Power Functions

Unit 4 Day 8

Warm-Up

1) $(a^7)(a^4) =$ _____

2) $(2p^3)(5p) =$ _____

3) $(x^4y^5)^2 =$ _____

4) $(2x^3y^4)^2 =$ _____

Add these in!

5) Given $f(x) = x^2 + 3x$,
evaluate $2f(x) - f(2x - 3)$.

6) Rewrite $f(x) = \sqrt[3]{8x - 8a}$ in graphing form, then describe the transformations from the parent graph.

Done early?

1st) check last night's HW answers on Blackboard

2nd) Finish Notes p. 22-24 (part of tonight's HW ☺)

Warm-Up Answers

1) $(a^7)(a^4) =$ $a^{7+4} = a^{11}$

2) $(2p^3)(5p) =$ $2 \cdot 5 \cdot p^{3+1} = 10p^4$

3) $(x^4y^5)^2 =$ $(x^{4 \cdot 2})(y^{5 \cdot 2}) = x^8y^{10}$
OR $(x^4y^5)(x^4y^5)$

4) $(2x^3y^4)^2 =$ $(2^2)(x^{3 \cdot 2})(y^{4 \cdot 2}) = 4x^6y^8$
OR $(2x^3y^4)(2x^3y^4)$

You'll do stuff related to these tonight in the HW! ☺

Warm-Up Answers

5) Given $f(x) = x^2 + 3x$,
evaluate $2f(x) - f(2x - 3)$.

$= 2(x^2 + 3x) - [(2x - 3)^2 + 3(2x - 3)]$
$= 2x^2 + 6x - [(2x - 3)(2x - 3) + 6x - 9]$
$= 2x^2 + 6x - [4x^2 - 12x + 9 + 6x - 9]$
$= 2x^2 + 6x - [4x^2 - 6x] = 2x^2 + 6x - 4x^2 + 6x$
$= -2x^2 + 12x$

6) Rewrite $f(x) = \sqrt[3]{8x - 8a}$ in graphing form, then describe the transformations from the parent graph.

$$f(x) = \sqrt[3]{8x - 8a} = \sqrt[3]{8(x - a)}$$

$$= \sqrt[3]{8} \bullet \sqrt[3]{(x - a)} = 2\sqrt[3]{x - a}$$

Vertical stretch by 2, translated right ***a*** units

Homework Answers Packet p. 11

Evaluate the piecewise function, $m(x)$. If there is no value for the given input, write, undefined.

4) $$m(x)=\begin{cases}-6 & \text{if } x<0\\ x+5 & \text{if } 0\le x\le 12\\ -2x+5 & \text{if } x>12\end{cases}$$

a) $m(0)$

use middle rule because
$x=0$ fits in to $0\le x\le 12$

$y=0+5$ $\boxed{5}$

b) $m(5)$

use middle rule because
$x=5$ fits in to $0\le x\le 12$

$y=5+5$ $\boxed{10}$

c) $m(14)$

use bottom rule because
$x=14$ fits in to $x>12$

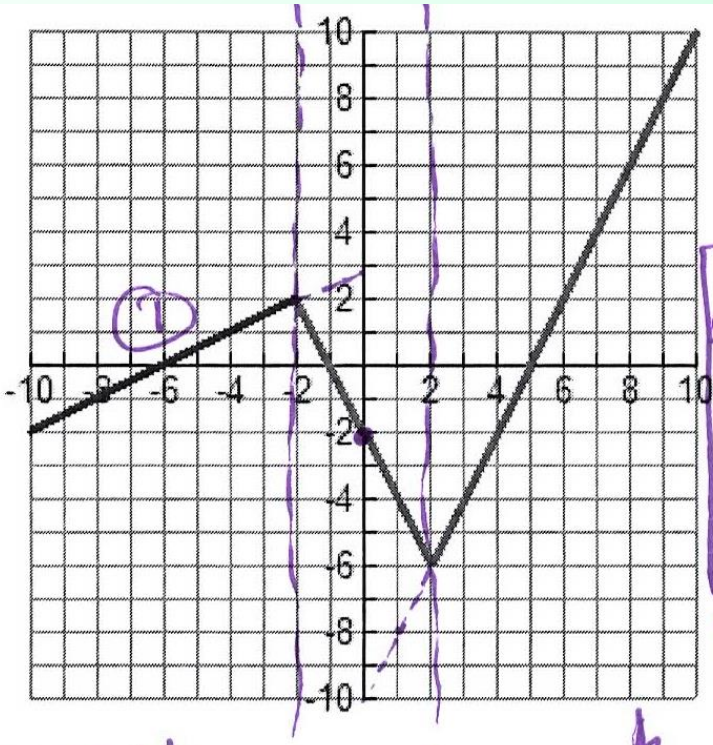
$-2(14)+5 = \boxed{-23}$

Homework Answers

Write the piecewise functions and their restrictions for the graphs below:

this part not on quiz

5)



$$f(x)=\begin{cases}\frac{1}{2}x+3, & x\le -2\\ -2x-2, & -2<x\le 2\\ 2x-10, & x>2\end{cases}$$

Due to the graph being continuous, any of the symbols could have the = to part on them

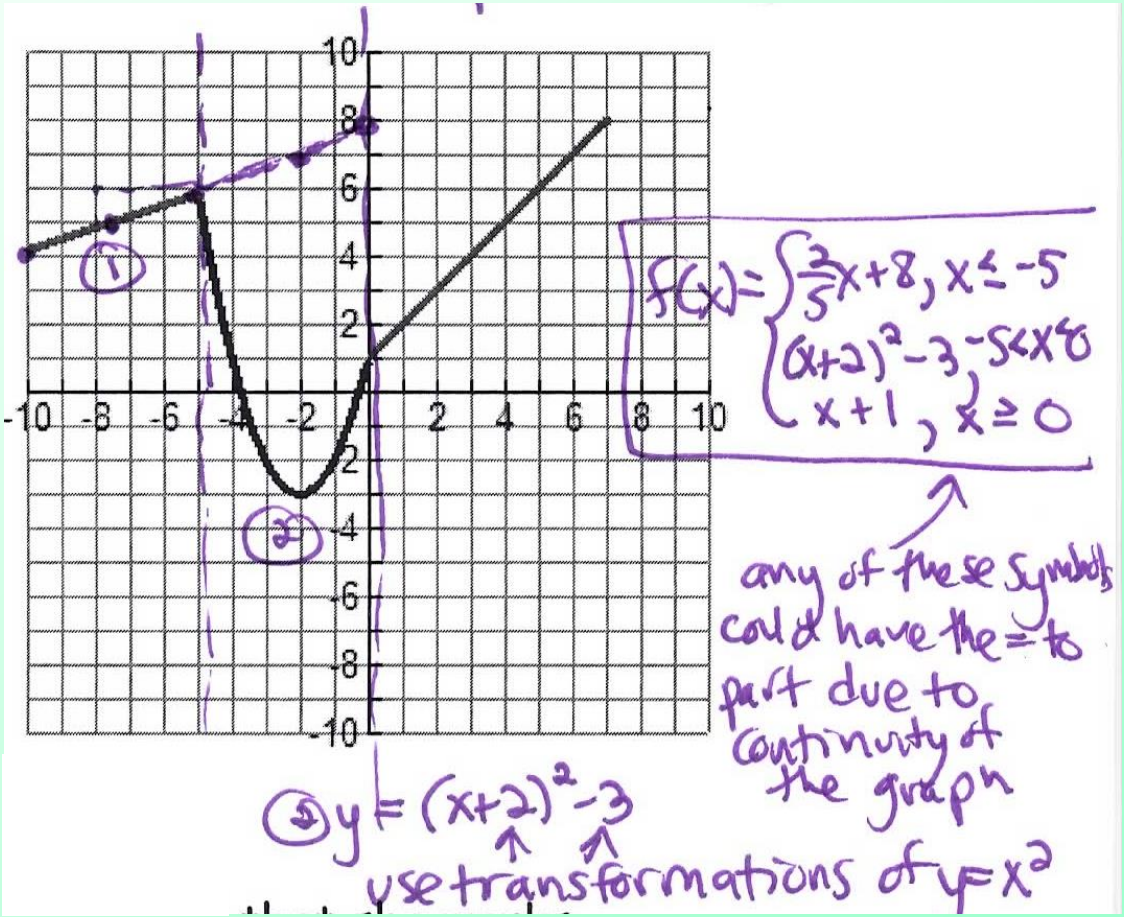
1) $y=mx+b$ $m=\frac{1}{2}$ $(0,b)=y-mt$ $=(0,3)$ if continue graph

Homework Answers

Write the piecewise functions and their restrictions for the graphs below:

this part not on quiz

6)



Homework Answers

this part not on quiz

7) Shelly earns $8 an hour. She earns $12 an hour for each hour over 40 that she works.

part like this not on quiz

a) Write piecewise functions that represent the money earned by Shelly for when she works regular hours and overtime hours.

b) Sketch a graph of Shelly's earnings versus the number of hours that she works up to 60 hours.

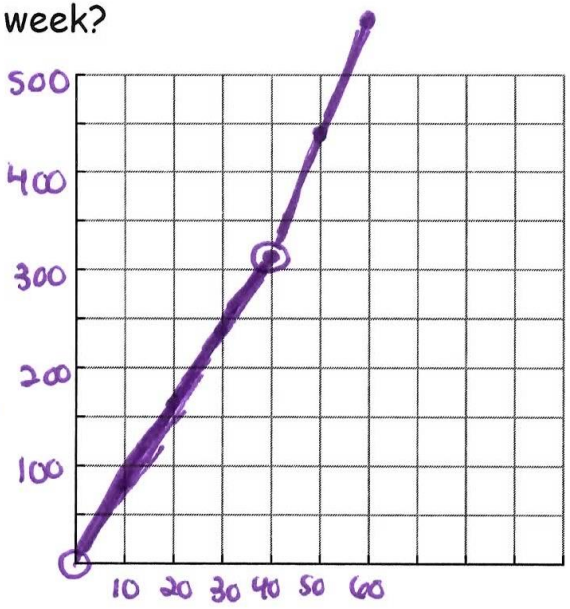
c) How much money will Shelly earn if she works 70 hours in one week?

a)
$$f(x) = \begin{cases} 8x, & 0 < x \le 40 \\ 320 + 12(x-40), & x > 40 \end{cases}$$

$8 \cdot 40$ ... makes $8 per hr for 1st 40 hours

12 per hr only for hrs over 40

b)



c) $320 + 12(70-40)$

$(8 \cdot 40)$ 12 (30 hrs over 40 hr limit)

680

Homework Answers p. 11

Evaluate the following given
$f(x) = 3x^2 - 1$, $g(x) = 4x$, and $h(x) = 2x + 5$

8) $g(-3) - f(-2)$ $= 4(-3) - (3(2)^2 - 1) = -12 - 11$ **$= -23$**

9) $3f(x) + 2g(x) - h(x) = 3(3x^2 - 1) + 2(4x) - (2x + 5)$
$= 9x^2 - 3 + 8x - 2x - 5 =$ **$9x^2 + 6x - 8$**

10) $h(3x - 2) = 2(3x - 2) + 5$
$= 6x - 4 + 5$ $=$ **$= 6x + 1$**

11) $f(x - 4) - 2f(x) = 3(x - 4)^2 - 1 - 2(3x^2 - 1)$
$= 3(x - 4)(x - 4) - 1 - 6x^2 + 2$
$= 3(x^2 - 8x + 16) - 6x^2 + 1$
$= 3x^2 - 24x + 48 - 6x^2 + 1$
$= -3x^2 - 24x + 49$

Tonight's Homework

- Packet p. 12

AND

- Print Next Packet

Notes:
Power Functions

Notes p. 25-28

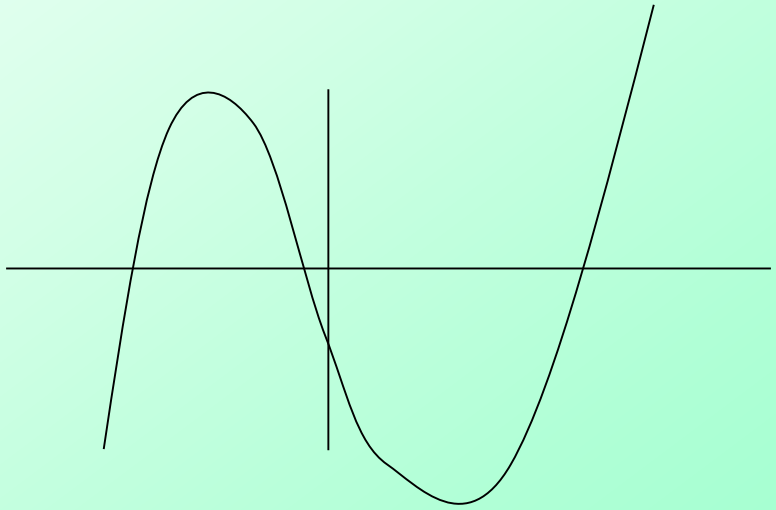
Fill in ALL the graphs, blanks, extra reminders and comments, etc in your notes! ☺

Power Function

- Definition $y = k \cdot x^p$

  for k < 1

- What effect will the *k* have?
  - Vertical stretch or compression

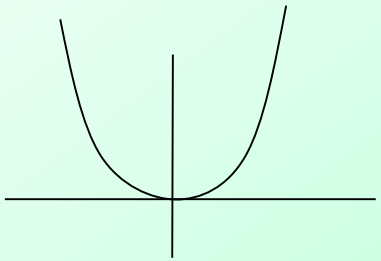


Note that "k" is a coefficient! This means "k" affects the whole equation, which is y-values. The "k" is NOT in the parentheses by x, so it doesn't affect x-values.

Special Power Functions…let's draw a reminder of their basic shapes! ☺

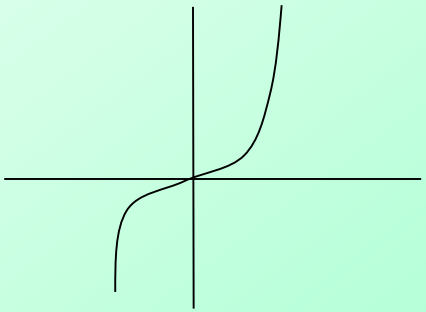
- Parabola

  $y = x^2$



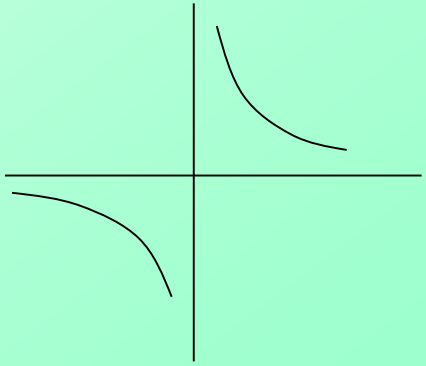
- Cubic function

  $y = x^3$



- Hyperbola

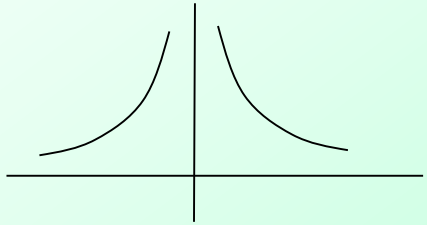
  $y = x^{-1}$ *same as* $y = \frac{1}{x}$



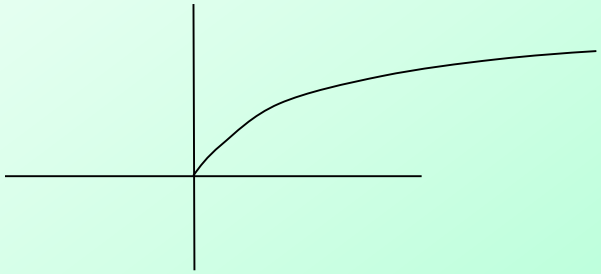
Draw pictures in your notes!

Special Power Functions…let's draw a reminder of their basic shapes! ☺

- $y = x^{-2}$



- $y = x^{\frac{1}{2}}$

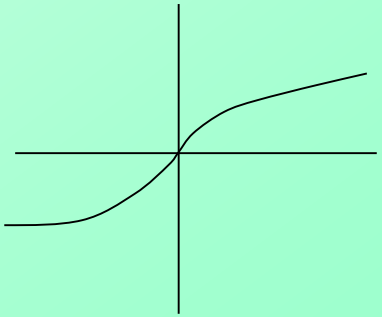


  - Square Root in disguise! ☺

Draw the pictures and write these reminders in your notes!

$$y = x^{\frac{1}{3}} = \sqrt[3]{x}$$

- Cube Root in disguise! ☺



Special Power Functions

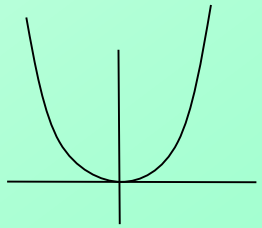
Most power functions are similar to one of the six just discussed

Which of the functions have symmetry? What kind of symmetry?

In your notes, draw the pictures! Also, write down the type of symmetry and examples !

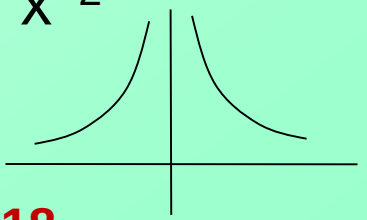
- x^p with positive even powers of p are similar to x^2
  - **Symmetry across y-axis**
  - **Examples**

    $y = 3x^2$, $y = -3x^2$, $y = -2x^4$, $y = -1/5x^{18}$



- x^p with negative even powers of p are similar to x^{-2}
  - **Symmetry across y-axis**
  - **Examples**

    $y = 3x^{-2}$, $y = -3x^{-2}$, $y = -2x^{-4}$, $y = -1/5x^{-18}$



Special Power Functions

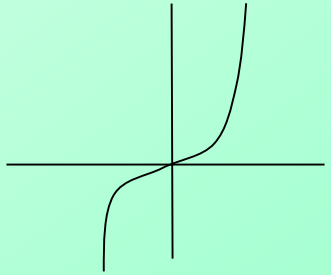
Most power functions are similar to one of the six just discussed

Which of the functions have symmetry? What kind of symmetry?

In your notes, draw the pictures! Also, write down the type of symmetry and examples !

- x^p with positive odd powers of p are similar to x^3
  - **Symmetry about the origin (or 180˚ rotation)**
    (symmetry across x-axis, then y-axis)
- **Examples**

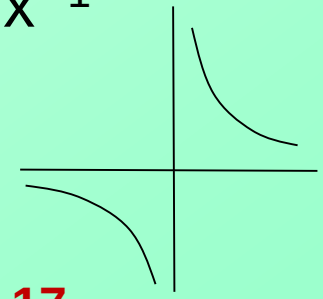
$$y = 2x^3,\ y = -2x^3,\ y = 3x^5,\ y = -1/5x^{17}$$



- x^p with negative odd powers of p are similar to x^{-1}
  - **Symmetry about the origin (or 180˚ rotation)**
    (symmetry across x-axis, then y-axis)

Examples

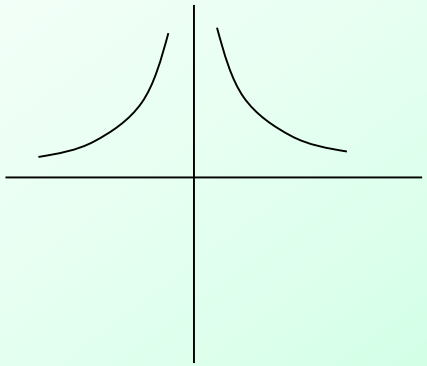
$$y = 2x^{-3},\ y = -2x^{-3},\ y = 3x^{-5},\ y = -1/5x^{-17}$$

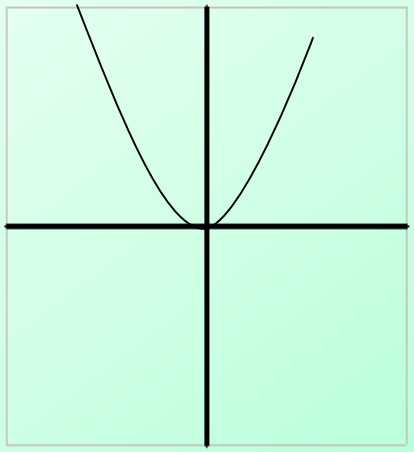


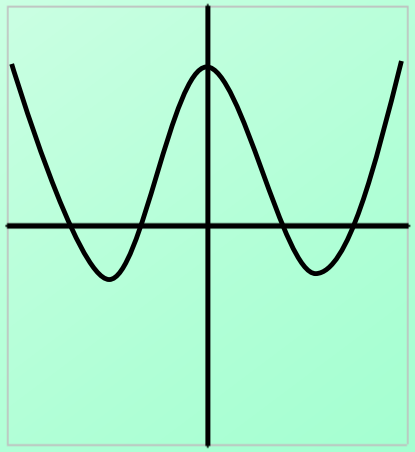
Remember that even functions are symmetric across the y-axis.

Draw pictures in your notes!

Examples:





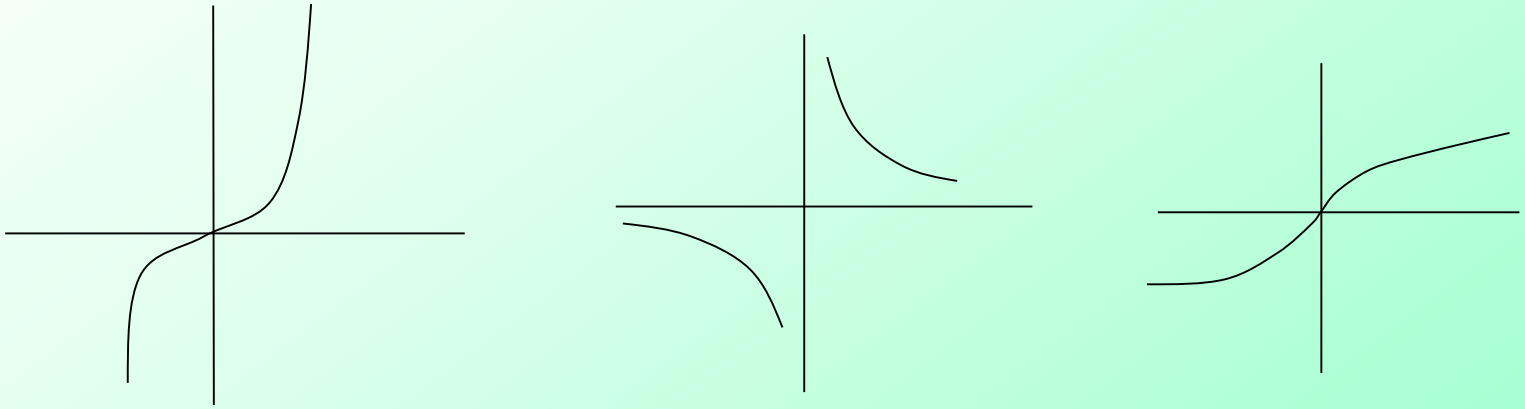


Be careful!! A function with an even degree (highest exponent) may or may not be an even function. A function with an odd degree may or may not be an odd function.

Remember that odd functions are symmetric about the origin.

Draw pictures in your notes!

Examples:



Be careful!! A function with an even degree (highest exponent) may or may not be an even function. A function with an odd degree may or may not be an odd function.

Direct Proportions

- The variable *y* is directly proportional to *x* when:

  y = k * x ⟵ This is a power function

  - k is some constant value
    - – called **Constant Of Proportionality**

- Alternatively $k = \frac{y}{x}$

  What is the Y-intercept?
  How can it be calculated?

- As x gets larger, y must also get larger

(cause and effect relationship between x & y)

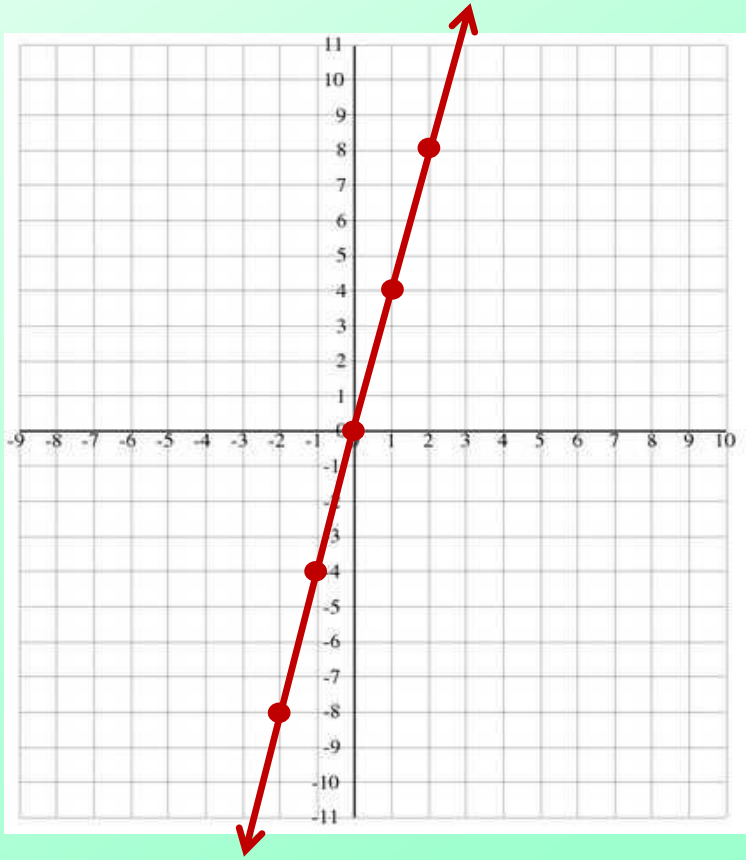
  - keeps the resulting *k* the same

Direct Proportions

- Examples:
  - Distance = rate x time
  - Force = mass x acceleration
  - Paycheck = (Standard hourly Pay/Wage)·(# of hrs worked)
  - Test grade = (5 pts/question)·(# of questions correct)
  - The harder you hit the baseball, the farther it travels
    - Distance hit is directly proportional to the force of the hit

Direct Proportion

- Suppose the constant of proportionality is 4
  - Then y = 4 * x
  - What does the graph of this function look like?



Remember
y = 4x is in y = mx + b form
so
b = y-int is (0, 0)
and
m = slope is 4 or 4/1

Inverse Proportion

- Another type of power function is an inverse proportion $y = \frac{k}{x}$ ← Again, this is a power function
  - k is some constant value
    - – called constant of proportionality
- Alternatively

  $y = k * x^{-1}$
- As x gets larger, y must get smaller to keep the resulting k the same

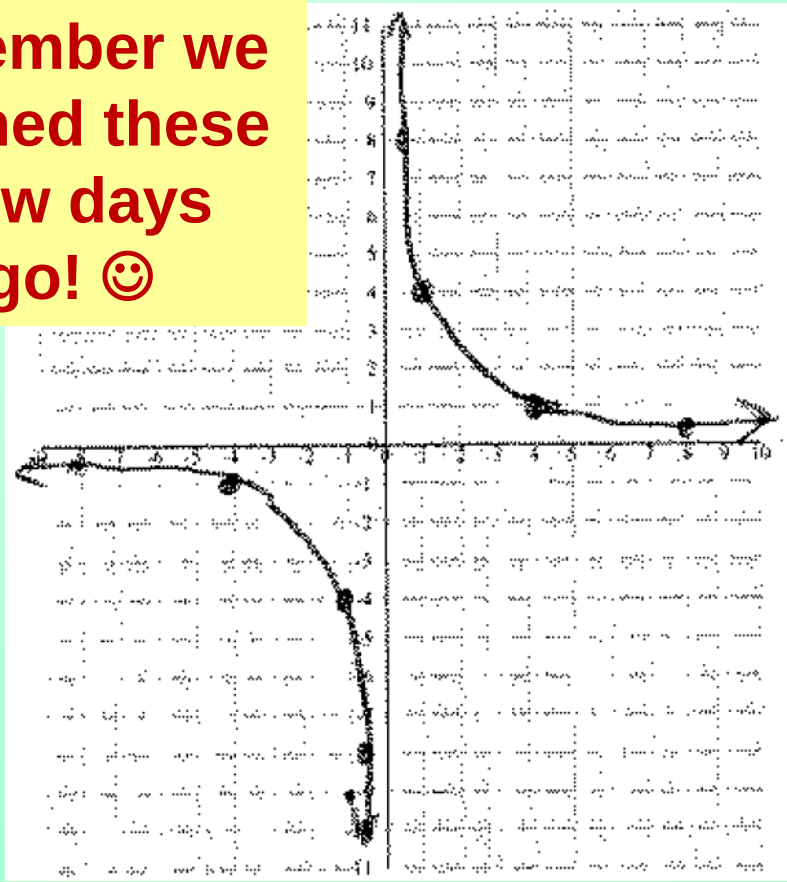
Inverse Proportion

- Example:
  - The time taken for a journey is inversely proportional to the speed of travel
  - The time needed to dig a hole is (approximately) inversely proportional to the number of people digging
  - Time to bake cookies is inversely proportional to oven temperature
    - If you bake cookies at a higher oven temperature, they take less time

Inverse Proportion

- **Remember** what the graph looks like
  - Let the constant or proportionality k = 4
  - Then $y = \frac{4}{x}$

Remember we graphed these a few days ago! ☺



Left Branch

-8	-½
-4	-1
-1	-4
-½	-8

Right Branch

8	½
4	1
1	4
½	8

Formulas for Power Functions

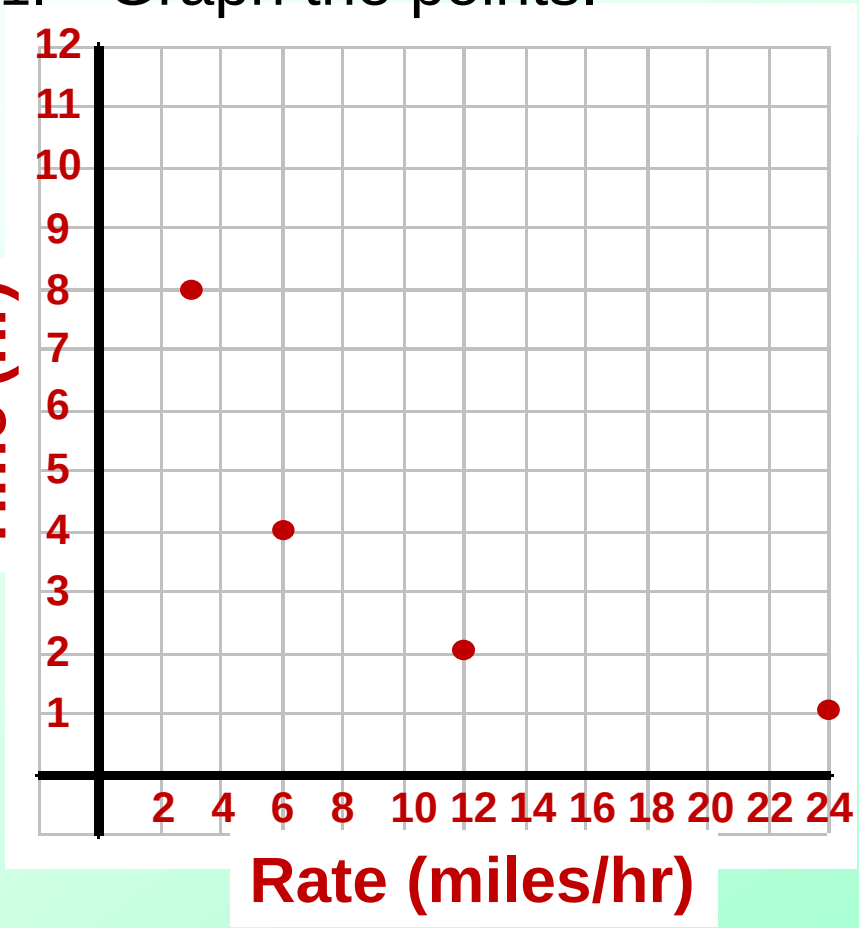
- Say that we are told that
  f(1) = 7 and f(3)=56
  - We can find f(x) when linear $y = mx + b$
  - We can find f(x) when it is $y = a(b)^t$

- Now we consider finding $f(x) = k\,x^p$
  - We'll use the calculator **for now**!

Application – Power Regression, Interpreting and Predicting values!

Rate (miles/hr)	1	3	6	9	12	18	24
Time (hr)		8	4		2		1

1. Graph the points.



NOTE: the pattern on the graph does NOT appear linear! It looks like part of a hyperbola. -> Clue that we probably have inverse variation.

Application – Power Regression!
Cycling for 24 miles

Rate (miles/hr)	1	3	6	9	12	18	24
Time (hr)		8	4		2		1

2. Find a power function that models the data.

- Stat → Edit → Input data into L1 and L2. Enter the values for which you have a **complete** ordered pair. (We'll fill in the blanks in the table later! ☺)
- Stat → CALC → A: PwrReg

L1	L2
3	8
6	4
12	2
24	1

To help with predictions, remember:

- You MUST store your equations in Y1. To get Y1, do **VARS, YVars, ENTER, ENTER**.
- For calculators with older operating system, do PwrReg, **L1**, **L2**, **Y1**
- For calculators with newer operating system, on the PwrReg screen, by StoreEq, do **Y1**

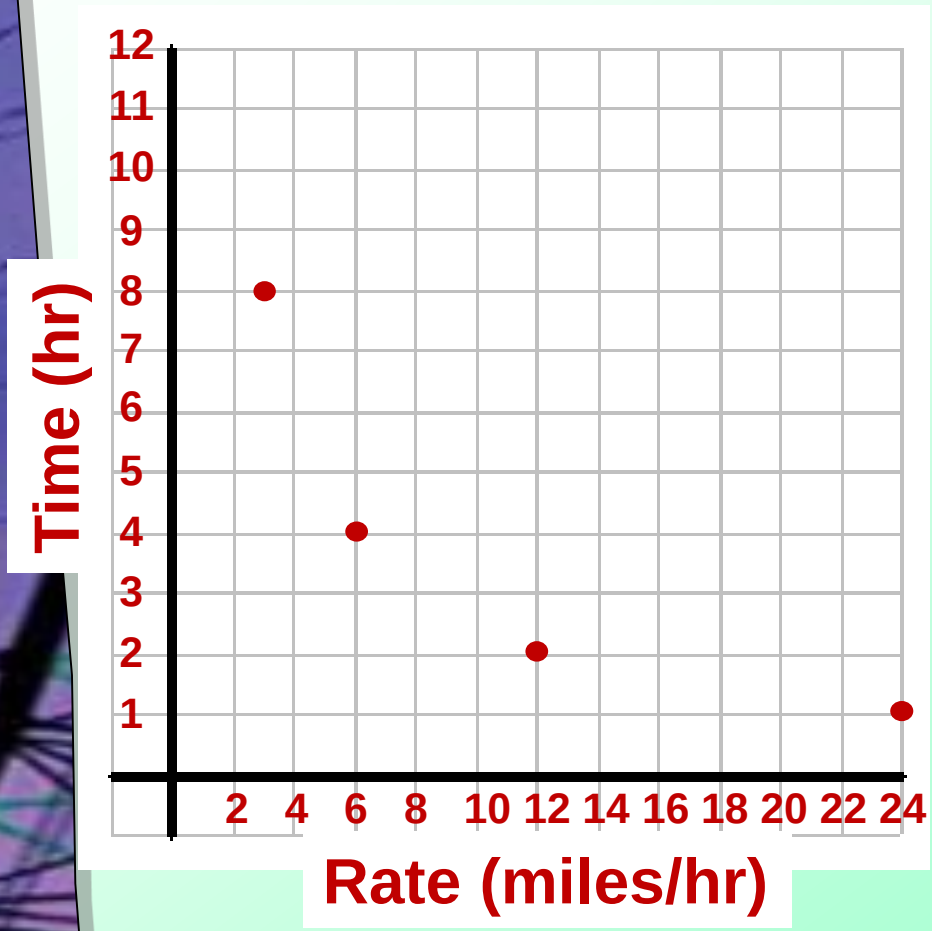
$y = 24 \cdot x^{-1}$

→

$y = 24/x$

Application – Power Regression!
Cycling for 24 miles

Rate (miles/hr)	1	3	6	9	12	18	24
Time (hr)		8	4		2		1



3. Determine whether the function is direct or inverse variation.

Inverse variation because as x-values increase, y-values decrease. (seen on the graph)

ALSO, inverse variation because

$y = 24/x$ is equivalent to

$y = 24 \cdot x^{-1}$

which fits $y = k \cdot x^{-1}$

Application – Power Regression!
Cycling for 24 miles

Rate (miles/hr)	1	3	6	9	12	18	24
Time (hr)	**24**	8	4	**2 2/3**	2	**1 1/3**	1

4. Fill in the missing values in the table

Get values from table in calculator (Press 2nd Graph)
OR do Y1(1) then Y1(9) etc on main screen
Don't see values in table? Didn't get the values above? Be sure you stored your equation in Y1 for problem 2 earlier!!

see values above

5. Determine the rate of cycling if a person biked for 6 hours.

Press y =, then enter y2 = 6
then find intersection
(Remember: 2nd Trace 5 ENTER ENTER ENTER.

4 miles per hour

Quiz Review

Not In Notes!

1) Write an equation, g(x), for the translation of $f(x) = \frac{8}{x}$ that has the asymptotes x = 7 and y = - 4.
   Also, describe the translation from f(x).
2) What are the domain and range of the translated function in #1?
3) Solve [3x – 2] = 6. Express your answer in set notation.
4) Rewrite y = - | 4x – 24 | + 3 in graphing form, then describe the transformations from the parent graph.
5) What are the domain and range of the translated function in #4?

Quiz Review Answers

1) Write an equation, g(x) for the translation of $f(x) = \frac{8}{x}$ that has the asymptotes x = 7 and y = - 4. Also, describe the translation from f(x).

$$g(x) = \frac{8}{x-7} - 4$$

g(x) is translated right 7 and down 4 from f(x)

2) What are the domain and range of the translated function in #1?

$$D: (-\infty, 7) \cup (7, \infty)$$

$$R: (-\infty, -4) \cup (-4, \infty)$$

3) Solve [3x – 2] = 6. Express your answer in set notation.

{x | 8/3 ≤ x < 3 }

Quiz Review

4) Rewrite $y = - | 4x – 24 | + 3$ in graphing form, then describe the transformations from the parent graph.

$y = - | 4 (x – 6) | + 3$ **$y = - 4 | x – 6 | + 3$**

Reflected over x-axis, vertical stretch by 4, translated right 6 and up 3

5) What are the domain and range of the translated function in #4?

$D: (-\infty, \infty)$

$R: (-\infty, 3]$

Tonight's Homework

- Packet p. 12

AND

- Print Next Packet