Day 5: Law of Sines and the Ambiguous Case

Warm-Up:

1. The straight line horizontal distance between a window in a school building and a skyscraper is 600ft. From a window in the school, the angle of elevation to the top of the skyscraper is 38 degrees and the angle of depression to the bottom of the tower is 24 degrees.
Approximately how tall is the skyscraper?

$\tan(38) = \frac{x}{600}$

$x = 600 \cdot \tan(38)$

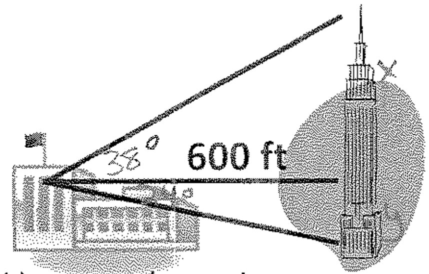
$x = 468.77$

$\tan(24) = \frac{y}{600}$

$y = 600 \cdot \tan(24)$

$y = 267.14$

$x + y = 468.77 + 267.14 = \boxed{735.91 \text{ ft}}$



2. A communications tower is located on a plot of flat land. It is supported by several guy wires. You measure that the longest guy wire is anchored to the ground 112 feet from the base of the tower and that it makes an 76° angle with the ground.
How long is the longest guy wire and at what height is it connected to the tower?

$\tan(76) = \frac{y}{112}$

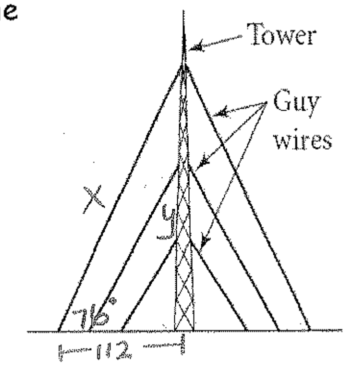
$y = 112 \tan(76)$

$y = \boxed{449.21 \text{ ft height}}$

$\cos(76) = \frac{112}{x}$

$x = \frac{112}{\cos(76)}$

$x = \boxed{462.96 \text{ ft length}}$



Notes Day 5: Law of Sines and the Ambiguous Case

With SSA situations, many *interesting* cases are possible. We will look at the 3 cases that occur given an acute angle.

If two sides and an angle opposite one of them is given (SSA), three possibilities can occur.

(1) No such triangle exists

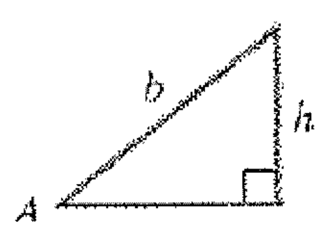
(2) Exactly one triangle exists

(3) Two different triangles exist

Before we look at the cases, let's use what we know about right triangles to set up the ratio for the following triangle.

sinA = $\frac{h}{b}$

When we solve for h, we get h = $b \cdot \sin A$



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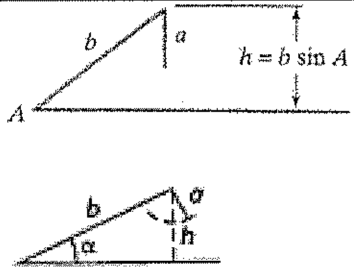
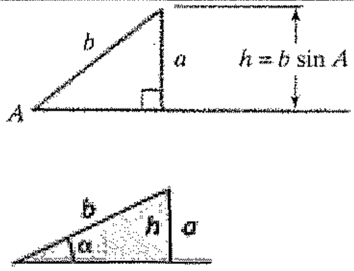
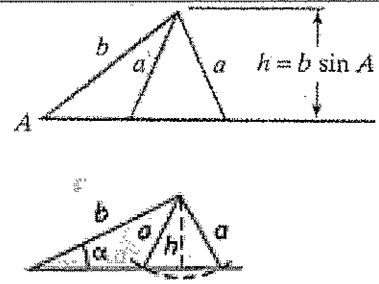
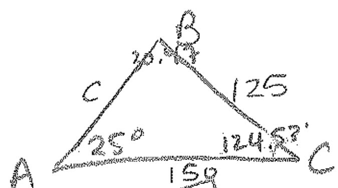
			
Number of Triangles Possible	0	1	2
Occurs when....	$a < h$ h = height of triangle	$a = h$	$a > h$
Why it occurs....	Side across from the angle is smaller than the height of the triangle	Just gives us one right Triangle	Ambiguous Case... The side across from the angle can "swing" to form an acute triangle AND an obtuse triangle

Summary:

If the side across from the given angle is smaller than the other side, then check for the ambiguous case!

Ex. 1: *SSA Ambiguous Case*

Solve $\triangle ABC$ if $m\angle A = 25°$, $a = 125$, and $b = 150$.



Case 1

$$\frac{\sin(25)}{125} = \frac{\sin B}{150}$$

$$\sin^{-1}\left(\frac{150\cdot\sin(25)}{125}\right) = B$$

$$\boxed{\angle B_1 = 30.47° \quad \angle C_1 = 124.53° \quad c = 243.8}$$

$$\frac{\sin(25)}{125} = \frac{\sin(124.53)}{c}$$

$$c\cdot\sin(25) = 125\sin(124.53)$$

$$c = \frac{125\sin(124.53)}{\sin(25)}$$

$c = 243.8$

Find suppliment of $\angle B_1$ is $\angle B_2$

$180 - 30.47 = 149.53$

$149.53 - 25\ (\angle A) = 5.47$

Case 2

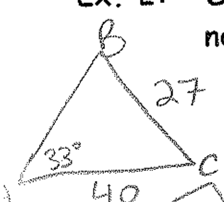
$$\boxed{\angle B_2 = 149.53° \quad \angle C_2 = 5.47°}$$

$$\frac{\sin(25)}{125} = \frac{\sin(5.47)}{c}$$

$$c = \frac{125\sin(5.47)}{\sin(25)}$$

$$\boxed{c_2 = 28.19}$$

Ex. 2: Solve a triangle when one side is 27 meters, another side is 40 meters and a non-included angle is 33°.



Case 1

$$\frac{\sin(33)}{27} = \frac{\sin B}{40}$$

$$B = \sin^{-1}\left(\frac{40\sin 33}{27}\right)$$

$$\boxed{\angle B_1 = 53.79° \quad \angle C_1 = 93.21° \quad c = 49.5}$$

$$\frac{\sin(33)}{27} = \frac{\sin(93.21)}{c}$$

$$c = \frac{27\sin(93.21)}{\sin(33)}$$

$c = 49.5$

Find suppliment of $\angle B_1$

$180 - 53.79 = 126.21$

Case 2

$$\boxed{\angle B_2 = 126.21° \quad \angle C_2 = 20.79°}$$

$180 - 126.21 = 53.79$

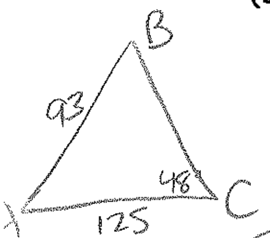
$\angle C_2 = 53.79 - 33\ (\angle A) = 20.79$

$$\frac{\sin(33)}{27} = \frac{\sin(20.79)}{c}$$

$$c_2 = \frac{27\sin(20.79)}{\sin(33)}$$

$$\boxed{c_2 = 17.6}$$

Ex. 3: Solve for all of the missing sides and angles given $m\angle C = 48$, $c = 93$, and $b = 125$. (Draw the triangle!)



$\frac{\sin(48)}{93} = \frac{\sin B}{125}$

$\angle B = \sin^{-1}\left(\frac{125\sin(48)}{93}\right)$

Case 1 $\boxed{\angle B_1 = 87.25° \quad \angle A_1 = 44.75° \quad a_1 = 88.1}$

$\frac{\sin(48)}{93} = \frac{\sin(44.75)}{a}$

$a = \frac{93\sin(44.75)}{\sin(48)}$

$a = 88.1$

$180 - \underset{\angle B}{87.25} = 92.75$

use supplement of $\angle B_1$ for $\angle B_2$ (case 2)

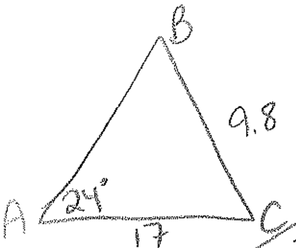
$\boxed{\angle B_2 = 92.75° \quad \angle A_2 = 39.25°}$ Case 2

$\frac{\sin(48)}{93} = \frac{\sin(39.25)}{a}$

$a = \frac{93\sin(39.25)}{\sin(48)}$

$\boxed{a_2 = 79.18}$

Ex. 4: Solve for all of the missing sides and angles given $m\angle A = 24$, $a = 9.8$, and $b = 17$. (Draw the triangle!)



$\frac{\sin(24)}{9.8} = \frac{\sin B_1}{17}$

$\angle B_1 = \sin^{-1}\left(\frac{17 \cdot \sin(24)}{9.8}\right)$

Case 1 $\boxed{\angle B_1 = 44.88° \quad \angle C_1 = 111.12° \quad c_1 = 22.48}$

$\frac{\sin(24)}{9.8} = \frac{\sin(111.12)}{c}$

$c = \frac{9.8\sin(111.12)}{\sin(24)}$

$c = 22.48$

$180 - 44.88 = 135.12$

$180 - \angle B_1 \qquad \angle B_2$

$\boxed{\angle B_2 = 135.12° \quad \angle C_2 = 20.88°}$ Case 2

$\frac{\sin(24)}{9.8} = \frac{\sin(20.88)}{c}$

$c = \frac{9.8\sin(20.88)}{\sin(24)}$

$\boxed{c_2 = 8.59}$

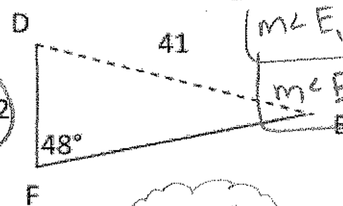
<u>Law of Sines Practice:</u>

2. For ΔDEF, $e = 52, f = 41$, and $m\angle F = 48°$. Find all possible $m\angle E$ to the nearest degree.

$\frac{\sin(48)}{41} = \frac{\sin(E)}{52}$

$E = \sin^{-1}\left(\frac{52\sin(48)}{41}\right)$

$= 70.48$



$\boxed{m\angle E_1 = 70.48°}$

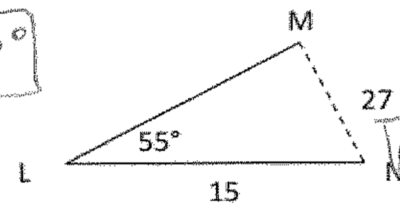
$\boxed{m\angle E_2 = 109.52°}$

$f < e$

1. For ΔLMN, $l = 27, m = 15$, and $m\angle L = 55°$. Find all possible $m\angle M$ to the nearest degree.

$\frac{\sin(55)}{27} = \frac{\sin M}{15}$

$M = \sin^{-1}\left(\frac{15\sin(55)}{27}\right)$



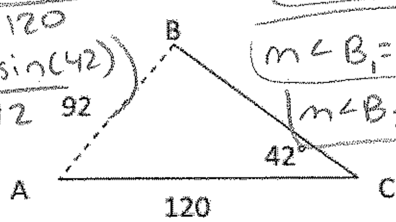
$\boxed{m\angle M_1 = 27.07°}$

4. For ΔABC, $b = 120, c = 92$, and $m\angle C = 42°$. How many triangles can be formed? Find m < B.

$\boxed{2\ \Delta\text{'s}}$

$\frac{\sin(42)}{92} = \frac{\sin B}{120}$

$B = \sin^{-1}\left(\frac{120\sin(42)}{92}\right)$



$\boxed{m\angle B_1 = 60.78}$

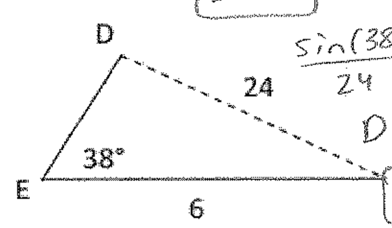
$\boxed{m\angle B_2 = 119.22}$

3. For ΔDEF, $d = 6, e = 24$, and $m\angle E = 38°$. How many Triangles can be formed? Find m < D.

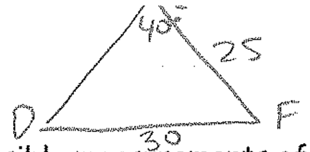
$\boxed{1\ \Delta}$

$\frac{\sin(38)}{24} = \frac{\sin D}{6}$

$D = \sin^{-1}\left(\frac{6\sin 38}{24}\right)$



$\boxed{m\angle D = 8.85°}$



5. For triangle DEF, d = 25, e = 30, and m∠ E = 40°. Find all possible measurements of f to the nearest whole number.

$\frac{\sin(40)}{30} = \frac{\sin D}{25}$

$D = \sin^{-1}\left(\frac{25\sin 40}{30}\right)$

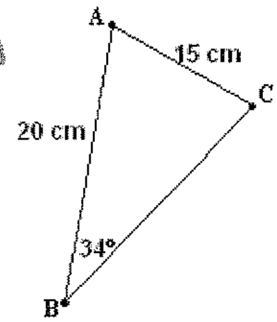
$\angle D = 32.39°$

$180 - 32.39 - 40 = 107.61$

$\frac{\sin(40)}{30} = \frac{\sin(107.61)}{f}$

$f = \frac{30\sin(107.61)}{\sin(40)} = \boxed{44.48}$

6. Given ΔABC with $\angle B = 34^0$, b = 15cm, and $c = 20cm.$, solve the triangle.



$\frac{\sin(34)}{15} = \frac{\sin C}{20}$

$\angle C = \sin^{-1}\left(\frac{20\sin(34)}{15}\right)$

Case 1

$\angle C_1 = 48.21°$
$\angle A_1 = 97.79°$
$a_1 = 26.58$

$\frac{\sin(34)}{15} = \frac{\sin(97.79)}{a}$

$a = \frac{15\sin(97.79)}{\sin(34)}$

$a = 26.58$

$180 - 48.21 = 131.79$

Case 2

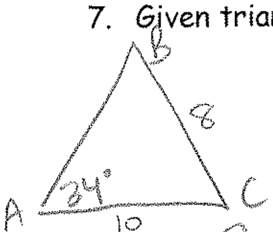
$\angle C_2 = 131.79$
$\angle A_2 = 14.21$

$\frac{\sin(34)}{15} = \frac{\sin(14.21)}{a}$

$a = \frac{15\sin(14.21)}{\sin(34)}$

$\boxed{a_2 = 6.58}$

7. Given triangle ABC, a = 8, b = 10, and m∠A = 34, solve the triangle.



Case 1

$\frac{\sin(34)}{8} = \frac{\sin B}{10}$

$\angle B = \sin^{-1}\left(\frac{10\sin(34)}{8}\right)$

$\angle B_1 = 44.35°$
$\angle C_1 = 101.65°$
$c_1 = 14.01$

$\frac{\sin(34)}{8} = \frac{\sin(101.65)}{c}$

$c_1 = \frac{8\sin(101.65)}{\sin(34)}$

$c_1 = 14.01$

$180 - 44.35 = 135.65$

Case 2

$\angle B_2 = 135.65°$
$\angle C_2 = 10.35°$

$\frac{\sin(34)}{8} = \frac{\sin(10.35)}{c}$

$c_2 = \frac{8\sin(10.35)}{\sin(34)}$

$\boxed{c_2 = 2.57}$

Day 6: Law of Cosines

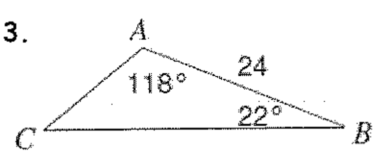
Warm-Up: Solve each proportion:

1) $\frac{2x - 3}{3} = \frac{10 - 4x}{2}$

2) $\frac{x + 3}{x + 2} = \frac{x - 1}{x - 4}$

Solve each triangle using Law of Sines.

3.



4. m∠C = 53°, m∠B = 44°, b = 7