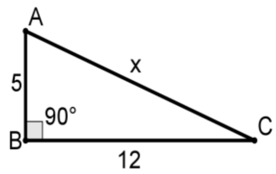
Day 1: Trigonometric Functions

Warm-Up:

Given the following triangles, find x.

1.



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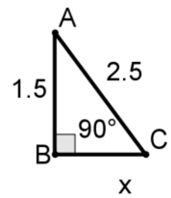
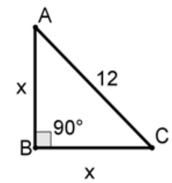


3.



Solve for the missing variables.

4. $x^2 - 12x = 45$

5. $y = \frac{1}{2}x - 5$
$3x + 8y = 2$

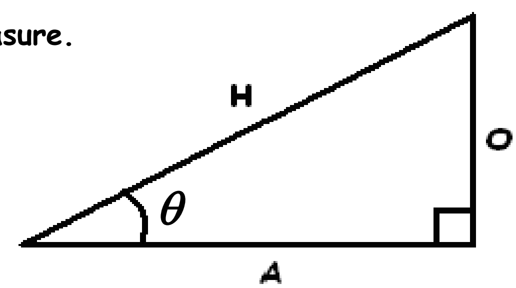
6. Simplify $(-5\sqrt{3})^2$

Notes Day 1: Trigonometric Functions

The trigonometric (trig) functions are ___________, ______________, and _______________.

These functions can be used to find _________ measures, knowing the ratio of the sides OR length of a __________, knowing one side and an angle measure.

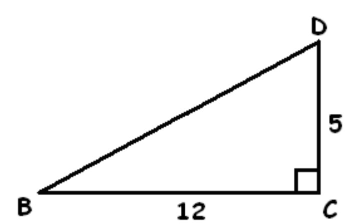
They are used only for ______________ triangles!



Sin θ = **Cos θ =** **Tan θ =**

O= **H=** **A=** **θ =**

SOH CAH TOA



Example 1: tan(∠B)

Example 2: tan(D)

Example 3: sin(D)

Example 4: cos(D)

Example 5: cos(B)

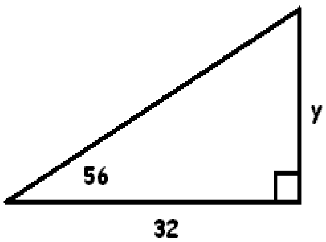
*** Puzzle – Practice Ratios**

Finding missing side lengths using Trigonometric Ratios

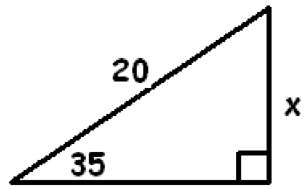
To solve for missing side lengths, set up the _____________, and put the trig function _________________, then cross-multiply to solve.

Use the trig ratios to find the length of the side labeled with a variable. All angle measures for these examples are in degrees. (Remember SOH CAH TOA)

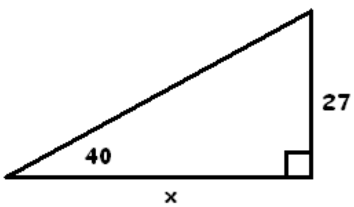
Example 1: Solve for y.



Example 2: Solve for x.

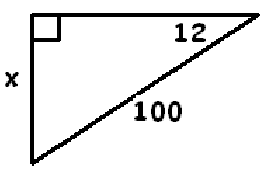


Example 3: Solve for x.



You try!!

1)



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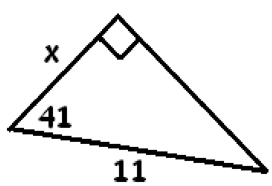

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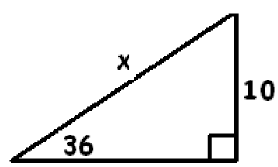

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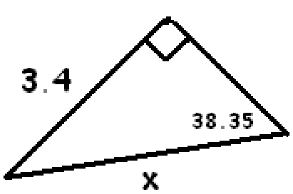

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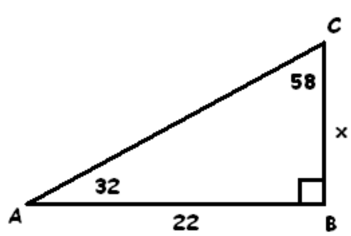
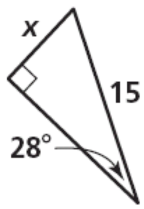


***Puzzle – Practice Finding Side Lengths**

Day 2: Finding Angles using Right Triangle Trigonometry

Warm-Up: Find the value of x. Round to the nearest hundredth.

1.



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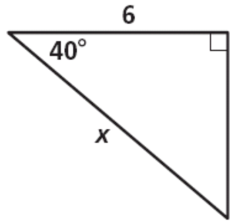

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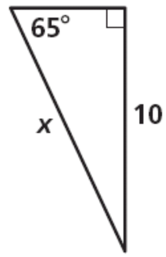

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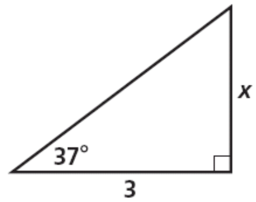
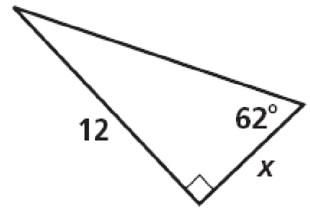


5.



Notes Day 2: Finding Angles using Right Triangle Trigonometry

<u>Finding missing angles with the Trigonometric Ratios</u>

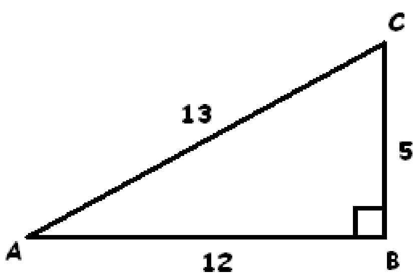
To find missing angle measures, set up the ______________________.

Then, you'll have to do the _____________________ of the trig function ___________________.

NOTE: the inverse of the trig function and the trig function itself cancel out!

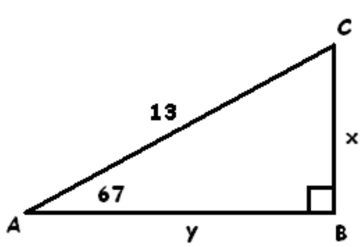
TIP: The inverse _______________ like the trig function with a _____________________.

Example 4: Find tan A and tan C.

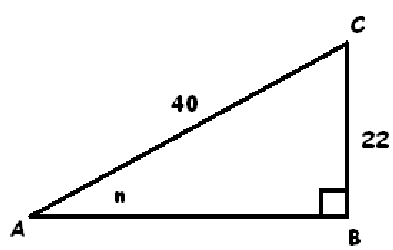


Example 5: Find A and C.

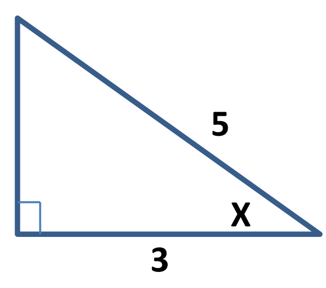
Example 6: Find x and y.



Example 7: Find n.



Ex 8: Find x.

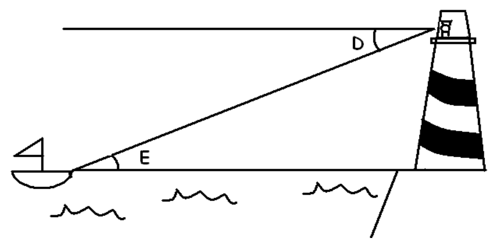


Notes Day 2: Angle of Elevation and Angle of Depression

The top of a lighthouse is 50 feet above sea level. Suppose a lighthouse operator sees a sailboat at an angle of 22° with a horizontal line straight out from his line of vision.

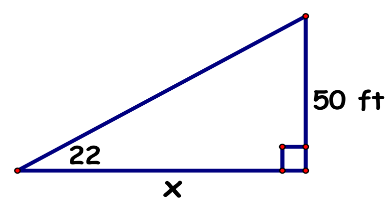
The angle between the horizontal line and the line of sight is called the ________________.

At the same time, a person in the boat looks up at an angle of ____ with the horizon and sees the operator in the lighthouse. This angle is called the ________________________.



NOTE: The measure of the angle of depression ____ the measure of the angle of elevation.

Example: The distance to the lighthouse from the sailboat can be found by



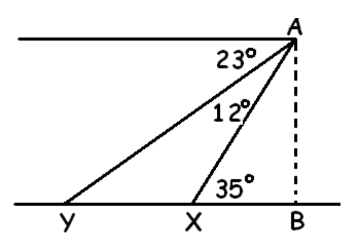
People at points X and Y see an airplane at A

The angle of elevation from X to A is _____________.

The angle of depression from A to X is _____________.

The angle of depression from A to Y is _____________.

The angle of elevation from Y to A is _____________.



Example:

Karen drives 25 km up a hill that is a grade of 14. What horizontal distance has she covered?

Day 2: Angle of Elevation and Angle of Depression Practice

For each problem: 1) Sketch a diagram.

2) Set up the equation.

3) Solve.

1) The leg opposite the 50 degree angle in a right triangle measures 8 meters. Find the length of the hypotenuse.

2) A cliff is 90 feet above the sea. From the cliff, the angle of depression to a boat measures 46 degrees. How far is the boat from the base of the cliff?

3) A ramp is 60 feet long. It rises a vertical distance of 8 feet. Find the angle of elevation.

4) A tree casts a 50-foot shadow while the angle of elevation of the sun is 48. How tall is the tree?

Day 3: Applications of Trigonometric Functions

Warm-Up: (Round to nearest tenth)

1. A tree casts a shadow 21m long while the sun's angle of elevation is 51°. What is the height of the tree?

2. A guy wire reaches from the top of a 120m TV tower to the ground making an angle of 63° with the ground. Find the length of the wire.

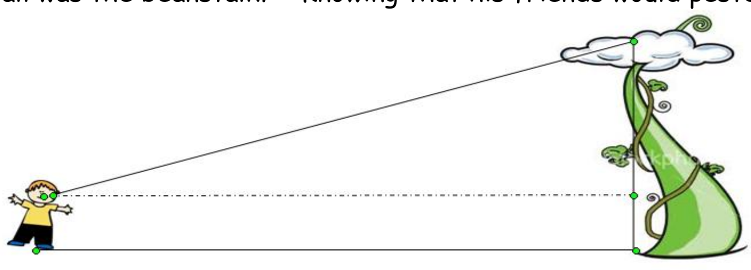
3. A 40 foot escalator rises to a height of 21 feet. What is the angle of inclination (elevation) of the escalator?

Notes Day 3 – More with Applications of Trigonometric Functions

Preparation for Clinometer Lab

Example:

Jack was bragging about climbing a beanstalk. One of his friends, tired of hearing the story for the umpteenth time asked, "Jack, how tall was the beanstalk?" Knowing that his friends would pester him forever, Jack decided to find out…



Jack stood 100 yards away from the point directly under where the beanstalk meets the clouds and used his clinometer to look at the top of the stalk (where it met the clouds). He measured the angle of elevation to be 27.5°. Using this information, what is the distance from the top of the bean stalk to Jack's line of sight?

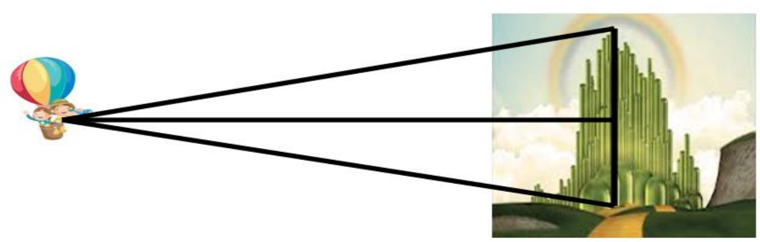
____

Jack then measured from his eyes to the ground (it was 48 inches). He then concluded that the stalk was ______________________ feet tall.

Example

While flying in a hot air balloon, Dorothy and the Wizard looked back at the Emerald City. Dorothy wondered, "How high was that lovely green castle?" Using her clinometer, she decided to find out! She knew (using her range finder) that the horizontal distance to the city was 150 yards.

Dorothy measured the angle of depression from the balloon to the base of the emerald castle to be 15° and the angle of elevation to the top of the castle to be 25°. Based on these measurements, how tall is the castle?



Day 4: Law of Sines, Area of Triangles with Sine

Warm-Up: Draw a picture, label the angle of elevation/depression and the height/length with words and numbers, and solve.

1. A tree 10 meters high cast a 17.3-meter shadow.
   Find the angle of elevation of the sun.

2. A car is traveling up a slight grade with an angle of elevation of 2°.
   After traveling 1 mile, what is the vertical change in feet?
   (1 mile = 5280 ft)

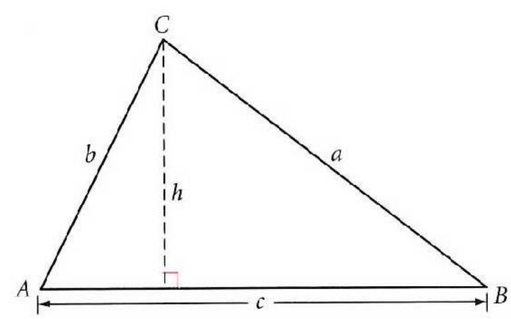
3. A person is standing 50 meters from a traffic light. If the angle of elevation from the person's feet to the top of the traffic light is 25°, find the height of the traffic light.

4. Two friends, each 5 foot tall, meet at a 120 foot tall flag pole, then walk in opposite directions. One measures her angle of elevation to the top of the pole to be 38 degrees, while the other friend finds his to be 42 degrees. How far apart are the friends?

Notes Day 4 Part 1 – Solving Oblique Triangles with Law of Sines

In trigonometry, the ______________________________ can be used to find missing parts of triangles that are ____________________ triangles.

Discovery for Solving Oblique Triangles!

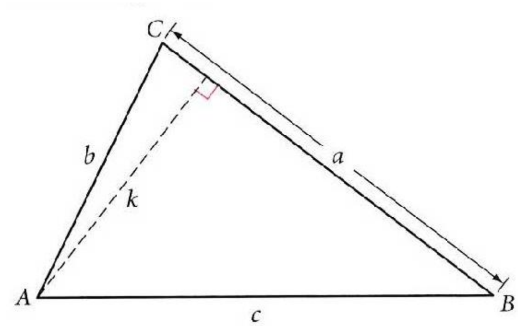


1) Set up the ratios for **sinA** and **sinB**.

2) Find h in terms of a and the sine of an angle.

3) Find h in terms of b and the sine of an angle.

4) Using Algebra, show that $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$



5) Find k in terms of c and the sine of an angle.

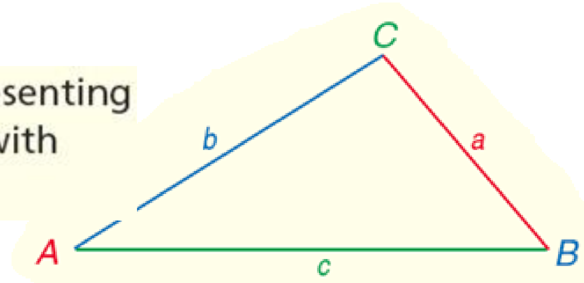
6) Find k in terms of b and the sine of an angle.

7) Using Algebra, show that $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

8) Combine steps 4 and 7 to complete the blanks in the following Law of Sines box.

Law of Sines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

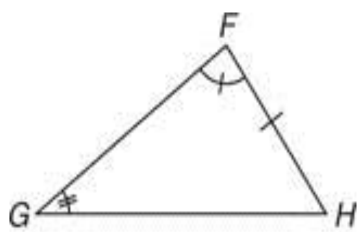


$$\frac{\sin A}{\underline{\quad}} = \frac{\underline{\quad}}{b} = \frac{\underline{\quad}}{\underline{\quad}}$$

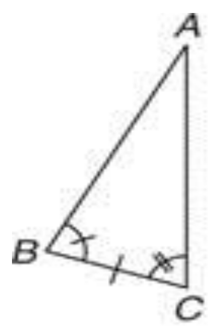
NOTE: To use the Law of Sines, we need an angle and a side _____________ from each other!!

Law of Sines is useful in these cases.

AAS

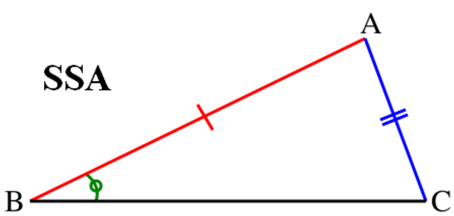


ASA



Law of Sines can also be used for SSA, but it is considered to be an *Ambiguous* case. We'll discuss this more later!

SSA

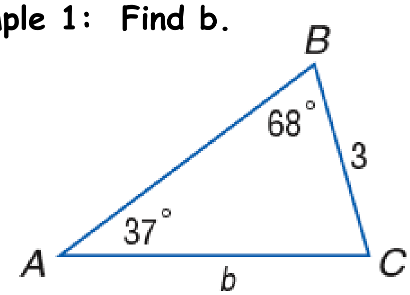


Define Ambiguous Case: Open to two or more interpretations.

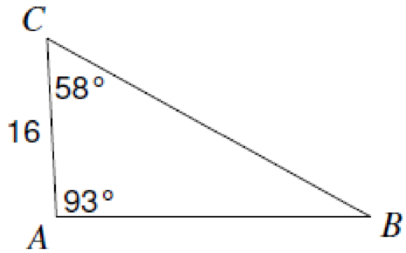
Such is in the case of certain solutions for Law Of Sines

- If you are given two angles and one side (ASA or AAS), the Law of Sines will nicely provide you with ONE solution for a missing side.
- Unfortunately, the Law of Sines has a problem dealing with SSA. If you are given two sides and one angle (where you must *find an angle*), the Law of Sines could possibly provide you with one or more solutions, or even no solution.

Example 1: Find b.



Example 2: Find B, a, and c.



The Law of Sines can be used to solve a triangle. <u>Solving a Triangle</u> means finding the measures of ________________________ and _______________________ of a triangle.

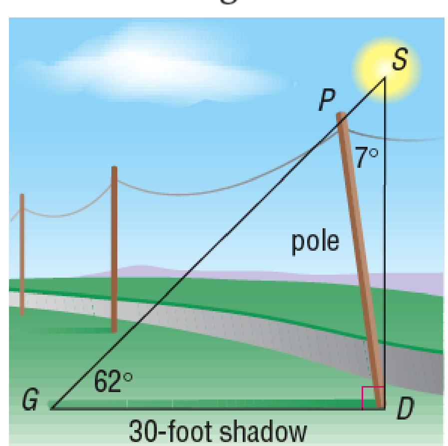
Example 3: Solve the Triangle.

Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.

Ex. 4 Word Problem

Indirect Measurement

When the angle of elevation to the sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.



Concept Summary — Law of Sines

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

8. $m\angle R = 66, m\angle Q = 59, p = 72$

9. $p = 32, r = 11, m\angle P = 105$

10. $m\angle P = 33, m\angle R = 58, q = 22$

11. $p = 28, q = 22, m\angle P = 120$

12. $m\angle P = 50, m\angle Q = 65, p = 12$

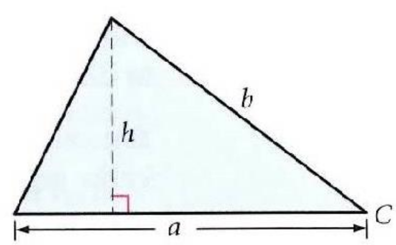
13. $q = 17.2, r = 9.8, m\angle Q = 110.7$

Notes Day 4 Part 2 – Finding Area of Oblique Triangles with Sine

Discovery for Area of Oblique Triangles!

1) What is the formula for the area of a triangle that you remember from other courses?

2) What is formula for the area of this triangle?



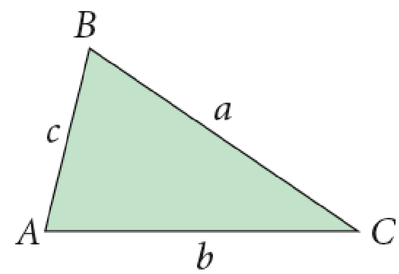
3) Find the ratio for sinC.

4) Suppose we only know the measures of a, b, and angle C – and that we do not know the measure of h. How could we find the area of the triangle? (Hint: use steps 2 & 3)

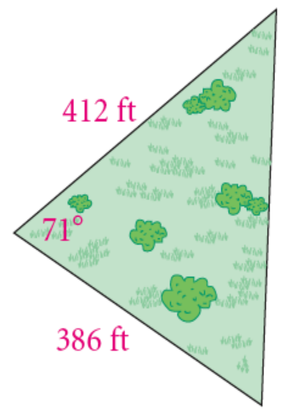
Theorem 9-1 **Area of a Triangle Given SAS**

The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of $\triangle ABC$ =

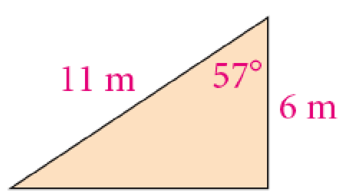


Surveying The surveyed lengths of two adjacent sides of a triangular plot of land are 412 ft and 386 ft. The angle between the sides is 71°. Find the area of the plot.



Find the area of each triangle. Give answers to the nearest tenth.

11.



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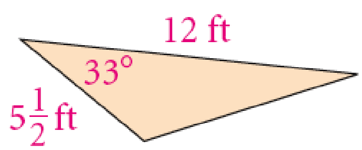

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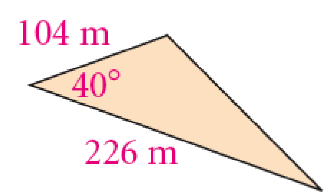

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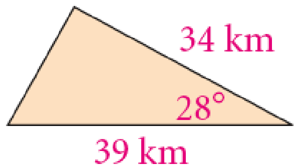

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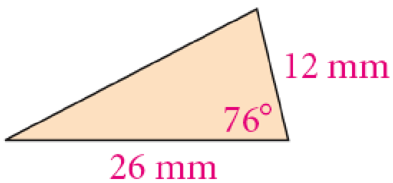

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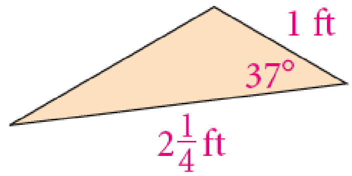


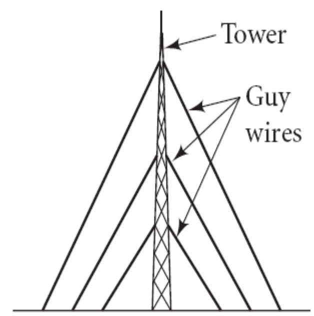
17. **Surveying** A surveyor marks off a triangular parcel of land. One side of the triangle extends 80 yd. A second side of 150 yd forms an angle of 67° with the first side. Determine the area of the parcel of land to the nearest square yard.

Day 5: Law of Sines and the Ambiguous Case

Warm-Up: Draw a picture and solve. Label the picture with numbers and words including the angle of elevation/depression and height/length.

1. The straight line horizontal distance between a window in a school building and a skyscraper is 600ft. From a window in the school, the angle of elevation to the top of the skyscraper is 38 degrees and the angle of depression to the bottom of the tower is 24 degrees.
Approximately how tall is the skyscraper?

2. A communications tower is located on a plot of flat land. It is supported by several guy wires. You measure that the longest guy wire is anchored to the ground 112 feet from the base of the tower and that it makes a 76° angle with the ground.
How long is the longest guy wire and at what height is it connected to the tower?



Notes Day 5: Law of Sines and the Ambiguous Case

With SSA situations, many *interesting* cases are possible. We will look at the 3 cases that occur given an acute angle.

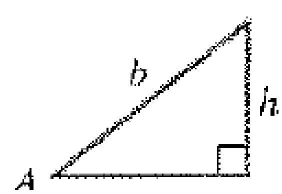
If two sides and an angle opposite one of them is given (SSA), three possibilities can occur.

(1) ____

(2) ____

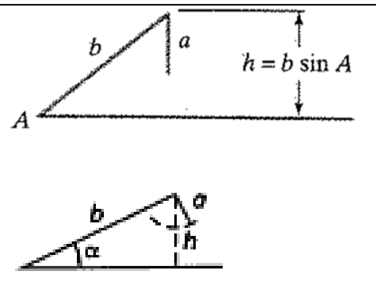
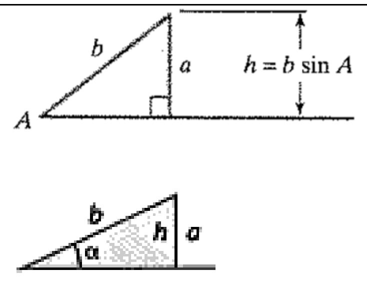
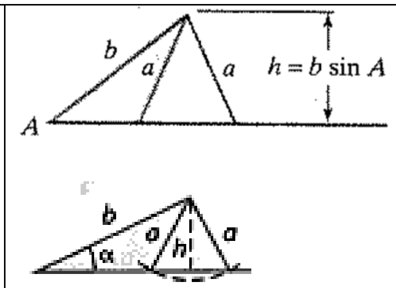
(3) ____

Before we look at the cases, let's use what we know about right triangles to set up the ratio for the following triangle.



sinA =

When we solve for h, we get h = ______________

Figures			
Number of Triangles Possible			
Occurs when….			
Why it occurs….	Side across from the angle is	Just gives us ________ ______________ Triangle	Ambiguous Case… The side across from the angle can "swing" to form an ___________ triangle AND an ___________ triangle

Summary:

If the side across from the given angle is ___________________ than the other side, then check for the ambiguous case!

Ex. 1: *SSA Ambiguous Case*

Solve △ABC if m∠A = 25°, a = 125, and b = 150. Round to the nearest tenth.

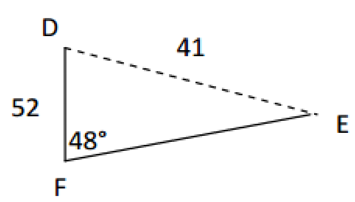
Ex. 2: Solve a triangle when one side is 27 meters, another side is 40 meters and a non-included angle across from the 27 meter side is 33°. Round to the nearest tenth.

Ex. 3: **Solve for all of the missing sides and angles given m∠C = 48, c = 93, and b = 125. (Draw the triangle!)** Round to the nearest tenth.

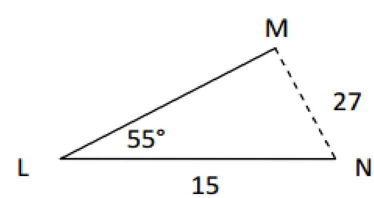
Ex. 4: **Solve for all of the missing sides and angles given m∠A = 24, a = 9.8, and b = 17. (Draw the triangle!)** Round to the nearest tenth.

Law of Sines Practice: Round to the nearest tenth.

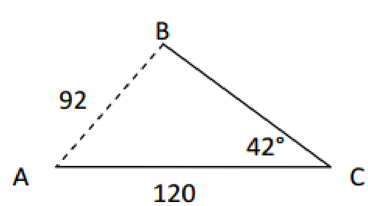
1. For ΔDEF, $e = 52, f = 41$, and $m\angle F = 48°$. Find all possible $m\angle E$ to the nearest degree.



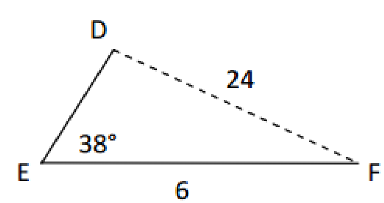
2. For ΔLMN, $l = 27, m = 15$, and $m\angle L = 55°$. Find all possible $m\angle M$ to the nearest degree.



3. For ΔABC, $b = 120, c = 92$, and $m\angle C = 42°$. How many triangles can be formed? **Find m < B.**

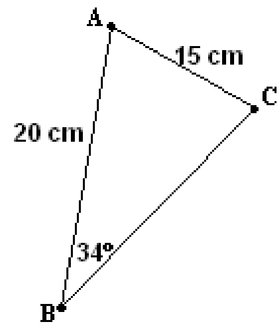


4. For ΔDEF, $d = 6, e = 24$, and $m\angle E = 38°$. How many Triangles can be formed? **Find m < D.**



5. For triangle DEF, d = 25, e = 30, and m$\angle$ E = 40°. Find all possible measurements of f to the nearest whole number.

6. Given ΔABC with $\angle B = 34^0$, b = 15cm, and $c = 20cm.$, solve the triangle. Round to the nearest tenth.



7. Given triangle ABC, a = 8, b = 10, and m$\angle$A = 34, solve the triangle. Round to the nearest tenth.

Day 6: Law of Cosines

Warm-Up ~

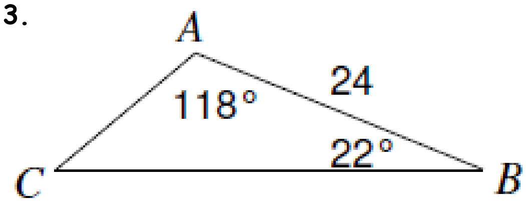
Solve each proportion:

1) $\dfrac{2x - 3}{3} = \dfrac{10 - 4x}{2}$

2) $\dfrac{x + 3}{x + 2} = \dfrac{x - 1}{x - 4}$

Solve each triangle using Law of Sines. Round to the nearest hundredth.

3.



4. $m\angle C = 53°$, $m\angle B = 44°$, $b = 7$

Notes Day 6: Law of Cosines

THE LAW OF COSINES Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

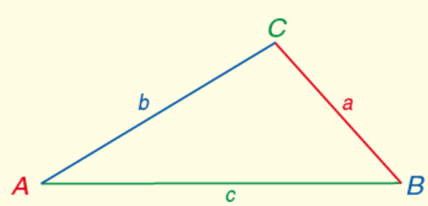
Key Concept *Law of Cosines*

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then the following equations are true.

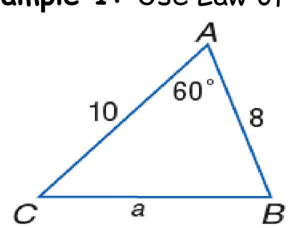
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

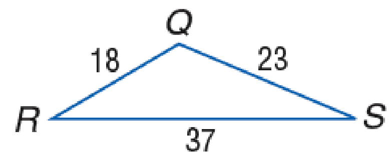


Example 1: Use Law of Cosines to find *a*.

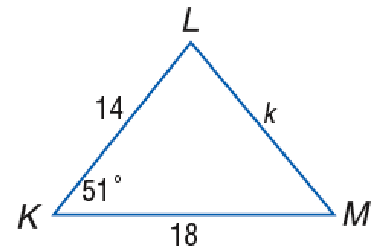


Show your work…

Example 2: Use Law of Cosines to find m∠R.

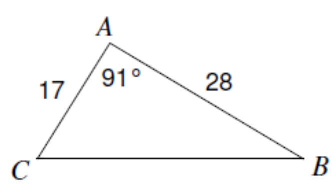


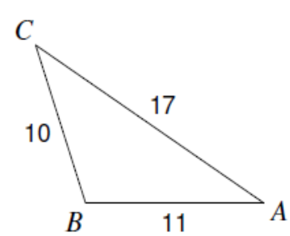
Example 3: Solve ΔKLM . Round angle measure to the nearest degree and side measure to the nearest tenth.



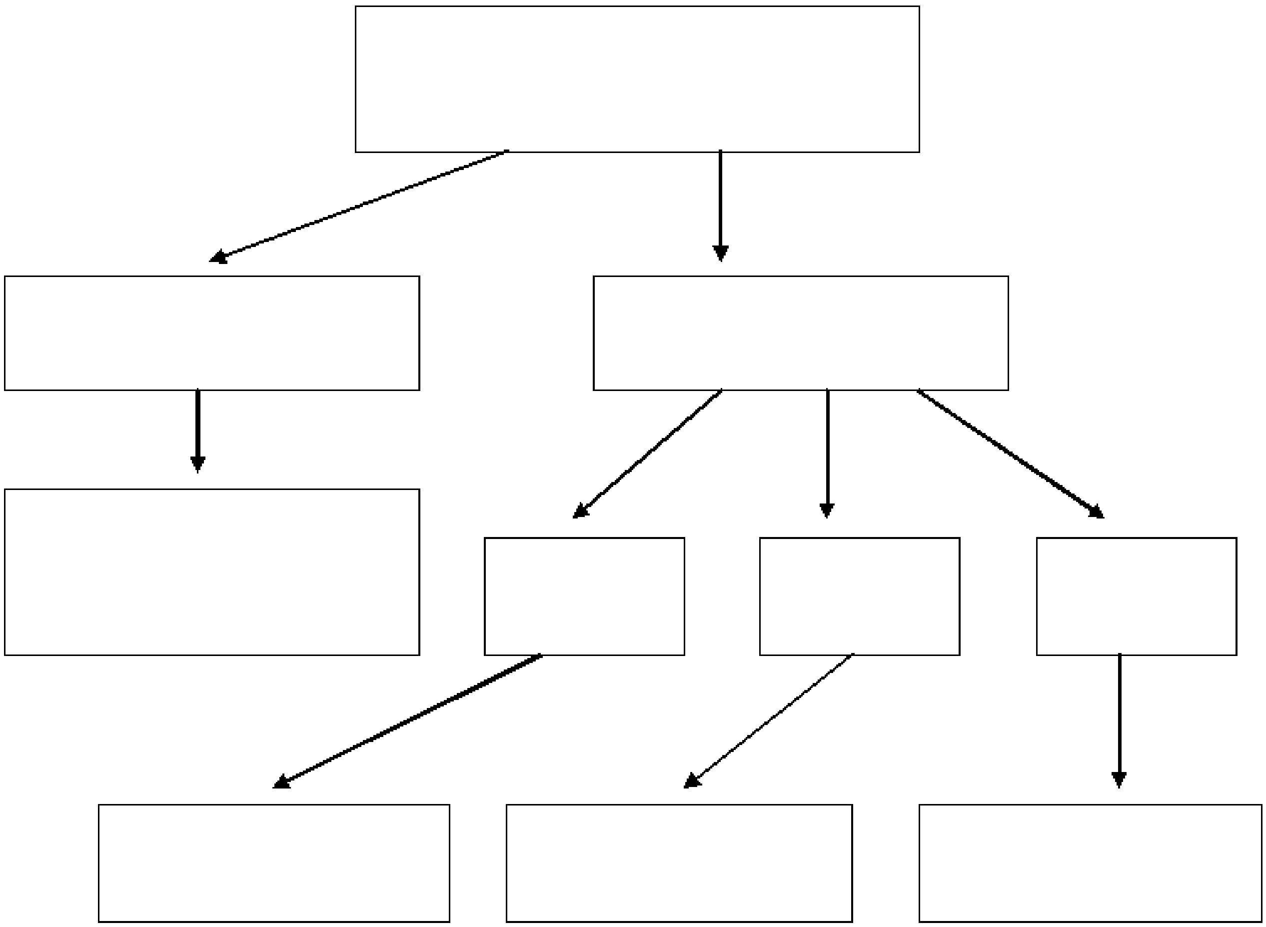
Example 4: Solve ΔABC if a=8, b=10, and c=5.

Examples 5, 6: YOU TRY – Solve each Triangle Using Law Of Cosines.





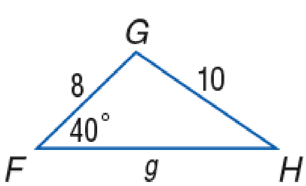
Which Formula Do I Use?



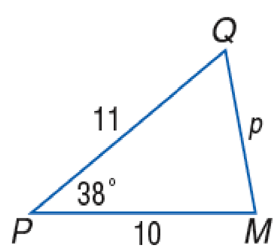
Practice ~ Show work on a separate piece of paper!!

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

19.



20.



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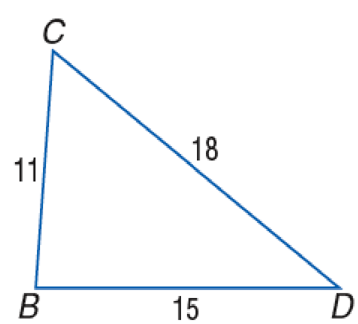

22. $\triangle ABC$: $m\angle A = 42, m\angle C = 77, c = 6$

23. $\triangle ABC$: $a = 10.3, b = 9.5, m\angle C = 37$

24. $\triangle ABC$: $a = 15, b = 19, c = 28$

25. $\triangle ABC$: $m\angle A = 53, m\angle C = 28, c = 14.9$