords cannot begin with 0)

$\underline{9}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$

no 0

$\boxed{90{,}000{,}000}$

B. Permutations—Another way to "count" possibilities

a. Two characteristics:
1. Order IS important
2. No item is used more than once

Supplemental Example *(not on your notesheet):* There are six "permutations", or arrangements, of the numbers 1, 2 and 3. What are they?

123 213 312
132 231 321

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123 132 213 231
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Example #1

How many ways can 10 cars park in 6 spaces? (The other four will have to wait for a parking spot.) ☺ (Use the Fundamental Counting Principle)

10 9 8 7 6 5

$\boxed{151200}$

b. Formula:

If we have a large number of items to choose from, the fundamental counting principle would be inefficient. Therefore, a formula would be useful

- First we need to look at **"factorials".**
  - Notation: n! stands for n factorial

Definition of n factorial:

For any integer n>0,

n! = n(n−1)(n−2)(n−3)…(3)(2)(1)

Supplemental Example:

$4! = 4\cdot3\cdot2\cdot1 = \boxed{24}$

If n=0, 0! = 1

Example #1 (revisited):

We could rewrite the computation in our example as follows:

$$10\cdot9\cdot8\cdot7\cdot6\cdot5 = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1}}{\cancel{4\cdot3\cdot2\cdot1}}$$

$$= \frac{10!}{4!}$$

Furthermore, notice that $= \frac{10!}{4!} = \frac{10!}{(10-6)!}$

So, the number of permutations (or arrangements) of 10 cars taken 6 at a time is 151200

- Generally, the ***Number of Permutations*** of *n* items taken *r* at a time,

$$_nP_r = \frac{n!}{(n-r)!}$$

- How to do on the calculator:

n $\boxed{MATH}$ $\boxed{PRB}$ $\boxed{2}$ r

type # on main screen

type on main screen

c. EXAMPLE #2

In a scrabble game, Jane picked the letters A,D,F,V,~~E~~ and I. How many permutations (or arrangements) of 4 letters are possible?

Fix

$${}_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

$$\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}}$$

$\boxed{360}$

More Practice Problems (Whiteboards or NOTES) ${}_nP_r = \dfrac{n!}{(n-r)!}$

1. Evaluate: (*By hand then using ${}_nP_r$ function on the calculator to check your answer.*)

a. ${}_{10}P_3$ $\boxed{720}$

b. ${}_9P_5$ $\boxed{15120}$

2. How many ways can runners in the 100 meter dash finish 1st (Gold Medal), 2nd (Silver) and 3rd (Bronze Medal) from 8 runners in the final? NOTE: This is a permutation because the people are finishing in a position. ORDER matters!

${}_8P_3$ $\boxed{336}$

C. Combinations

a. Two characteristics:

1. Order DOES NOT matter
2. No item is used more than once

Supplemental Example: How many "combinations" of the numbers 1, 2 and 3 are possible?

123

EXAMPLE:

While creating a playlist on your I pod you can choose 4 songs from an album of ~~8~~ 6 songs. If you can choose a given song only once, how many different combinations are possible? (List all the possibilities)

Fix

Call songs ABCDEF

ABCD ABCE ABCF ACDE ACDF ACEF
ABDE ABDF ADEF
ABEF

BCDE BCDF BCEF CDEF
BDEF

$\boxed{15}$

b. Formula:

Making a list to determine the number of combinations can be time consuming. Like permutations, there is a general formula for finding the number of possible combinations.

- ***Number of Combinations*** of n items taken r items at a time is

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

$r!$ takes out repeats from permutation list that are really the same

- How to do on the calculator:

n MATH PRB 3: ${}_nC_r$ r

type on main screen

Practice (Whiteboards) $_nC_r = \frac{n!}{(n-r)! \cdot r!}$

1. Evaluate:
   a. $_4C_2$ b. $_7C_3$ c. $_8C_8$

$\boxed{6}$ $\boxed{35}$ $\boxed{1}$

Practice (Whiteboards) $_nC_r = \frac{n!}{(n-r)! \cdot r!}$

2. A local restaurant is offering a 3 item lunch special. If you can choose 3 or fewer items from a total of 7 choices, how many possible combinations can you select?

$_7C_3 + {_7C_2} + {_7C_1} + {_7C_0}$

$35 + 21 + 7 + 1$ $\boxed{64}$

$_nC_r = \frac{n!}{(n-r)! \cdot r!}$ $_nP_r = \frac{n!}{(n-r)!}$

Review: Indicate if the situation following is a Permutation or Combination. Then, solve.

a. In a bingo game 30 people are playing for charity. There are prizes for 1st through 4th. How many ways can we award the prizes?

Permutation or Combination

$_{30}P_4$

$\boxed{657720}$

$_nC_r = \frac{n!}{(n-r)! \cdot r!}$ $_nP_r = \frac{n!}{(n-r)!}$

b. From a 30-person club, in how many ways can a President, Treasurer and Secretary be chosen?

Permutation or Combination

$_{30}P_3$ $\boxed{24360}$

$_nC_r = \frac{n!}{(n-r)! \cdot r!}$ $_nP_r = \frac{n!}{(n-r)!}$

c. In a bingo game 30 people are playing for charity. There are 2 \$50 prizes. In how many ways can prizes be awarded?

Permutation or Combination

$_{30}C_2$ $\boxed{435}$

$_nC_r = \frac{n!}{(n-r)! \cdot r!}$ $_nP_r = \frac{n!}{(n-r)!}$

d. How many 3-digit passwords can be formed with the numbers 1, 2,3,4,5 and 6 if no repetition is allowed?

Permutation or Combination

$_6P_3 = \boxed{120}$

$6 \cdot 5 \cdot 4$

$${}_nC_r = \frac{n!}{(n-r)!\bullet r!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

e. Converse is offering a limited edition of shoes. They are individually made for you and you chose 4 different colors from a total of 25 colors. How ~~many shoes~~ are possible?

Permutation or Combination

${}_{25}C_4$ $\boxed{12650}$

$${}_nC_r = \frac{n!}{(n-r)!\bullet r!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

f. A fast food chain is offering a \$5 box special. You can choose 5 items from a list of 8 items on a special menu. In how many ways could you fill the box?

Permutation or Combination

${}_8C_5$ $\boxed{56}$

For Fall '13 Changed to "5 or fewer"

${}_8C_5 + {}_8C_4 + {}_8C_3 + {}_8C_2 + {}_8C_1 + {}_8C_0$

pick 5 pick 4 pick 3 pick 2 pick 1 pick 0

NOTES!!

- The probability of an event occurring is

$$P(E) = \frac{\text{\# of outcomes in event E}}{\text{\# of outcomes in sample space S}} = \frac{n(E)}{n(S)}$$

Notation for probability of an event.

- Example: The probability of getting "heads" in a coin toss is P(heads)=

$$\frac{\text{\# of desired outcomes}}{\text{\# of total outcomes}} = \frac{\text{heads}}{\text{heads and tails}} = \frac{1}{2}$$

- Sample Space:

A list of all possible outcomes with NO repetition

TIPS for making Sample Spaces:

* Use the Counting Principle to determine the number of outcomes
* Create your sample space in an organized way
* Use a tree diagram for help determining all the outcomes

Example 1:

When tossing a coin twice, the sample space would be:

AH HT
TH TT

**HINT: First, determine how many items should be in our sample space.

$\frac{2}{\text{1st toss}} \cdot \frac{2}{\text{2nd toss}}$

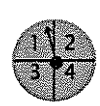
4 possibilities

- Example 2:

When using this spinner twice in a row,

a) the sample space would be:

**a table is a good way to do a sample space here



**HINT: First, determine how many items should be in our sample space.

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

b) What is the probability the sum of the spins is 5?

$\frac{4}{16} = \boxed{\frac{1}{4}}$