Day 9: Modeling Advanced Functions

Warm-Up:

Released Exam Items. Show your work to complete these problems. Do NOT just circle an answer!

1. The equation $s = 2\sqrt{5x}$ can be used to estimate speed, s, of a car in miles per hour, given the length in feet, x, of the tire marks it leaves on the ground. A car traveling 90 miles per hour came to a sudden stop. According to the equation, how long would the tire marks be for this car?

   A. 355 feet B. 380 feet C. 405 feet D. 430 feet

2. Which function is even?

   A. f(x) = (x + 2)(x – 2) B. f(x) = x(x + 2)
   C. f(x) = (x + 1)(x – 2) D. f(x) = (x – 1)(x – 1)

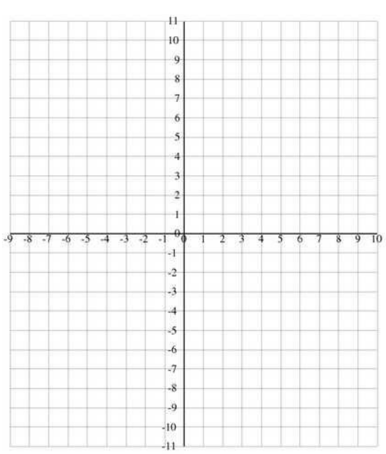
3. A marathon is roughly 26.2 miles long. Which equation could be used to determine the time, t, it takes to run a marathon as a function of the average speed, s, of the runner where t is in hours and s is in miles per hour?

   A. t = 26.2 – 26.2 s B. t = 26.2 – s /26.2
   C. t = 26.2 s D. t = 26.2 / s

Practice Graphing Inverse Variation… Do a table for each branch and completely graph the function! Also, indicate the horizontal and vertical asymptotes, domain, and range for each function.

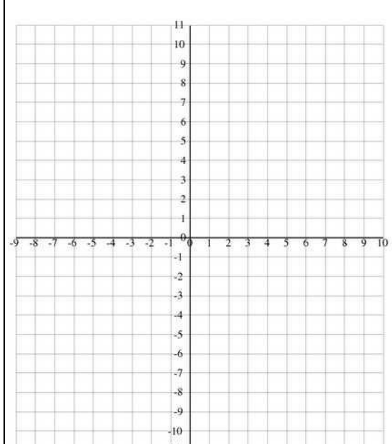
4. $y = \frac{1}{x}$

HA: ______
VA: ______
Domain: _________
Range: _________



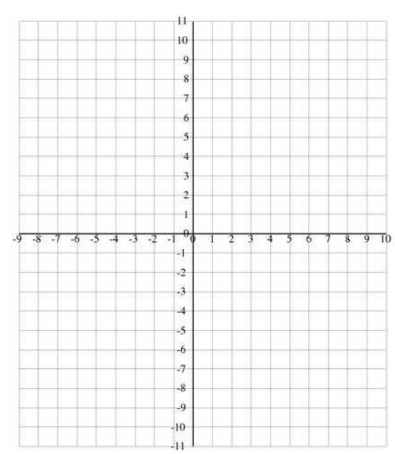
5. $y = \frac{2}{x}$

HA: ______
VA: ______
Domain: _________
Range: _________



6. $y = \frac{-5}{x}$

HA: ______
VA: ______
Domain: _________
Range: _________



Notes: Inverse Variation

A relationship that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$, is an **inverse variation**. The constant k is the constant of variation.

Multiplying both sides of $y = \frac{k}{x}$ by x gives ______________________. So, the product of x and y in an inverse variation is __________________________.

Inverse Variations

WORDS	NUMBERS	ALGEBRA
y varies inversely as x.	$y = \frac{3}{x}$	$y = \frac{k}{x}$
y is inversely proportional to x.	$xy = 3$	$xy = k \; (k \neq 0)$

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether xy is a constant for each ordered pair.

Example: Tell whether the relationship is an inverse variation. Explain. If it is an inverse variation, write the equation.

1.

x	y
1	30
2	15
3	10

2.

x	y
1	5
2	10
4	20

3. 2xy = 28

4.

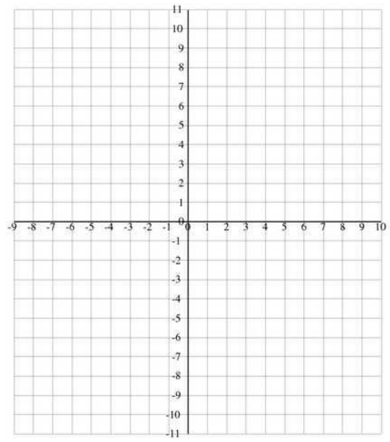
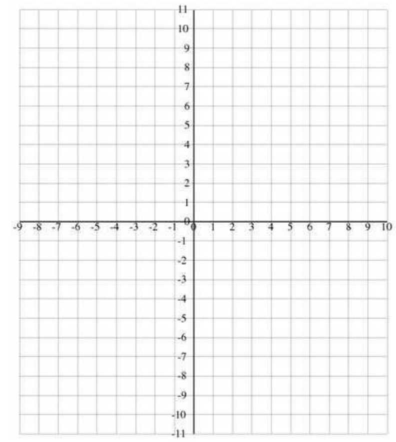
X	y
-12	24
1	-2
8	-16

5.

x	y
3	3
9	1
18	0.5

6. 2x + y = 10

Examples:

1. Write and graph the inverse variation in which *y* = 0.5 when *x* = –12. 	2. Write and graph the inverse variation in which y =1/2 when *x* = 10 

3. The inverse variation *xy* = 350 relates the constant speed *x* in mi/h to the time *y* in hours that it takes to travel 350 miles. Determine a reasonable domain and range and then graph this inverse variation.

4. The inverse variation *xy* = 100 represents the relationship between the pressure *x* in atmospheres (atm) and the volume *y* in mm^3 of a certain gas. Determine a reasonable domain and range and then graph this inverse variation.

Product Rule for Inverse Variation

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

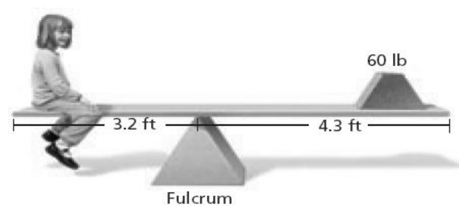
Examples:

5. Let $x_1 = 5$, $x_2 = 3$, and $y_2 = 10$. Let y vary inversely as x. Find y_1.

6. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y vary inversely as x. Find y_2.

7. Boyle's law states that the pressure of a quantity of gas *x* varies inversely as the volume of the gas *y*. The volume of gas inside a container is 400 in^3 and the pressure is 25 psi. What is the pressure when the volume is compressed to 125 in^3?

8. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight. The diagram shows a balanced lever. How much does the child weigh?



Notes: Joint Variation

Joint Variation

- Occurs when 1 quantity varies directly as the ___________________________of 2 or more other quantities.
- Form ___________________________, $x \neq 0$, $z \neq 0$

Ex: The area of a trapezoid varies jointly as the height h and the sum of its bases b_1 and b_2. Find the equation of joint variation if $A = 48$ in, h= 8 in, $b_1 = 5$in, and $b_2 = 7$ in.

Write an equation for the following…

- y varies directly with x and inversely with z^2.
- y varies inversely with x^3.
- y varies directly with x^2 and inversely with z.
- z varies jointly with x^2 and y.
- y varies inversely with x and z.

Practice: Tell whether x and y show direct variation, inverse variation, or neither.

1.) $xy = \frac{1}{4}$ 2.) $2x + y = 4$ 3.) $\frac{y}{x} = 12$ 4.) $y = \frac{1}{x}$

Write the function that models each relationship. Find z when x = 6 and y = 4.

5. z varies jointly with x and y. When x = 7 and y = 2, z = 28.

6. z varies directly with x and inversely with the cube of y. When x = 8 and y = 2, z = 3.

Word Problems:

1. The speed of the current in a whirlpool varies inversely with the distance from the whirlpool's center. The Lofoten Maelstrom is a whirlpool located off the coast of Norway. At a distance of 3000 meters from the center, the speed of the current is about 0.1 meters per second.

a. Find the equation for this scenario.

b. What's the speed of the whirlpool when 50 meters from the center?

2. In building a brick wall, the amount of time it takes to complete the wall varies directly with the number of bricks in the wall and varies inversely with the number of bricklayers that are working together. A wall containing 1200 bricks, using 3 bricklayers, takes 18 hours to build. How long would it take to build a wall of 4500 bricks if 5 bricklayers worked on it?

Day 10: Solving Rational Equations

Warm-Up: Simplify without a calculator!

1. $\frac{5}{12}-\frac{1}{12}=$

3. $\frac{4}{5}+\frac{1}{7}=$

2. $\frac{6}{4}-\frac{3}{7}=$

4. $\frac{2}{3}+\frac{5}{6}=$

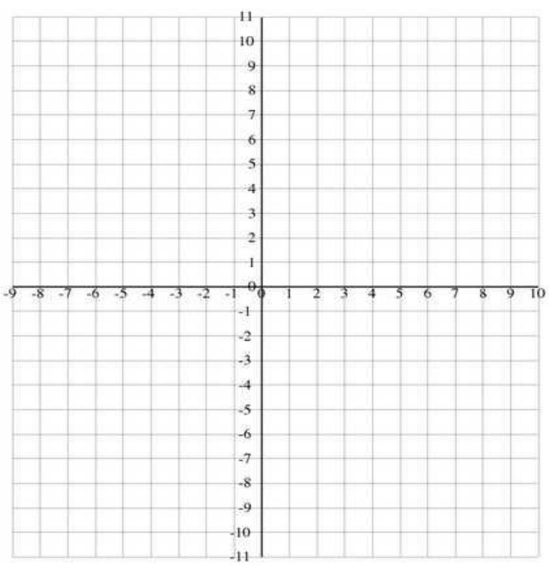
5. Suppose that y varies inversely as x^2 and that y = 6 when x = 9.

a) Find the equation that represents the relationship of x and y.

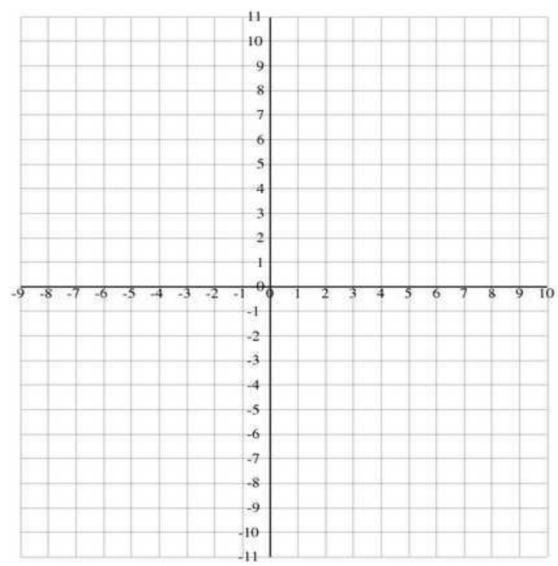
b) Find the value of y when x = 3.

Create a table and completely graph numbers 6 and 7 BY HAND (i.e. no calculator!)

6. $y=\sqrt[3]{x+2}$



7. $y=\sqrt{x-1}$



9. Find the domain and the range of the following function: $f(x)=|4x-4c|-7$

Notes Day 10: Solving Rational Equations

A _________________________ is an equation that contains one or more rational expressions. It can have a variable in the number and/or the denominator. *Our* goal when solving rational equations is to eliminate the fractions and solve the equation for the variable!

Recall that when you graph a rational function, there is a vertical asymptote. This is an x-value that the graph approaches but **NEVER** touches. When you solve rational equations, there are some values for x that must be excluded from the domain because they will make the denominator equal to zero, and dividing by zero is **undefined**.

Any number that causes the denominator to equal zero is called an ______________________.

To find excluded values, **factor the denominator** (if possible), then set the factors equal to zero and solve for the variable; the solutions are excluded values. When solving rational equations, if **all solutions of the rational equation are excluded values** then there is _______________ to the rational equation!

To solve simple rational equations, the cross product property can be utilized to eliminate the fraction leaving a linear equation to solve. **REMEMBER**: Check your final answers to make sure they are not an excluded value!

Examples: Using the cross product property, solve the following equations. Do not forget to determine the excluded values.

1. $\frac{6}{x} = \frac{3}{7}$ EV: __________

2. $\frac{4}{x-7} = \frac{6}{x}$ EV: __________

3. $\frac{-5}{x+4} = \frac{1}{x+4}$ EV: __________

4. $\frac{6}{x+5} = \frac{x}{6}$ EV: __________

Examples: Multiply through by the LCD to solve the following equations. Do not forget to determine the excluded values.

5. $\frac{2}{x} - 3 = \frac{8}{x}$ EV: __________

6. $\frac{7x}{x-3} + 4 = \frac{x+1}{x-3}$ EV: _________

Examples: Solve the following equations. Do not forget to determine the excluded values.

7. $\frac{8}{x+8}=\frac{x}{x+2}$ EV: __________

8. $\frac{4}{x+2}+3=\frac{9}{x+2}$ EV: __________

9. $\frac{3x}{x-1}-2=\frac{10}{x-1}$ EV: __________

10. $\frac{12}{x+2}=\frac{7}{x-3}$ EV: __________

Solving Simple Rational Equations Practice

Solve the rational equation. Do not forget to determine the excluded values.

1. $\frac{3}{x}=\frac{2}{x+4}$ EV: __________

2. $\frac{x+1}{2x+5}=\frac{2}{x}$ EV: __________

3. $\frac{3}{x+2}+5=\frac{4}{x+2}$ EV: __________

4. $\frac{6}{x-3}=\frac{x}{18}$ EV: __________

5. $\frac{2x}{x+4}-3=\frac{-12}{x+4}$ EV: __________

6. $\frac{14}{2-x}=\frac{2}{x}$ EV: __________

Day 11: Solving Harder Rational Equations

Warm-up

1. $\frac{x+2}{x+1} - x = \frac{-6}{x+1}$ EV: __________

2. $\frac{4}{x-5} = \frac{2}{x+8}$ EV: ______

3. $\frac{2}{x-4} + 2 = \frac{6}{x-4}$ EV: __________

4. $\frac{x}{x+24} = \frac{2}{x}$ EV: ______

5. The volume, V, of a certain gas varies inversely with the amount of pressure, P, placed on it. The volume of this gas is 175 cm^3 when 3.2 kg/cm^2 of pressure is placed on it. What amount of pressure must be placed on 400 cm^3 of this gas?

6. The time, *t*, in hours, that it takes *x* people to plant *n* trees varies directly with the number of trees and inversely with the number of people. Suppose 6 people can plant 12 trees in 3 hours. How many people are needed to plant 28 trees in 5 hours and 15 minutes?

Notes Day 11: Solving Harder Rational Equations

Example 1: $\frac{x-4}{4} + \frac{x}{3} = 6.$

Steps:
1. Factor the denominator (if possible).
2. Find the LCD.
2. Multiply each side by the LCD.
3. Simplify.
4. Solve for x!

Example 2: $\frac{3}{2x} - \frac{2x}{x+1} = -2$

Note that x ≠ -1 and x ≠ 0. The LCD of the fractions is 2x(x + 1)

Multiply each side of the equation by 2x(x + 1).

Example 3: $\frac{4}{x+3}-\frac{3}{x^2+6x+9}=1$

Least Common Denominator OR LCD

$\frac{x}{3}+\frac{2}{5}=7 \Rightarrow \quad LCD=15 \Rightarrow$

$\left((15)\frac{x}{3}\right)+\left((15)\frac{2}{5}\right)=7(15) \Rightarrow$

$5(x)+3(2)=105 \Rightarrow$

$x=19.8$

Example 4: $\frac{6}{x}-\frac{9}{x-1}=\frac{1}{4}$

Example 5: $\frac{2m}{m-1}+\frac{m-5}{m^2-1}=1$

Solving Rational Equations Practice

Please complete work on a separate sheet of paper!

1. $\frac{2a-3}{6}=\frac{2a}{3}+\frac{1}{2}$

2. $\frac{2b-3}{7}-\frac{b}{2}=\frac{b+3}{14}$

6. $\frac{4x}{3x-2}+\frac{2x}{3x+2}=2$

7. $\frac{5}{5-p}-\frac{p^2}{5-p}=-2$

3. $\frac{3}{5x}+\frac{7}{2x}=1$

4. $\frac{5k}{k+2}+\frac{2}{k}=5$

5. $\frac{m}{m+1}+\frac{5}{m-1}=1$

8. $\frac{2a-3}{a-3}-2=\frac{12}{a+3}$

9. $\frac{2b-5}{b-2}-2=\frac{3}{b+2}$

10. $\frac{4}{k^2-8k+12}=\frac{k}{k-2}+\frac{1}{k-6}$

Day 12: Advanced Functions Review

Warm-up:

Find the domain and range of the following functions. Then, tell how they are changed from their parent graph. (Hint: Remember that the order of transformations can be important).

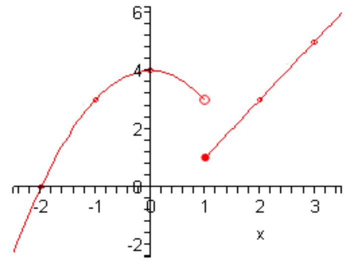
1) $f(x)=2[\![x+3]\!]-4$

2) $f(x)=\sqrt[3]{8x-16}-5$

3) $f(x)=-\sqrt{9x+54}+2$

4) $f(x)=-3|x-7|+1$

5) Write a Piecewise Function for the graph shown. Then tell its domain and range. (Hint: use graph paper!)



Complete the following Released Exam problems

6. The amount of time it takes to build a road varies inversely with the number of workers building the road. Suppose it takes 50 workers 8 months to build the road. Write an equation that could be used to determine how long it would take *n* workers to build the road. (Be sure to define the variables. How much faster would 60 workers build the road than 50 workers?

7. The force, *F*, acting on a charged object varies inversely to the square of its distance, *r* , from another charged object. When the two objects are 0.64 meters apart, the force acting on them is 8.2 Newtons. *Approximately* how much force would the object feel if it is at a distance of 0.77 meters from another object? Round to the tenths place.

Given $f(x) = x^2 - 3x + 2$, find:

8. $f(x-4)$

9. $f(x+2) - 3f(x)$