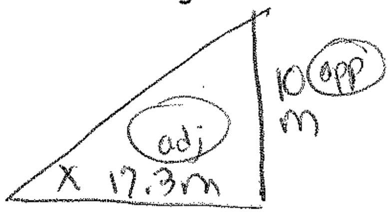
Day 4: Law of Sines, Area of Triangles with Sine

Warm-Up:

1. A tree 10 meters high cast a 17.3 meter shadow. Find the angle of elevation of the sun. 30°

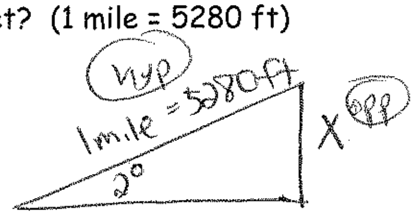


$\tan(x) = \frac{10}{17.3}$

$x = \tan^{-1}\left(\frac{10}{17.3}\right)$

$\boxed{x = 30^\circ}$

2. A car is traveling up a slight grade with an angle of elevation of 2°. After traveling 1 mile, what is the vertical change in feet? (1 mile = 5280 ft)

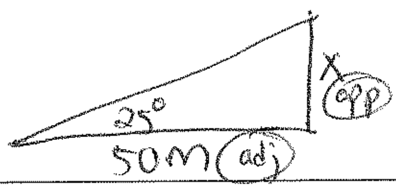


$\sin(2) = \frac{x}{5280}$

$x = 5280\sin(2)$

$\boxed{184.3 \text{ ft}}$

3. A person is standing 50 meters from a traffic light. If the angle of elevation from the person's feet to the top of the traffic light is 25°, find the height of the traffic light. $\boxed{23.3 \text{ m}}$



$\tan(25) = \frac{x}{50}$

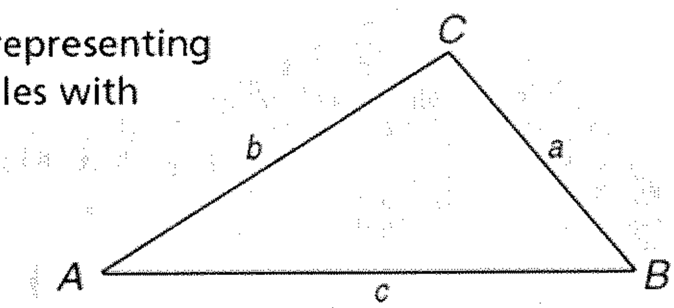
$x = 50\tan(25)$

Notes 9.1 and 9.2 – Trigonometric Functions

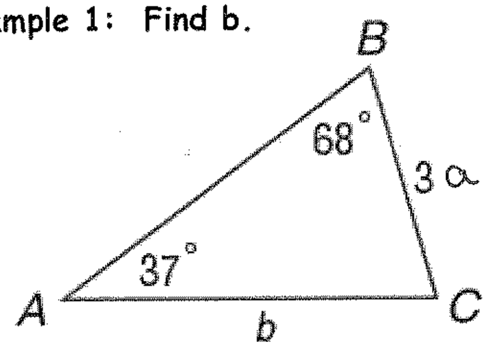
In trigonometry, the law of sines **can be used to find missing parts of triangles that are** oblique **triangles.**

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$



Example 1: Find b.



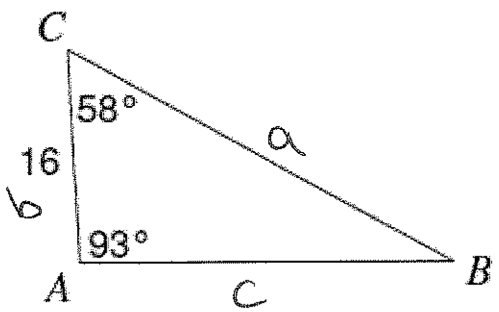
$\frac{\sin(37)}{3} = \frac{\sin(68)}{b}$ substitute in info

$b\sin(37) = 3\sin(68)$ cross multiply

$\frac{b\cancel{\sin(37)}}{\cancel{\sin(37)}} = \frac{3\sin(68)}{\sin(37)}$ solve

$\boxed{b = 4.6}$

Example 2: Find B, a, and c.



$\frac{\sin(93)}{a} = \frac{\sin(B)}{16} = \frac{\sin(58)}{c}$

you need an angle & side across from each other BUT that's not given

① Get last ∠ → All ∠s in △ = sum 180°

$m\angle A + m\angle B + m\angle C = 180$

$93 + m\angle B + 58 = 180$ → $15\cancel{0} + m\angle B = 180$

$\boxed{m\angle B = 29}$

② $\frac{\sin(93)}{a} = \frac{\sin(29)}{16}$ → $\frac{16\sin(93)}{\sin(29)} = \frac{a\cancel{\sin(29)}}{\cancel{\sin(29)}}$

$\boxed{33.0 = a}$

③ $\frac{\sin(58)}{c} = \frac{\sin(29)}{16}$ → $\frac{c\cancel{\sin(29)}}{\cancel{\sin(29)}} = \frac{16\sin(58)}{\sin(29)}$

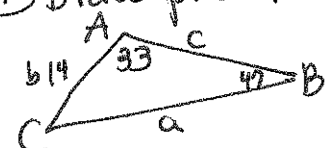
$\boxed{c = 28.0}$

The Law of Sines can be used to solve a triangle. Solving a Triangle means finding the measures of all the angles and all the sides of a triangle.

Example 3: Solve the Triangle.

Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.

① Draw pic 1st, if not given



② $\frac{\sin(33)}{a} = \frac{\sin(47)}{14}$

$\frac{a\cancel{\sin(47)}}{\cancel{\sin(47)}} = \frac{14\sin(33)}{\sin(47)}$

$\boxed{a = 10.4}$

③ $m\angle A + m\angle B + m\angle C = 180$

$33 + 47 + m\angle C = 180$

$80 + m\angle C = 180$

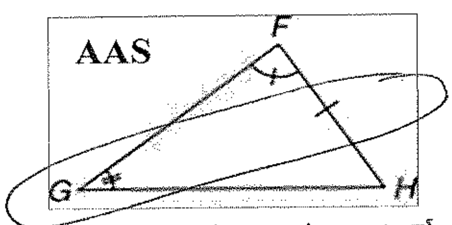
$\boxed{m\angle C = 100}$

④ $\frac{\sin(100)}{c} = \frac{\sin(47)}{14}$

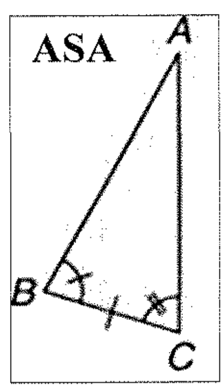
$\frac{c\cancel{\sin(47)}}{\cancel{\sin(47)}} = \frac{14\sin(100)}{\sin(47)}$

$\boxed{c = 18.9}$

Law of Sines is useful in these cases.

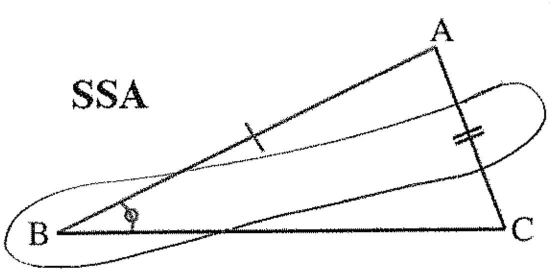


need angle & side across from each other to do law of sines



If subtract given angles from 180, you can get last angle. Then you have angle & side across from each other.

Law of Sines can also be used in this case, but it is ambiguous.



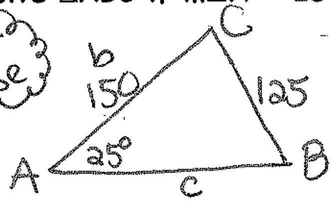
have ∠ & side across from each other BUT can have more than one triangle in this case

Practice Break Notes p10 #8, 10, 12

Ex. 4 SSA Ambiguous Case

Solve △ABC if m∠A = 25°, a = 125, and b = 150.

1st case



① $\frac{\sin(25)}{125} = \frac{\sin B}{150}$

$\frac{150\sin(25)}{125} = \frac{125\sin B}{125}$

$.5071 = \sin B$

$\sin^{-1}(.5071) = \cancel{\sin^{-1}}(\cancel{\sin} B)$

$\boxed{30.5° = B}$

② $m\angle A + m\angle B + m\angle C = 180$

$25 + 30.5 + m\angle C = 180$

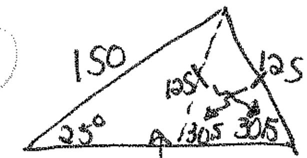
$\boxed{m\angle C = 124.5°}$

③ $\frac{\sin(124.5)}{c} = \frac{\sin(25)}{125}$

$\frac{c\cancel{\sin(25)}}{\cancel{\sin(25)}} = \frac{125\sin(124.5)}{\sin(25)}$

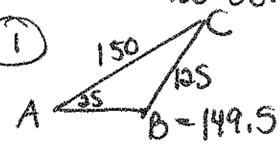
$\boxed{c = 243.8}$

OR 2nd case



$180 - 30.5 = 149.5$

①



$\boxed{m\angle B = 149.5}$

② $m\angle C + 149.5 + 25 = 180$

$\boxed{m\angle C = 5.5°}$

③ $\frac{\sin(5.5)}{c} = \frac{\sin(25)}{125}$

$\frac{c\sin(25)}{\sin(25)} = \frac{125\sin(5.5)}{\sin(25)}$

$\boxed{c = 28.3}$

Ex. 5 Word Problem

Indirect Measurement

When the angle of elevation to the sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.

③ $\frac{\sin(62)}{x} = \frac{\sin(35)}{30}$

$\frac{x\cancel{\sin(35)}}{\cancel{\sin(35)}} = \frac{30\sin(62)}{\sin(35)}$

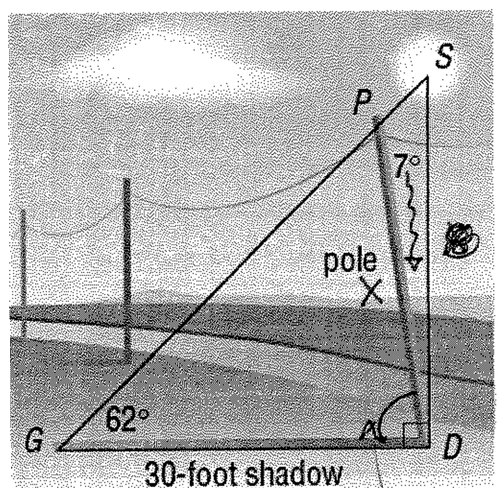
$\boxed{x = 46.2 \text{ ft}}$

② $62 + 83 + m\angle P = 180$

$145 + m\angle P = 180$

$-145 \quad -145$

$\boxed{m\angle P = 35}$

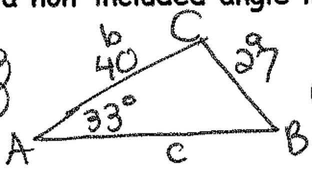


① $90 - 7 = 83$

Ex. 6 Another SSA Ambiguous Case

Solve a triangle when one side is 27 meters, another side is 40 meters and a non-included angle is 33°.

1st case



① label △ with letters

② $\frac{\sin(33)}{27} = \frac{\sin(B)}{40}$

$\frac{27\sin(B)}{27} = \frac{40\sin(33)}{27}$

$\sin B = .8069$

$B = \sin^{-1}(.8069)$

$\boxed{B = 53.8°}$

③ $m\angle A + m\angle B + m\angle C = 180$

$33 + 53.8 + m\angle C = 180$

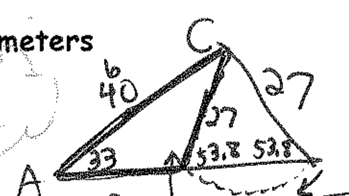
$\boxed{m\angle C = 93.2°}$

④ $\frac{\sin(93.2)}{c} = \frac{\sin(33)}{27}$

$\frac{c\cancel{\sin(33)}}{\sin(33)} = \frac{27\sin(93.2)}{\sin(33)}$

$\boxed{c = 49.5}$

OR 2nd case



① $180 - 53.8 = \boxed{126.2 = B}$

② $180 - 33 - 126.2 = \boxed{C = 20.8}$

③ $\frac{\sin(20.8)}{c} = \frac{\sin(33)}{27}$

$\frac{27\sin(20.8)}{\sin(33)} = \frac{c\cancel{\sin(33)}}{\cancel{\sin(33)}} = \boxed{17.6 = c}$

Concept Summary — Law of Sines

The Law of Sines can be used to solve a triangle in the following cases.

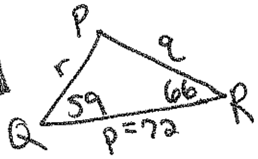
Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

8. $m\angle R = 66, m\angle Q = 59, p = 72$
9. $p = 32, r = 11, m\angle P = 105$ **SSA**
10. $m\angle P = 33, m\angle R = 58, q = 22$
11. $p = 28, q = 22, m\angle P = 120$ **SSA**
12. $m\angle P = 50, m\angle Q = 65, p = 12$
13. $q = 17.2, r = 9.8, m\angle Q = 110.7$ **SSA**

$\boxed{8}$



① $59+66+m\angle P=180$

$\boxed{m\angle P=55}$

② $\dfrac{\sin(55)}{72}=\dfrac{\sin(59)}{q}$

$\dfrac{q\cancel{\sin(55)}}{\cancel{\sin(55)}}=\dfrac{72\sin(59)}{\sin(55)}$

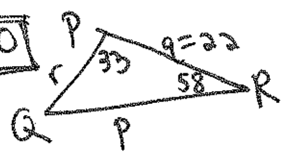
$\boxed{q=75.3}$

③ $\dfrac{\sin(55)}{72}=\dfrac{\sin(66)}{r}$

$\dfrac{r\cancel{\sin(55)}}{\cancel{\sin(55)}}=\dfrac{72\sin(66)}{\sin(55)}$

$\boxed{r=80.3}$

$\boxed{10}$



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12. $m\angle P = 50, m\angle Q = 65, p = 12$
13. $q = 17.2, r = 9.8, m\angle Q = 110.7$ SSA

(8) $m\angle P = 55°$ $q \approx 75.3$ $r \approx 80.3$

(9) $m\angle R \approx 19°$ $m\angle Q = 56°$ $q \approx 27.5$

(10) $m\angle Q = 89°$ $p \approx 12.0$ $r \approx 18.7$

(11) $m\angle Q \approx 43°$ $m\angle R \approx 17$ $r \approx 9.5$

(12) $m\angle R = 65°$ $q \approx 14.2$ $r \approx 14.2$

(13) $m\angle P \approx 37°$ $p \approx 11.1$ $m\angle R \approx 32°$