Day 8: Modeling Advanced Functions

Warm-Up:

Released Exam Items. Show your work to complete these problems. Do NOT just circle an answer!

1. The equation $s = 2\sqrt{5x}$ can be used to estimate speed, s, of a car in miles per hour, given the length in feet, x, of the tire marks it leaves on the ground. A car traveling 90 miles per hour came to a sudden stop. According to the equation, how long would the tire marks be for this car? → $s = 90$

$90 = 2\sqrt{5x}$ $\frac{90}{2} = \sqrt{5x}$ $45 = \sqrt{5x}$ $(45)^2 = (\sqrt{5x})^2$

A. 355 feet B. 380 feet C. 405 feet D. 430 feet

$\frac{2025}{5} = \frac{5x}{5}$

$405 = x$ ft

2. Which function is even? *Even functions are symmetric across the y-axis

A. $f(x) = (x + 2)(x - 2)$ has roots $x = -2, 2$

B. $f(x) = x(x + 2)$

C. $f(x) = (x + 1)(x - 2)$

D. $f(x) = (x - 1)(x - 1)$

3. A marathon is roughly 26.2 miles long. Which equation could be used to determine the time, t, it takes to run a marathon as a function of the average speed, s, of the runner where t is in hours and s is in miles per hour?

distance = rate • time

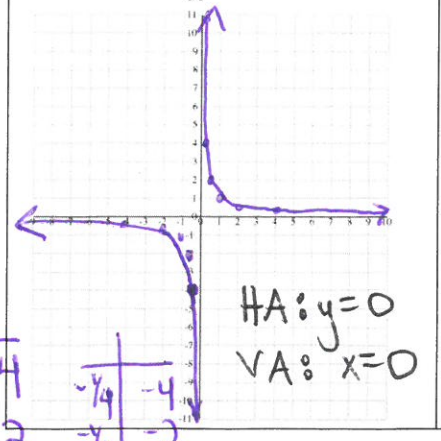
$\frac{26.2}{s} = \frac{s\,t}{s}$ $\frac{26.2}{s} = t$

A. $t = 26.2 - 26.2\,s$ B. $t = 26.2 - s/26.2$

C. $t = 26.2\,s$ D. $t = 26.2 / s$

Practice Graphing Inverse Variation… Do a table for each branch and completely graph the function!

4. $y = \frac{1}{x}$

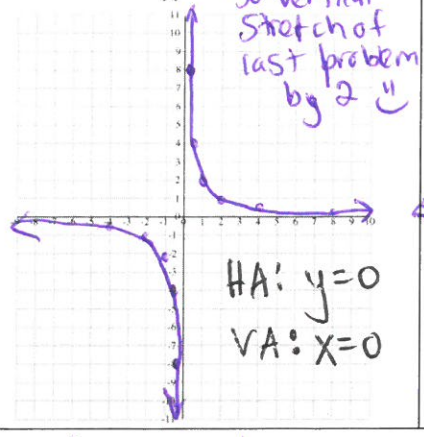


HA: $y = 0$

VA: $x = 0$

1/4	4	-1/4	-4
1/2	2	-1/2	-2
1	1	-1	-1
2	1/2	-2	-1/2
4	1/4	-4	-1/4

5. $y = \frac{2}{x}$ Note: $y = 2 \cdot \frac{1}{x}$ So vertical stretch of last problem by 2 "

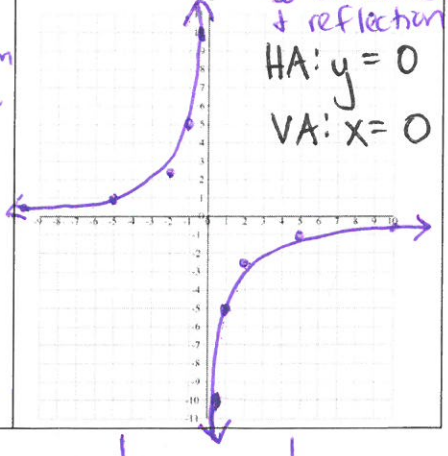


HA: $y = 0$

VA: $x = 0$

1/4	8	-1/4	-8
1/2	4	-1/2	-4
1	2	-1	-2
2	1	-2	-1
4	1/2	-4	-1/2

6. $y = \frac{-5}{x}$ Note: $y = -5 \cdot \frac{1}{x}$ So vertical stretch by 5 & reflection over x-axis versus $y = \frac{1}{x}$



HA: $y = 0$

VA: $x = 0$

-1/2	10	1/2	-10
-1	5	1	-5
-2	2.5	2	-2.5
-5	1	5	-1
-10	1/2	10	-1/2

Notes: Inverse Variation

A relationship that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$, is an **inverse variation**. The constant k is the constant of variation.

Multiplying both sides of $y = \frac{k}{x}$ by x gives $y \cdot x = k$. So, the product of x and y in an inverse variation is k (a constant).

Inverse Variations

WORDS	NUMBERS	ALGEBRA
y varies inversely as x.	$y = \frac{3}{x}$	$y = \frac{k}{x}$
y is inversely proportional to x.	$xy = 3$	$xy = k (k \neq 0)$

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether xy is a constant for each ordered pair.

Example: Tell whether the relationship is an inverse variation. Explain. If it is an inverse variation, write the equation.

1.

x	y	xy
1	30	30
2	15	30
3	10	30

$k = 30$, a constant

$y = \frac{30}{x}$

Yes. Inverse because $xy = k$, a constant of 30

2.

x	y	xy
1	5	5
2	10	20
4	20	80

Not inverse because $xy \neq k$, a constant

$y = 5x$ direct → review from Math 1

3. $\frac{2xy}{2} = \frac{28}{2}$

$xy = 14 = k$

$y = \frac{14}{x}$

Yes. Inverse because $xy = k = 14$, a constant

4.

x	y	xy
-12	24	-288
1	-2	-2
8	-16	-128

You Try

Not inverse variation because $xy \neq k$, a constant

It's Direct $y = -2x$ from Math 1

5.

x	y	xy
3	3	9
9	1	9
18	0.5	9

You Try

$xy = k = 9$

$y = \frac{9}{x}$

Yes inverse variation because $xy = k$, a constant

6. $2x + y = 10$

x	$y = 10 - 2x$	xy
1	$8 = 10 - 2(1)$	8
2	$6 = 10 - 2(2)$	12
3	$4 = 10 - 2(3)$	12
5	$0 = 10 - 2(5)$	0

You Try

Not inverse variation because $xy \neq$ constant

Linear $y = -2x + 10$ from Math 1

Examples:

1. Write and graph the inverse variation in which $y = 0.5$ when $x = -12$.

Steps
① Find K ($K = xy$)
② Write new equation as $y = \frac{K}{x}$
③ Graph on grid

$K = xy$
$K = (-12)(.5)$
$K = -6$
$y = \frac{-6}{x}$

x	y
-1	6
-2	3
-3	2
-6	1
-12	½

x	y
1	-6
2	-3
3	-2
6	-1
12	-½

VA: $x = 0$
HA: $y = 0$
$y = \frac{-6}{x}$
HA: $y = 0$
VA: $x = 0$
because can't ÷ by 0
D & R $(-\infty, 0) \cup (0, \infty)$

You Try

2. Write and graph the inverse variation in which $y = 1/2$ when $x = 10$

① Find K
$K = xy$
$K = 10(\frac{1}{2})$
$K = 5$
② Write equation
$y = \frac{5}{x}$
③ graph

x	y
1	5
2	2.5
5	1
10	½

x	y
-1	-5
-2	-2.5
-5	-1
-10	-½

HA $y = 0$
VA $x = 0$
D & R $(-\infty, 0) \cup (0, \infty)$

3. The inverse variation $xy = 350$ relates the constant speed x in mi/h to the time y in hours that it takes to travel 350 miles. Determine a reasonable domain and range and then graph this inverse variation.

$y = \frac{350}{x}$

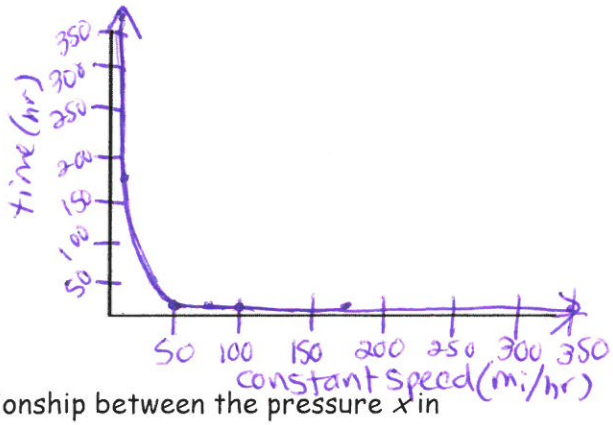
Practical Domain: $x > 0$ $(0, \infty)$

Practical Range: $y > 0$ $(0, \infty)$

x	y
100	3.5
50	7
70	5
350	1
175	2

x	y
-100	-3.5
-50	-7
-70	-5
-350	-1
-175	-2

no negative speed or time



4. The inverse variation $xy = 100$ represents the relationship between the pressure x in atmospheres (atm) and the volume y in mm^3 of a certain gas. Determine a reasonable domain and range and then graph this inverse variation.

You Try

Practical Domain: $x > 0$ $(0, \infty)$

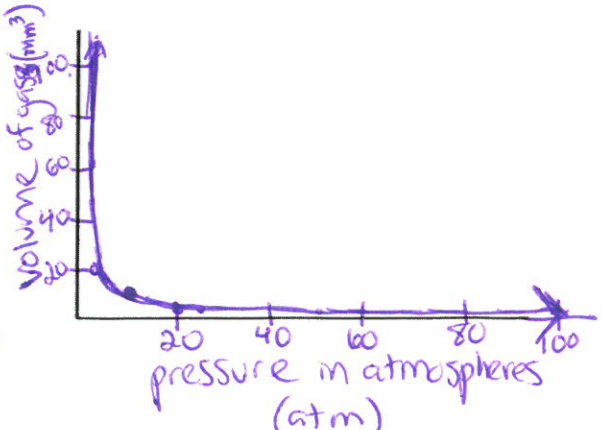
Practical Range: $y > 0$ $(0, \infty)$

$y = \frac{100}{x}$

x	y
10	10
20	5
25	4
50	2
100	1
200	½

x	y
-10	-10
-20	-5
-25	-4
-50	-2
-100	-1
-200	-½

no negative speed or time



Product Rule for Inverse Variation

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

↳ Remember $xy = k$ so $x_1y_1 = k$ and $x_2y_2 = k$ therefore substituting $x_1y_1 = x_2y_2$

Examples:

5. Let $x_1 = 5$, $x_2 = 3$, and $y_2 = 10$. Let y vary inversely as x. Find y_1.

Together 1st

$x_1y_1 = x_2y_2$

$5y_1 = (3)(10)$

$\frac{5y_1}{5} = \frac{30}{5}$ → $y_1 = 6$

↳ means $xy = k$ every time

6. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y vary inversely as x. Find y_2.

3rd You Try (after #7 together)

↳ $xy = k$

$x_1y_1 = x_2y_2$

$\frac{(2)(-6)}{-4} = \frac{(-4)y_2}{-4}$ → $y_2 = 3$

7. Boyle's law states that the pressure of a quantity of gas x varies inversely as the volume of the gas y. The volume of gas inside a container is 400 in^3 and the pressure is 25 psi. What is the pressure when the volume is compressed to 125 in^3?

Together 2nd

need to find y_2

y_2 y_1 x_1

$x_1y_1 = x_2y_2$

$\frac{(400)(25)}{125} = \frac{125(x_2)}{125}$

$x_2 = 80$ psi = pressure when volume is 125 in^3

4th You try (after #6)

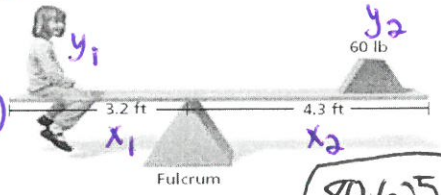
8. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight. The diagram shows a balanced lever. How much does the child weigh?

$x_1y_1 = x_2y_2$

$(3.2)y_1 = (4.3)(60)$

$\frac{3.2y_1}{3.2} = \frac{258}{3.2}$

$y_1 = 80.625$



80.625 lbs = weight of child

Notes: Joint Variation

Joint Variation

- Occurs when 1 quantity varies directly as the product of 2 or more other quantities.
- Form $z = kxy$, $x \neq 0$, $z \neq 0$

z x y

Ex: The area of a trapezoid varies jointly as the height h and the sum of its bases b_1 and b_2. Find the equation of joint variation if A = 48 in, h = 8 in, b_1 = 5in, and b_2 = 7 in.

*Hint: part before "varies" is the part off by itself in the equation

$z = kxy$

$A = kh(b_1 + b_2)$

$48 = k(8)(5+7)$

$48 = k(96)$

$\frac{48}{96} = \frac{k(96)}{96}$

$k = \frac{1}{2}$

$A = \frac{1}{2}h(b_1 + b_2)$

Reminders ⟹ Direct $y=kx$ Joint $z=kxy$ Inverse $y=\frac{k}{x}$

Write an equation for the following...

- y varies directly with x and inversely with z^2. $y=\frac{kx}{z^2}$
- y varies inversely with x^3. $y=\frac{k}{x^3}$
- y varies directly with x^2 and inversely with z. $y=\frac{kx^2}{z}$
- z varies jointly with x^2 and y. $z=kx^2y$
- y varies inversely with x and z. $y=\frac{k}{xz}$

Practice: Tell whether x and y show direct variation, inverse variation, or neither.

1.) $xy=\frac{1}{4}$ $y=\frac{1/4}{x}$ Inverse $k=\frac{1}{4}$

2.) $2x+y=4$ Neither linear $y=-2x+4$

3.) $\frac{y}{x}=12$ $y=12x$ Direct $k=12$

4.) $y=\frac{1}{x}$ Inverse $k=1$

It's like a part (a) in each problem → Write the function that models each relationship. Find z when x = 6 and y = 4. (It's like part (b) in each problem → answer overall question; x=6, y=4)

5. z varies jointly with x and y. When x = 7 and y = 2, z = 28. (a) find equation

(a) $z=kxy$; $28=k(7)(2)$; $\frac{28}{14}=\frac{14k}{14}$; $k=2$; $z=2xy$

(b) $z=2(6)(4)$; $z=48$

6. z varies directly with x and inversely with the cube of y. When x = 8 and y = 2, z = 3 (a) $z=\frac{kx}{y^3}$; $3=\frac{k(8)}{(2)^3}$; $k=3$; $z=\frac{3x}{y^3}$

(b) $z=\frac{3(6)}{4^3}$; $z=\frac{9}{32}$

Word Problems:

2nd Try

1. The speed of the current in a whirlpool varies inversely with the distance from the whirlpool's center. The Lofoten Maelstrom is a whirlpool located off the coast of Norway. At a distance of 3000 meters from the center, the speed of the current is about 0.1 meters per second.

a. Find the equation for this scenario. $S=\frac{k}{d}$ s = speed, d = distance; $0.1=\frac{k}{3000}$; $k=300$; $S=\frac{300}{d}$

b. What's the speed of the whirlpool when 50 meters from the center? $d=50$; $S=\frac{300}{50}$; $S=6\ \frac{\text{meters}}{\text{sec}}$

1st Together

2. In building a brick wall, the amount of time it takes to complete the wall varies directly with the number of bricks in the wall and varies inversely with the number of bricklayers that are working together. A wall containing 1200 bricks, using 3 bricklayers, takes 18 hours to build. How long would it take to build a wall of 4500 bricks if 5 bricklayers worked on it? (b) answer overall question

(a) find equation for the scenario: $t=\frac{kb}{p}$; t = time, b = # bricks, p = # people; $18=\frac{k(1200)}{3}$; $k=.045$; $t=\frac{.045b}{p}$

(b) $t=\frac{.045(4500)}{5}$; 40.5 hours