Inverse & Joint Variations

Unit 4 Day 9

Warm-Up: Released Exam Items & Practice.

Show your work to complete these problems. Do NOT just circle an answer!

1. The equation $2\sqrt{5x}$ can be used to estimate speed, s, of a car in miles per hour, given the length in feet, x, of the tire marks it leaves on the ground. A car traveling 90 miles per hour came to a sudden stop. According to the equation, how long would the tire marks be for this car?

A. 355 feet B. 380 feet C. 405 feet D. 430 feet

2. Which function is even?

A. $f(x) = (x + 2)(x – 2)$ B. $f(x) = x(x + 2)$

C. $f(x) = (x + 1)(x – 2)$ D. $f(x) = (x – 1)(x – 1)$

3. A marathon is roughly 26.2 miles long. Which equation could be used to determine the time, t, it takes to run a marathon as a function of the average speed, s, of the runner where t is in hours and s is in miles per hour?

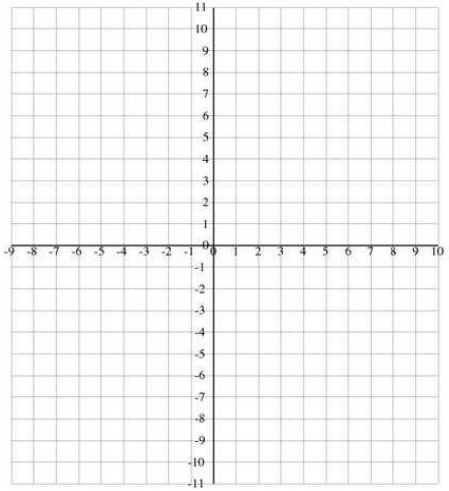
A. $t = 26.2 – 26.2\ s$ B. $t = 26.2 – s\ /26.2$

C. $t = 26.2\ s$ D. $t = 26.2\ /\ s$

Warm – Up Continues →

Warm-Up Continues….Practice Graphing Inverse Variation.
Do a table for each branch and completely graph the function!
Also, indicate the horizontal and vertical asymptotes, domain, and range for each function.

4. $y = \frac{1}{x}$

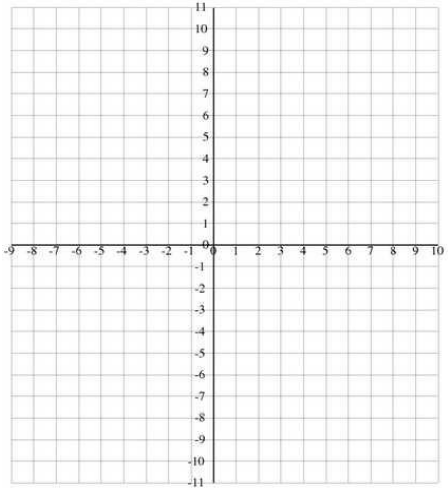


x	y
¼	4
½	2
1	1
2	½
4	¼

x	y
-¼	-4
-½	-2
-1	-1
-2	-½
-4	-¼

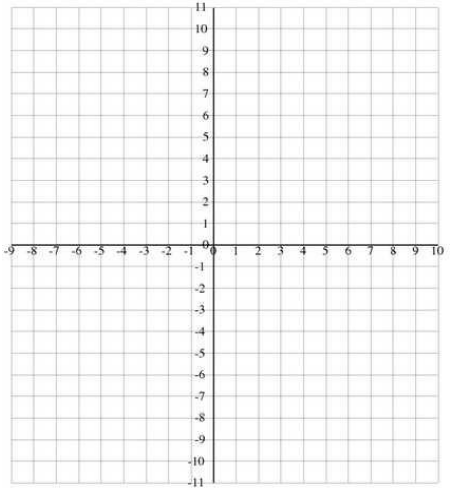
Looks familiar, right?

5. $y = \frac{2}{x}$



HINT: You may want to try these by hand… good test preparation! ☺

6. $y = \frac{-5}{x}$



Warm-Up: Released Exam Items & Practice.

Show your work to complete these problems. Do NOT just circle an answer!

1. The equation can be used to estimate speed, s, of a car in miles per hour, given the length in feet, x, of the tire marks it leaves on the ground. A car traveling 90 miles per hour came to a sudden stop. According to the equation, how long would the tire marks be for this car?

A. 355 feet B. 380 feet C. 405 feet D. 430 feet

2. Which function is even?

A. $f(x) = (x + 2)(x - 2)$ B. $f(x) = x(x + 2)$

C. $f(x) = (x + 1)(x - 2)$ D. $f(x) = (x - 1)(x - 1)$

3. A marathon is roughly 26.2 miles long. Which equation could be used to determine the time, t, it takes to run a marathon as a function of the average speed, s, of the runner where t is in hours and s is in miles per hour?

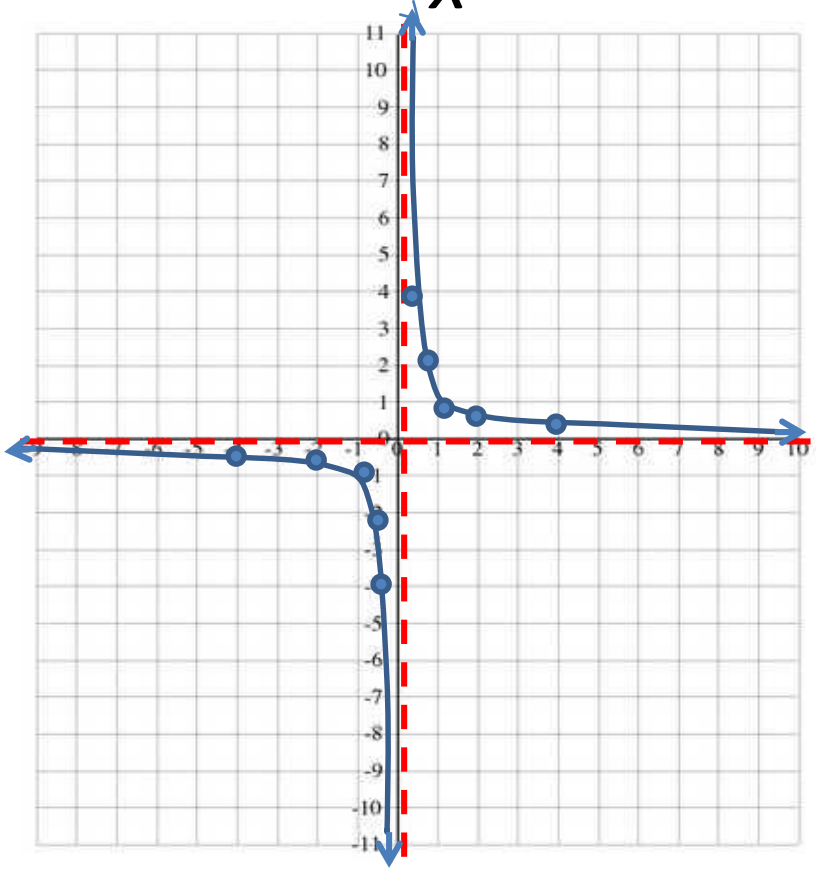
A. $t = 26.2 - 26.2\ s$ B. $t = 26.2 - s\ /26.2$

C. $t = 26.2\ s$ D. $t = 26.2\ /\ s$

Warm – Up Continues →

4. Graphing Inverse Variation…

$$y = \frac{1}{x}$$



Make a table of values for each branch!

x	y
¼	4
½	2
1	1
2	½
4	¼

x	y
-¼	-4
-½	-2
-1	-1
-2	-½
-4	-¼

Looks familiar, right?

HA: $y = 0$

VA: $x = 0$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Plot at least 5 exact points per branch!

Graphing Inverse Variation.....

5. $y = \frac{2}{x}$

HA: y = 0

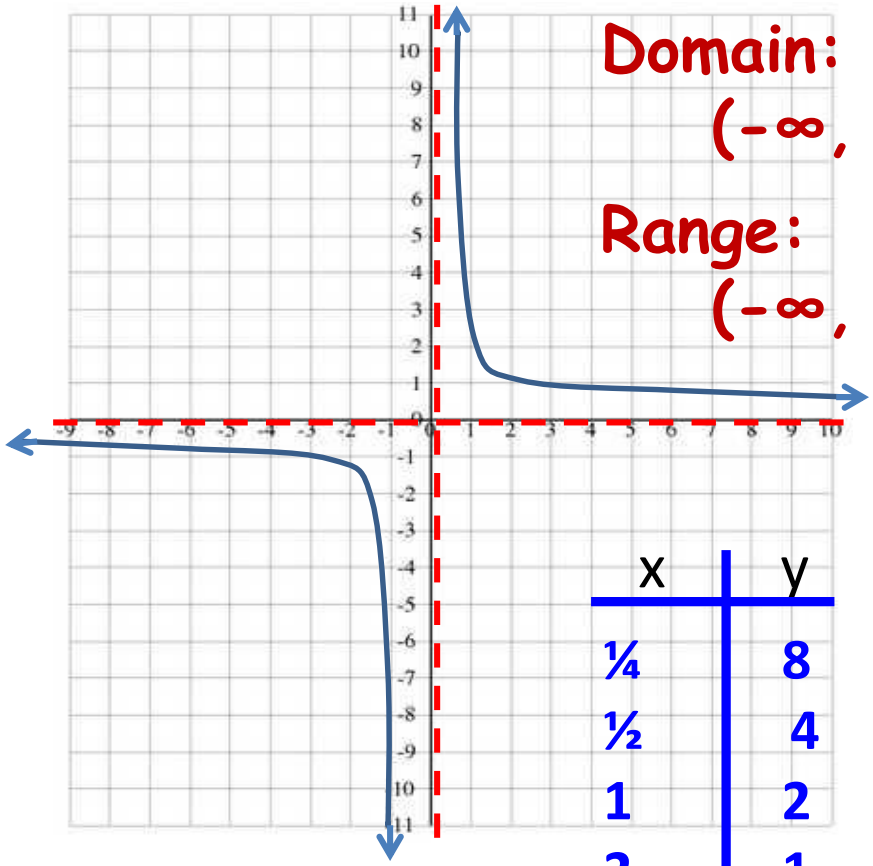
VA: x = 0

Domain:

(-∞, 0) U (0, ∞)

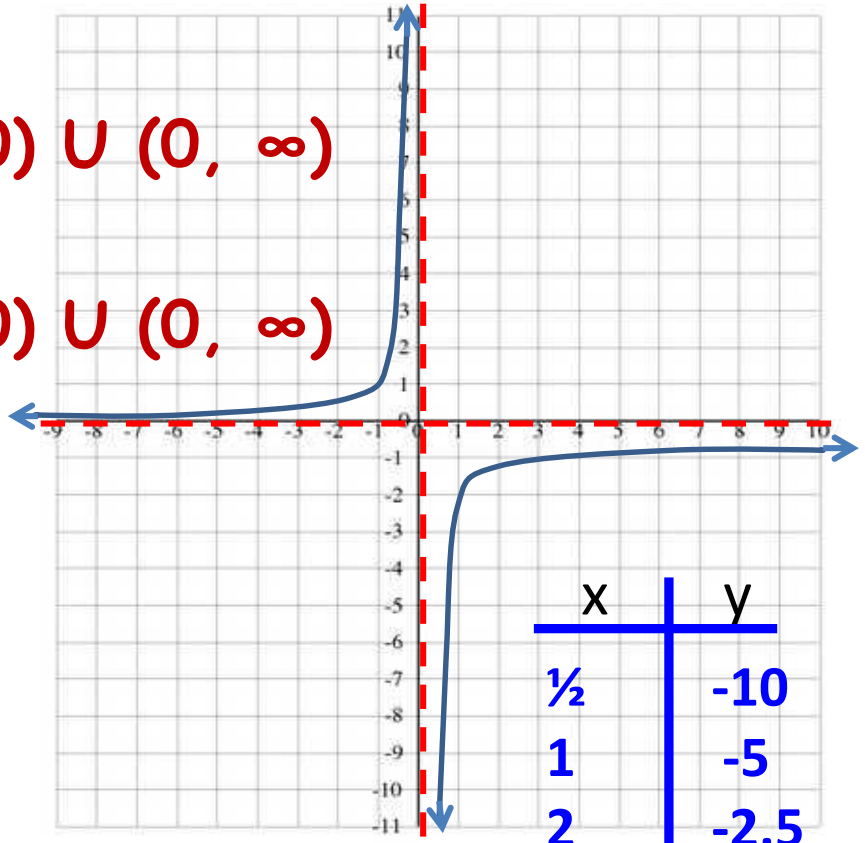
Range:

(-∞, 0) U (0, ∞)



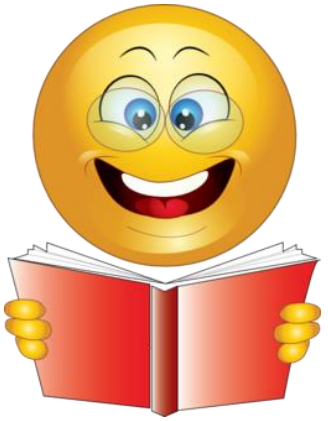
x	y
¼	8
½	4
1	2
2	1
4	½

6. $y = \frac{-5}{x}$



x	y
½	-10
1	-5
2	-2.5
5	-1
10	-½

Tonight's Homework
Packet p. 13 – 15



Remember:

Quiz Tomorrow!

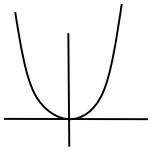
We also have a lesson, so tonight

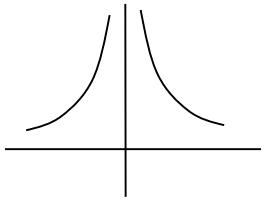
***check HW online AND**

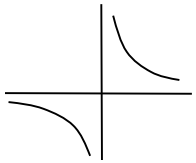
***work the warm-up in the notes**

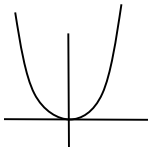
Homework Answers

1. $y = 7.25 \bullet x^{1}$ Direct because x^{1} 6.25 miles
2. $y = 144 \bullet x^{-1}$ Indirect because x^{-1} 2.4 people
   OR $y = 144/x$
3. $y = 9 \bullet x^{1}$ Direct because x^{1} 111.1 hours

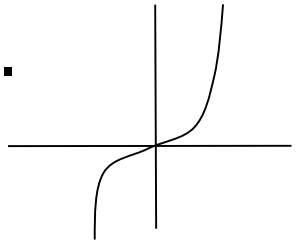
4. 
   $y = 5x^{4}$ Symmetry over y-axis Even Function
   Most similar to $y = x^{2}$ because positive even exponent

5. 
   $y = 3x^{-6}$ Symmetry over y-axis Even Function
   Most similar to $y = x^{-2}$ because negative even exponent

6. 
   $y = ½ x^{-5}$ Symmetry about origin Odd Function
   Most similar to $y = x^{-1}$ because negative odd exponent

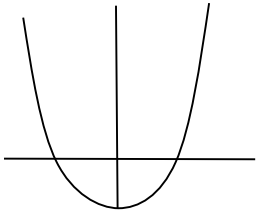
7. 
   $y = 1/6x^{8}$ Symmetry over y-axis Even Function
   Most similar to $y = x^{2}$ because positive even exponent

Homework Answers

8. 

$y = 8x^9$ Symmetry about the origin Even Function

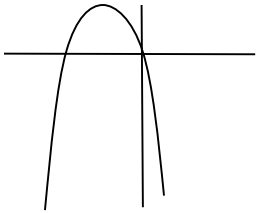
Most similar to $y = x^3$ because positive odd exponent

9. 

$y = 4(x - 3)(x + 3)$ **Note: This is a quadratic with roots at x = 3, -3 NOT a power function**

Symmetry over y-axis Even Function

Most similar to $y = x^2$ because positive even exponent

10. 

$y = -2x(x + 3)$ **Note: This is a quadratic with roots at x = 0, -3 NOT a power function**

No Symmetry Neither even nor odd Function

Most similar to $y = x^2$ because positive even exponent

Notes Day 9

Inverse & Joint Variations

Inverse Variations

A relationship that can be written in the form $y = \frac{k}{x}$, where

k is a nonzero constant and $x \neq 0$, is an **inverse variation**. The constant k is the constant of variation.

Multiplying both sides of by x gives **y•x = k**.
So, the product of x and y in an inverse variation is **k , the constant**.

Inverse Variations

WORDS	NUMBERS	ALGEBRA
y varies inversely as x.	$y = \frac{3}{x}$	$y = \frac{k}{x}$
y is inversely proportional to x.	$xy = 3$	$xy = k (k \neq 0)$

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether *xy* is a constant for each ordered pair.

Example: Tell whether the relationship is an inverse variation. Explain. If it is an inverse variation, write the equation.

1.

x	y
1	30
2	15
3	10

xy
30
30
30

$y = \frac{30}{x}$

Yes, inverse variation

xy = 30 = k (constant)

2.

x	y
1	5
2	10
4	20

xy
5
20
80

NOT inverse variation

xy ≠ k (constant)

3. 2xy = 28

Yes, inverse variation

xy = 14 = k (constant)

$y = \frac{14}{x}$

You Try!! **Remember:** You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether *xy* is a constant for each ordered pair.

Tell whether the relationship is an inverse variation. Explain. If it is an inverse variation, write the equation.

4.

x	y	xy
-12	24	-288
1	-2	-2
8	-16	-128

NOT inverse variation

xy ≠ k (constant)

5.

x	y	xy
3	3	9
9	1	9
18	0.5	9

Yes, inverse variation

xy = 9 = k (constant)

$y = \frac{9}{x}$

6. 2x + y = 10

$y = 10 - 2X$

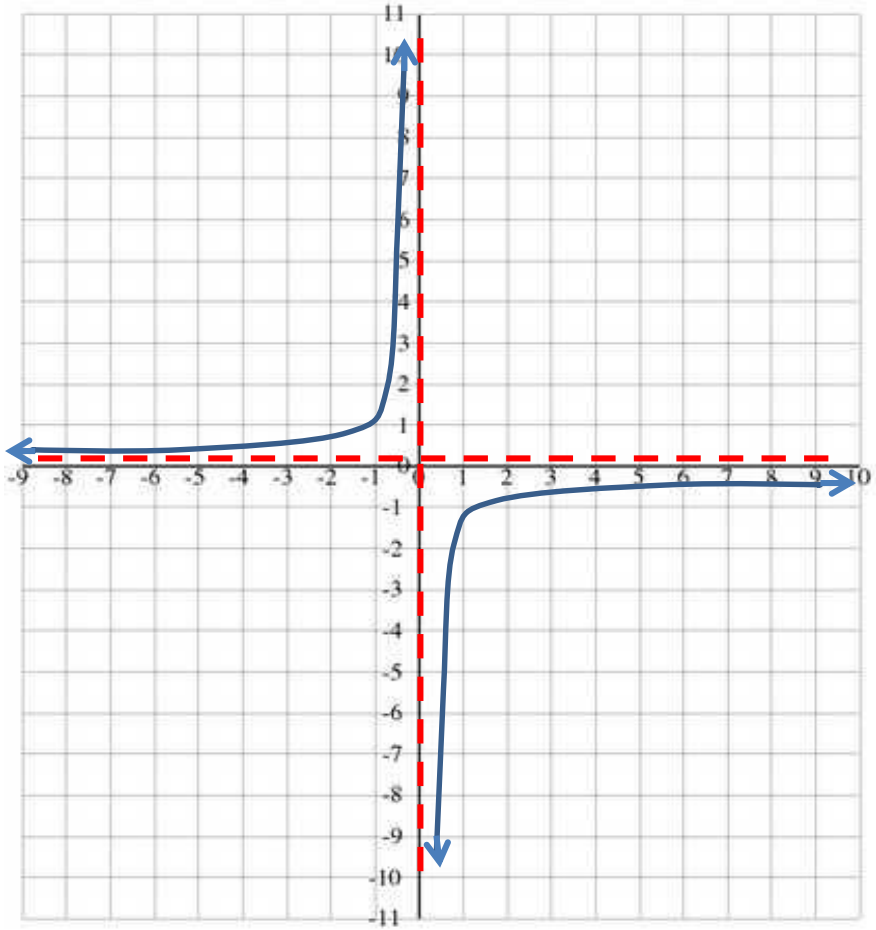
x	y	xy
1	8	8
2	6	12
3	4	12
5	0	0

NOT inverse variation

xy ≠ k (constant)

Examples:

Write and graph the inverse variation in which $y = 0.5$ when $x = -12$.



Steps:

1. **Find K. (k = xy)**
2. **Write new equation as y =k/x**
3. **Graph on grid.**

$$y = \frac{k}{x}$$

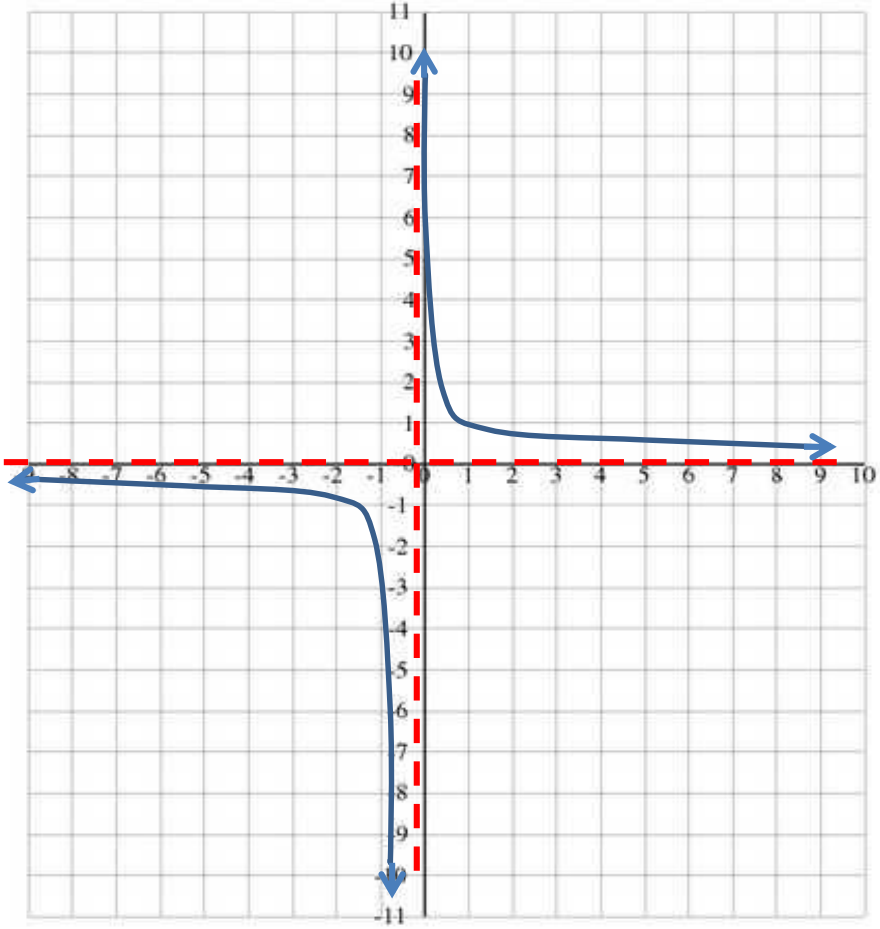
$$\frac{0.5}{1} = \frac{k}{-12}$$

$$k = -6$$

$$y = \frac{-6}{x}$$

You Try!

Write and graph the inverse variation in which y =1/2 when *x* = 10



Steps:

1. **Find K. (k = xy)**
2. **Write new equation as y =k/x**
3. **Graph on grid.**

$$y = \frac{k}{x}$$

$$\frac{0.5}{1} = \frac{k}{10}$$

$$k = 5$$

$$y = \frac{5}{x}$$

Examples

The inverse variation $xy = 350$ relates the constant speed x in mi/h to the time y in hours that it takes to travel 350 miles. Determine a reasonable domain and range and then graph this inverse variation.

$$y = \frac{350}{x}$$

Practical Domain: $(0, \infty)$

Practical Range: $(0, \infty)$

x	y
1	**350**
2	**175**
5	**70**
10	**35**
50	**7**
100	**3.5**

You Try!

The inverse variation $xy = 100$ represents the relationship between the pressure x in atmospheres (atm) and the volume y in mm^3 of a certain gas. Determine a reasonable domain and range and then graph this inverse variation.

$$y = \frac{100}{x}$$

Practical Domain: $(0, \infty)$

Practical Range: $(0, \infty)$

x	y
1	**100**
2	**50**
5	**20**
10	**10**
25	**4**
50	**2**

Product Rule for Inverse Variation

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

Examples

5. Let $x_1 = 5$, $x_2 = 3$, and $y_2 = 10$. Let y vary inversely as x. Find y_1.

$$x_1y_1 = x_2y_2$$

So… xy = k every time!

$$5 \cdot y_1 = 3 \cdot 10$$

$$5y_1 = 30$$

$$y_1 = 6$$

Product Rule for Inverse Variation

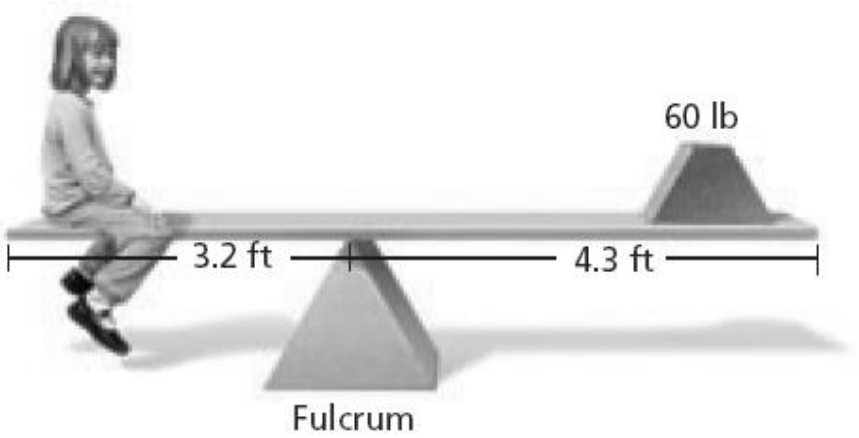
If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

You Try!

6. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y vary inversely as x. Find y_2.

7. Boyle's law states that the pressure of a quantity of gas *x* varies inversely as the volume of the gas *y*. The volume of gas inside a container is 400 in^3 and the pressure is 25 psi. What is the pressure when the volume is compressed to 125 in^3?

8. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight.
The diagram shows a balanced lever. How much does the child weigh?



ANSWERS!

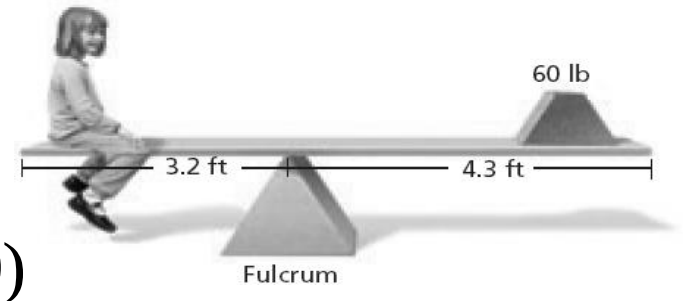
6. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y vary inversely as x. Find y_2.

$$x_1 y_1 = x_2 y_2 \qquad 2 \bullet -6 = -4 \bullet y_2$$
$$-12 = -4y_2$$
$$y_2 = 3$$

7. Boyle's law states that the pressure of a quantity of gas *x* varies inversely as the volume of the gas *y*. The volume of gas inside a container is 400 in^3 and the pressure is 25 psi. What is the pressure when the volume is compressed to 125 in^3?

$$x_1 y_1 = x_2 y_2 \qquad 400 \bullet 25 = 125 \bullet y_2$$
$$10000 = 125y_2$$
$$y_2 = 80 \text{ psi}$$

8. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight.
The diagram shows a balanced lever. How much does the child weigh?



$$3.2 \bullet y_1 = 4.3(60)$$
$$x_1 y_1 = x_2 y_2 \qquad 3.2y_1 = 258$$
$$y_1 = 80.625 \; lbs$$

Joint Variation

Occurs when 1 quantity varies directly as the **product** of 2 or more other quantities.

Form **$z = k x y$**, $x \neq 0$, $z \neq 0$

Ex: The area of a trapezoid varies jointly as the height h and the sum of its bases b_1 and b_2. Find the equation of joint variation if A = 48 in^2, h= 8 in, b_1 = 5in, and b_2 = 7 in.

$A = k\, h\, (b_1 + b_2)$

$48 = k\,(8)\,(5 + 7)$

$48 = k\,(96)$

$\frac{1}{2} = k$

$A = \frac{1}{2}\, h\, (b_1 + b_2)$

Write an equation for the following…

- y varies directly with x and inversely with z^2. $y = \frac{k\,x}{z^2}$
- y varies inversely with x^3. $y = \frac{k}{x^3}$
- y varies directly with x^2 and inversely with z. $y = \frac{k\,x^2}{z}$
- z varies jointly with x^2 and y. $z = kx^2y$
- y varies inversely with x and z. $y = \frac{k}{xz}$

You Try!

Tell whether x and y show direct variation, inverse variation, or neither.

1.) $xy = 1/4$ **Inverse** **k = 1/4**

2.) $2x + y = 4$ **Neither**

3.) $\frac{y}{x} = 12$ **Direct** **k = 12**

4.) $y = \frac{1}{x}$ **Inverse** **k = 1**

Write the function that models each relationship. Find z when x = 6 and y = 4.

5. z varies jointly with x and y. When x = 7 and y = 2, z = 28.

z = k x y

28 = k (7) (2)

2 = k

z = 2 x y

z = 2 (6) (4)

z = 48

6. z varies directly with x and inversely with the cube of y. When x = 8 and y = 2, z = 3.

$z = \frac{k\,x}{y^3}$ **$3 = \frac{k\,(8)}{(2)^3}$** **3 = k**

$z = \frac{3\,x}{y^3}$

$z = \frac{3\,(6)}{(4)^3}$

$z = \frac{9}{32}$

Word Problems

1. The speed of the current in a whirlpool varies inversely with the distance from the whirlpool's center. The Lofoten Maelstrom is a whirlpool located off the coast of Norway. At a distance of 3000 meters from the center, the speed of the current is about 0.1 meters per second.
   a. Find the equation for this scenario.

$s = \frac{k}{d}$ $\quad 0.1 = \frac{k}{3000}$ $\quad 300 = k$ $\quad \boxed{s = \frac{300}{d}}$

   b. What's the speed of the whirlpool when 50 meters from the center?

$s = \frac{300}{50}$ $\quad \boxed{s = 6 \text{ meters/sec}}$

Word Problems You Try!!

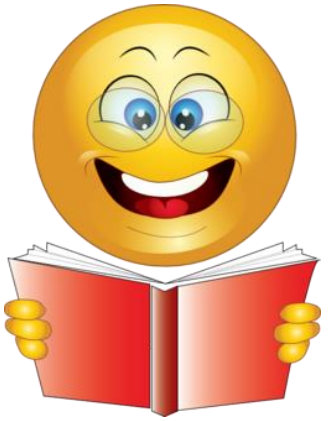
2. In building a brick wall, the amount of time it takes to complete the wall varies directly with the number of bricks in the wall and varies inversely with the number of bricklayers that are working together. A wall containing 1200 bricks, using 3 bricklayers, takes 18 hours to build. How long would it take to build a wall of 4500 bricks if 5 bricklayers worked on it?

$t = \frac{k\,b}{p}$ where t = time, b = # bricks, p = # people working

$18 = \frac{k\,(1200)}{3}$ **$0.045 = k$** **$t = \frac{0.045\,b}{p}$**

$t = \frac{0.045\,(4500)}{5}$ **t = 40.5 hours**

Tonight's Homework
Packet p. 13 – 15



Remember:

Quiz Tomorrow!

We also have a lesson, so tonight

***check HW online AND**

***work the warm-up in the notes**