Day 9A: Solving Rational Equations

Warm-Up:

1. Simplify: $\frac{5}{12} - \frac{1}{12} = \frac{4}{12} = \boxed{\frac{1}{3}}$

3. Simplify: $\frac{4}{5} + \frac{1}{7} = \frac{4 \cdot 7}{5 \cdot 7} + \frac{1 \cdot 5}{7 \cdot 5}$

   LCD = 35 $\frac{28}{35} + \frac{5}{35} \boxed{\frac{33}{35}}$

2. Simplify: $\frac{6}{4} - \frac{3}{7} = \frac{6 \cdot 7}{4 \cdot 7} - \frac{3 \cdot 4}{7 \cdot 4}$

   LCD = 28 $\frac{42}{28} - \frac{12}{28} = \frac{30}{28} \boxed{\frac{15}{14}}$

4. Simplify: $\frac{2}{3} + \frac{5}{6}$ $\frac{2 \cdot 2}{3 \cdot 2} + \frac{5}{6}$

   LCD = 6 $\frac{4}{6} + \frac{5}{6} = \frac{9}{6} = \boxed{\frac{3}{2}}$

Day 9A: Solving Rational Equations

A __rational equation__ is an equation that contains one or more rational expressions. It can have a variable in the numerator and/or the denominator. Our goal when solving a rational equation is to eliminate the fractions and solve the equation for the variable!

Recall that when you graph a rational function, there is a vertical asymptote. This is an x-value that the graph *approaches* but NEVER touches. When you solve rational equations, there are some values for x that must be excluded from the domain because they will make the denominator equal to zero, and dividing by zero is undefined. Any number that causes the denominator to equal zero is called an __excluded value__. To find the excluded values, set the denominator equal to zero and solve for the variable; the solutions are the excluded values. When solving rational equations, if **all solutions of the rational equation are excluded values** then there is **no solution** to the rational equation!

To solve __simple__ rational equations, the cross product property can be utilized to eliminate the fraction leaving a linear equation to solve. **REMEMBER:** Check your final answers to make sure they are not an excluded value!

(margin note:) Excluded values are x-values that CANNOT be part of the solution because they would give division by zero!!

Examples: Using the cross product property, solve the following equations. Do not forget to determine the excluded values.

1 fraction = 1 fraction so cross multiply!!

1. $\frac{6}{x} = \frac{3}{7}$ EV: $x \neq 0$ *(solve $x \neq 0$ to get EV)*

   $6 \cdot 7 = 3 \cdot x$

   $\frac{42}{3} = \frac{3x}{3}$ $\boxed{x = 14}$ ✓ not an EValue Excluded

2. $\frac{4}{x-7} = \frac{6}{x}$ EV: $x \neq 0, 7$ *(Solve $x-7 \neq 0$ and $x \neq 0$ to get EV)*

   $4 \cdot x = 6(x - 7)$ *(#use parentheses if more than 1 piece in numerator or denominator)*

   $4x = 6x - 42$ $\boxed{x = 21}$

   $-6x \quad -6x$ ✓ not an EV

   $-2x = -42$

3. $\frac{-5}{x+4} = \frac{1}{x+4}$ EV: $x \neq -4$ *(Solve $x+4 \neq 0$ to get EV)*

   $-5(x+4) = 1(x+4)$

   $-5x - 20 = x + 4$ $\frac{-6x}{-6} = \frac{24}{-6}$

   $-x \quad +20 \quad -x \quad +20$ $x = -4$ but EV so $\boxed{\text{No Solution}}$

4. $\frac{6}{x+5} = \frac{x}{6}$ EV: $x \neq -5$ *(Solve $x+5 \neq 0$ to get EV)*

   (#quadratic so set = 0 then factor)

   $6 \cdot 6 = x(x+5)$ $0 = (x+9)(x-4)$

   $36 = x^2 + 5x - 36$ $x+9=0 \quad x-4=0$

   -36 $\boxed{x = -9, 4}$ ✓ not an EV

   $0 = x^2 + 5x - 36$

   (watch for x^2... then it's a quadratic)

→ Be sure to multiply the LCD to ALL the parts on BOTH sides!!

Examples: Multiply through by the LCD to solve the following equations. Do not forget to determine the excluded values.

Need to use LCD because constant added onto fraction... so can't just cross multiply!

5. $\frac{2}{x} - 3 = \frac{8}{x}$ EV: $x \neq 0$

LCD: x

$\frac{2 \cdot x}{x} - 3 \cdot x = \frac{8 \cdot x}{x}$

$2 - 3x = 8$

$\frac{-3x = 6}{-3 \quad -3}$

$x = -2$ ✓ not an EV

You Try!

6. $\frac{7x}{x-3} + 4 = \frac{x+1}{x-3}$ EV: $x \neq 3$

LCD: $x-3$

$\frac{7x \cdot (x-3)}{x-3} + 4(x-3) = \frac{x+1(x-3)}{x-3}$

$7x + 4(x-3) = x+1$

$7x + 4x - 12 = x+1$

$11x - 12 = x+1$
$-x \quad +12 \quad -x \quad +12$

$10x = 13$

$x = \frac{13}{10}$ ✓ not an EV

Examples: Solve the rational equation. Do not forget to determine the excluded values.

1 fraction = 1 fraction so no LCD is needed!!

7. $\frac{8}{x+8} = \frac{x}{x+2}$ EV: $x \neq -8, -2$

$8(x+2) = x(x+8)$

$8x + 16 = x^2 + 8x$
$-8x - 16 \qquad -8x - 16$

$0 = x^2 - 16$

quadratic so set = 0 then factor

$0 = (x-4)(x+4)$

$x = 4, -4$

$x^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

8. $\frac{4}{x+2} + 3 = \frac{9}{x+2}$ EV: $x \neq -2$

LCD needed because LCD = $x+2$

$\frac{4 \cdot (x+2)}{x+2} + 3(x+2) = \frac{9(x+2)}{x+2}$

$4 + 3(x+2) = 9$

$4 + 3x + 6 = 9$

$3x + 10 = 9$

$3x = -1$

$x = -\frac{1}{3}$

9. $\frac{3x}{x-1} - 2 = \frac{10}{x-1}$ EV: $x \neq 1$

LCD: $x-1$ needed because

$\frac{3x(x-1)}{x-1} - 2(x-1) = \frac{10(x-1)}{x-1}$

$3x - 2x + 2 = 10$

$x + 2 = 10$

$x = 8$

10. $\frac{12}{x+2} = \frac{7}{x-3}$ EV: $x \neq -2, 3$

$\frac{1 \text{ fraction}}{} = \frac{1 \text{ fraction}}{}$ so NO LCD is needed!!

$12(x-3) = 7(x+2)$

$12x - 36 = 7x + 14$
$-7x \quad +36 \quad -7x \quad +36$

$5x = 50$ $x = 10$

Day 10: Solving Rational Equations Practice

Solve the rational equation. Do not forget to determine the excluded values.

1. $\frac{3}{x} = \frac{2}{x+4}$ EV: $x \neq 0, -4$

$3(x+4) = 2x$

$3x + 12 = 2x$
$-2x \qquad -2x$

$x = -12$

2. $\frac{x+1}{2x+5} = \frac{2}{x}$ EV: $x \neq -\frac{5}{2}, 0$

$x(x+1) = 2(2x+5)$

$x^2 + x = 4x + 10$
$-4x - 10 \quad -4x - 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

3. $\frac{3}{x+2} + 5 = \frac{4}{x+2}$ EV: $x \neq -2$

LCD: $x+2$

$\frac{3(x+2)}{x+2} + 5(x+2) = \frac{4(x+2)}{x+2}$

$3 + 5x + 10 = 4$

$5x + 13 = 4$
$-13 \quad -13$

$5x = -9$

$x = -\frac{9}{5}$

4. $\frac{6}{x-3} = \frac{x}{18}$ EV: $x \neq 3$

$x(x-3) = 6 \cdot 18$

$x^2 - 3x = 108$

$x^2 - 3x - 108 = 0$

$(x-12)(x+9) = 0$

$x = 12, -9$

$-12 \cdot 9 = -108$
$-12 + 9 = -3$

5. $\frac{2x}{x+4} - 3 = \frac{-12}{x+4}$ EV: $x \neq -4$

LCD: $x+4$

$\frac{2x(x+4)}{x+4} - 3(x+4) = \frac{-12(x+4)}{x+4}$ → $-1x = 0$

$\boxed{x=0}$

$2x - 3x - 12 = -12$

$-1x - 12 = -12$

$+12 \quad +12$

6. $\frac{14}{2-x} = \frac{2}{x}$ EV: $x \neq 0, 2$

$14x = 2(2-x)$

$14x = 4 - 2x$

$+2x \quad +2x$

$\frac{16x}{16} = \frac{4}{16}$ $\boxed{x = 1/4}$

Day 10: Solving Harder Rational Equations

Warm-up:

1. $\frac{x+2}{x+1} - x = \frac{-6}{x+1}$ EV: $x \neq -1$ $x+1 \neq 0$

LCD: $x+1$

$x+2 - x(x+1) = -6$

$x+2 - x^2 - x = -6$

$2 - x^2 = -6$

$-x^2 = -8$

$x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm\sqrt{8}$

$\boxed{x = \pm 2\sqrt{2}}$

2. $\frac{4}{x-5} = \frac{2}{x+8}$ EV: $x \neq 5, 8$

no addition or subtraction ... just 1 term per side ... so cross multiply

$4(x+8) = 2(x-5)$

$4x+32 = 2x-10$

$-32 \quad -32$

$-2x \quad -2x$

$2x = -42$

$\boxed{x = -21}$

3. $\frac{2}{x-4} + 2 = \frac{6}{x-4}$ EV: $x \neq 4$

LCD: $x-4$

$2 + 2(x-4) = 6$

$2 + 2x - 8 = 6$

$2x - 6 = 6$

$2x = 12$

$\boxed{x = 6}$

4. $\frac{x}{x+24} = \frac{2}{x}$ EV: $x \neq -24, 0$

$x^2 = 2(x+24)$

$x^2 = 2x + 48$

$-2x \quad -2x \quad -48$

-48

$0 = x^2 - 2x - 48$

$(x-8)(x+6) =$

$\boxed{x = 8, -6}$

Day 10: Solving Harder Rational Equations

No EV because no x's in denominator

Example 1: $\frac{12(x-4)}{4} + \frac{12(x)}{3} = (6)^{12}$ LCD: 12   from 4·3

$\frac{12(x-4)}{4} + \frac{12x}{3} = 72$

$3(x-4) + 4x = 72$

$3x - 12 + 4x = 72$

$7x - 12 = 72$

$7x = 84$

$x = 12$

Steps:
1. Find the LCD.
2. Multiply each side by the LCD.
3. Simplify.
4. Solve for x!