

Unit 4 Day 7

More With Piecewise Functions

Warm-up

$$f(x) = \begin{cases} -2x - 3, & x \leq -4 \\ x^2 - 4, & -4 < x < 3 \\ \frac{1}{2}x + 1, & x \geq 3 \end{cases}$$

1. $f(2) =$

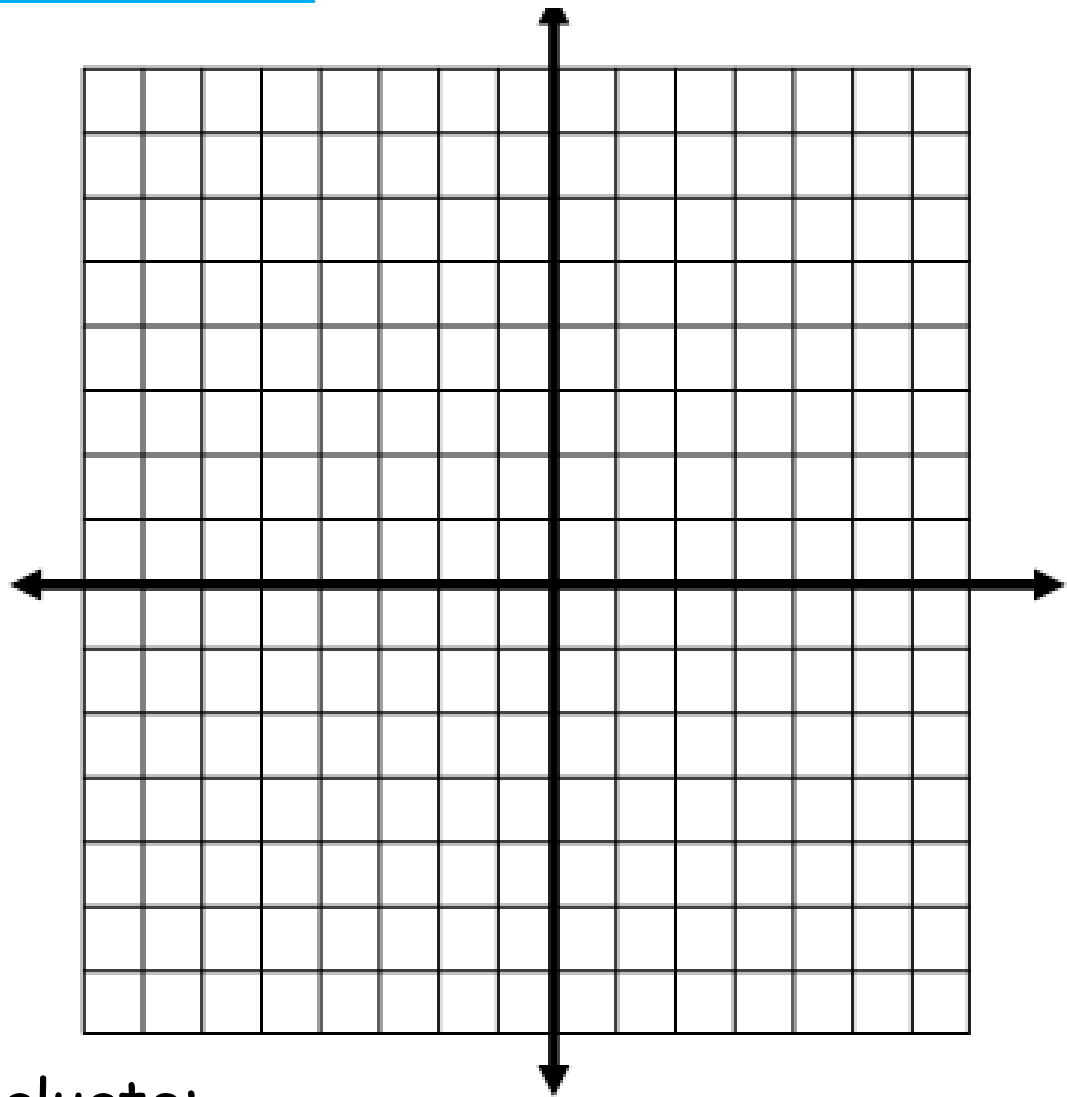
2. $f(-4) =$

3. $f(8) =$

4. Domain: _____

5. Range: _____

6. Graph the function.



Given $g(x) = x^2 - 4x + 5$, evaluate:

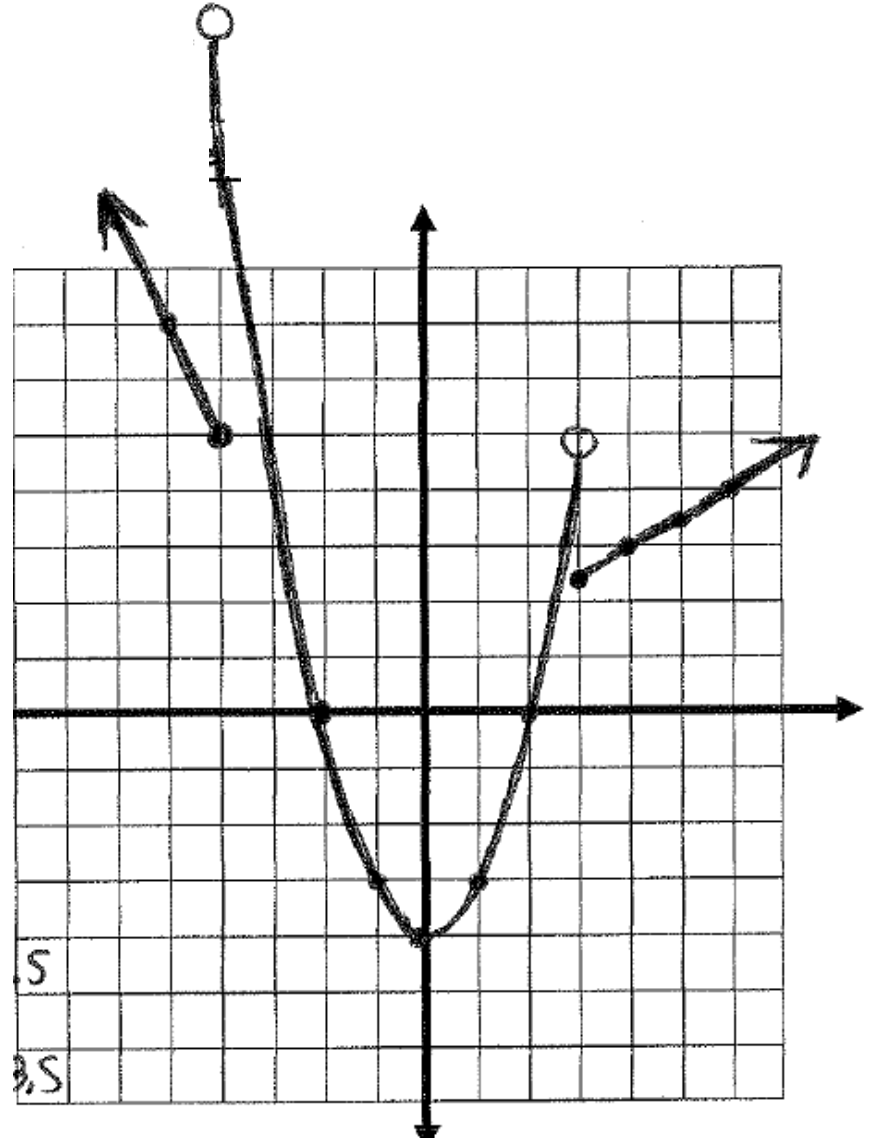
7) $g(2x - 3)$

8) $g(x - 3) - 2g(x)$

Warm-up Answers

$$f(x) = \begin{cases} -2x - 3, & x \leq -4 \\ x^2 - 4, & -4 < x < 3 \\ \frac{1}{2}x + 1, & x \geq 3 \end{cases}$$

1. $f(2) = (2)^2 - 4 = 0$ (middle rule)
2. $f(-4) = -2(-4) - 3 = 5$ (top rule)
3. $f(8) = \frac{1}{2}(8) + 1 = 5$
(bottom rule)
4. Domain: $(-\infty, \infty)$
5. Range: $[-4, \infty)$
6. Graph the function.



Warm-up Answers

Given $g(x) = x^2 - 4x + 5$, evaluate:

$$\begin{aligned} 7) \quad g(2x - 3) &= (2x - 3)^2 - 4(2x - 3) + 5 \\ &= (2x - 3)(2x - 3) - 8x + 12 + 5 \\ &= 4x^2 - 12x + 9 - 8x + 17 \\ &= 4x^2 - 20x + 26 \end{aligned}$$

$$\begin{aligned} 8) \quad g(x - 3) - 2g(x) &= (x - 3)^2 - 4(x - 3) + 5 - 2(x^2 - 4x + 5) \\ &= (x - 3)(x - 3) - 4x + 12 + 5 - 2x^2 + 8x - 10 \\ &= x^2 - 6x + 9 - 4x + 17 - 2x^2 + 8x - 10 \\ &= -x^2 - 2x + 16 \end{aligned}$$

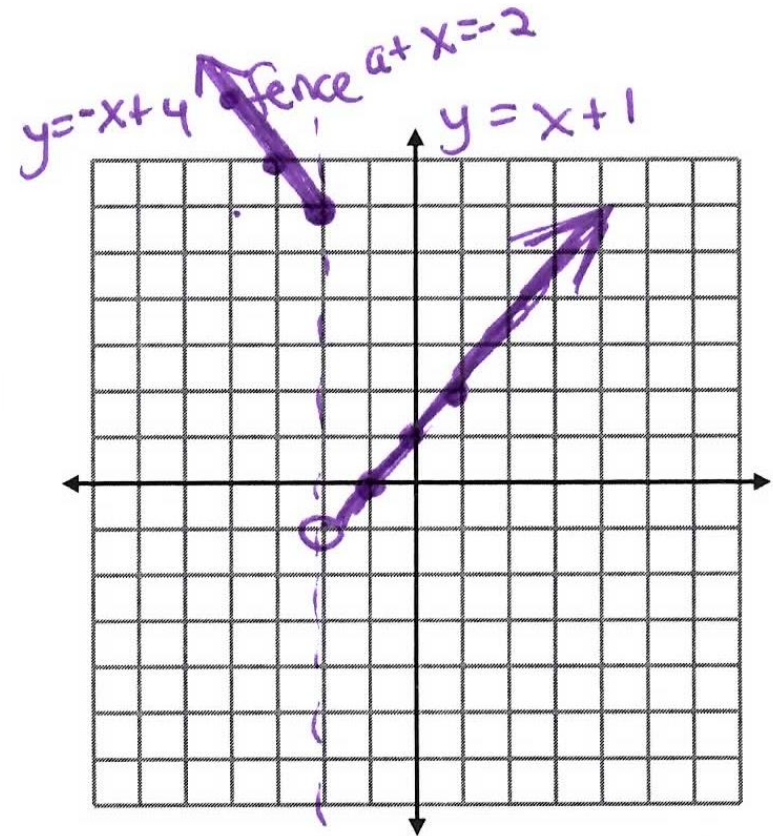
Homework Answers Packet p. 10

$$1) g(x) = \begin{cases} -x + 4 & \text{if } x \leq -2 \\ x + 1 & \text{if } x > -2 \end{cases}$$

	$y = -x + 4$	
closed	-2	6
	-3	7
	-4	8
	-5	9

$-(-2)+4$
 $-(-3)+4$
 $-(-4)+4$
 $-(-5)+4$

open	-2	-1	-2+1
	-1	0	-1+1
	0	1	0+1
	1	2	1+1



Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Homework Answers Packet p. 10

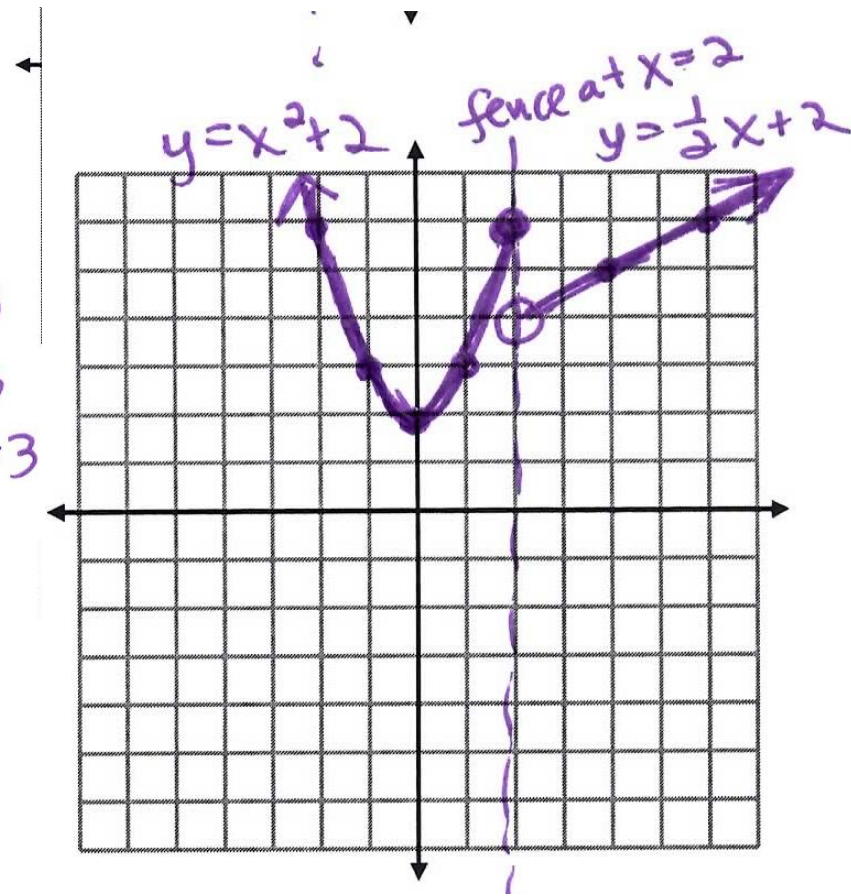
$$2) f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 2 \\ \frac{1}{2}x + 3 & \text{if } x > 2 \end{cases}$$

	$y = x^2 + 2$	
closed	2	6
	1	3
	0	2
	-1	3
	-2	6

$$\text{if } x \leq 2$$

$$\text{if } x > 2$$

	$y = \frac{1}{2}x + 3$	
open	2	4
	4	5
	6	6



Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

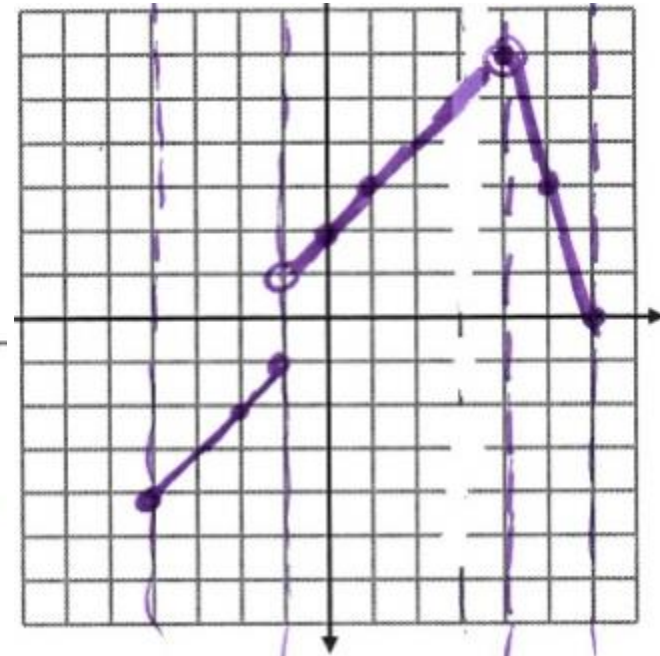
Homework Answers Packet p. 10

$$3) f(x) = \begin{cases} x & \text{if } -4 \leq x \leq -1 \\ x+2 & \text{if } -1 < x < 4 \\ -3x+18 & \text{if } 4 \leq x \leq 6 \end{cases}$$

	$y=x$
closed -4	-4
-2	-2
closed -1	-1

	$y=x+2$
open -1	1
0	2
1	3
open 4	6

	$y=-3x+18$
closed 4	6
5	3
closed 6	0



Domain: $[-4, 6]$

Range: $[-4, -1] \cup [0, 6]$



Tonight's Homework

Packet p. 11

Remember to write Domain &
Range
using Interval Notation

Notes on Applications

NOTES p. 21



Applications!

NOTES p. 21

1. When a diabetic takes long-acting insulin, the insulin reaches its peak effect on the blood sugar level in about three hours. This effect remains fairly constant for 5 hours, then declines, and is very low until the next injection. In a typical patient, the level of insulin might be modeled by the following function.

$$f(t) = \begin{cases} 40t + 100 & \text{if } 0 \leq t \leq 3 \\ 220 & \text{if } 3 < t \leq 8 \\ -80t + 860 & \text{if } 8 < t \leq 10 \\ 60 & \text{if } 10 < t \leq 24 \end{cases}$$

Here, $f(t)$ represents the blood sugar level at time t hours after the time of the injection. If a patient takes insulin at 6 am, find the blood sugar level at each of the following times.

a. 7 am

b. 11 am

c. 3 pm

d. 5 pm

$$7 - 6 = 1 \text{ hour}$$

$$0 \leq 1 \leq 3$$

$$40(1) + 100 = 140$$

$$11 - 6 = 5 \text{ hours}$$

$$3 < 5 \leq 8$$

$$220$$

$$15:00 - 6:00 = 9 \text{ hours}$$

$$8 < 9 \leq 10$$

$$-80(9) + 860 = 140$$

$$17:00 - 6:00 = 11 \text{ hours}$$

$$10 < 11 \leq 24$$

$$60$$

You Try the table!

Applications

2. Lisa makes \$4/hr baby-sitting before midnight and \$6/hr after midnight. She begins her job at 7 PM.

a. Complete the table below for the total amount of money Lisa makes.

Time	8PM	9PM	10PM	11:30PM	12:00AM	12:30AM	1:00AM	1:30AM	2:00AM
Hours worked	1	2	3	4.5	5	5.5	6	6.5	7
Money Earned	4	8	12	18	20	23	26	29	32

b. If we want to fill out the entries after midnight in the table above, we need to realize that the function is piecewise; that is, Lisa is paid at two different rates, one for the time she baby-sits before midnight, and another for the time she babysits after midnight.

Since the rate changes at $t = 5$, we need two different rules: one for $t \leq 5$ and one for $t > 5$.

$$f(t) = \begin{cases} 4x & \text{for } 0 \leq x \leq 5 \\ 6x - 10 & \text{for } 5 < x \end{cases}$$

**Together, let's do
the function!**

You Try! NOTES p. 22 if time allows

3. I really want to write the letter M on my graph paper using $y=mx+b$ form...

From $x = -6$ to $x = -4$, use the equation $y = 2x + 12$

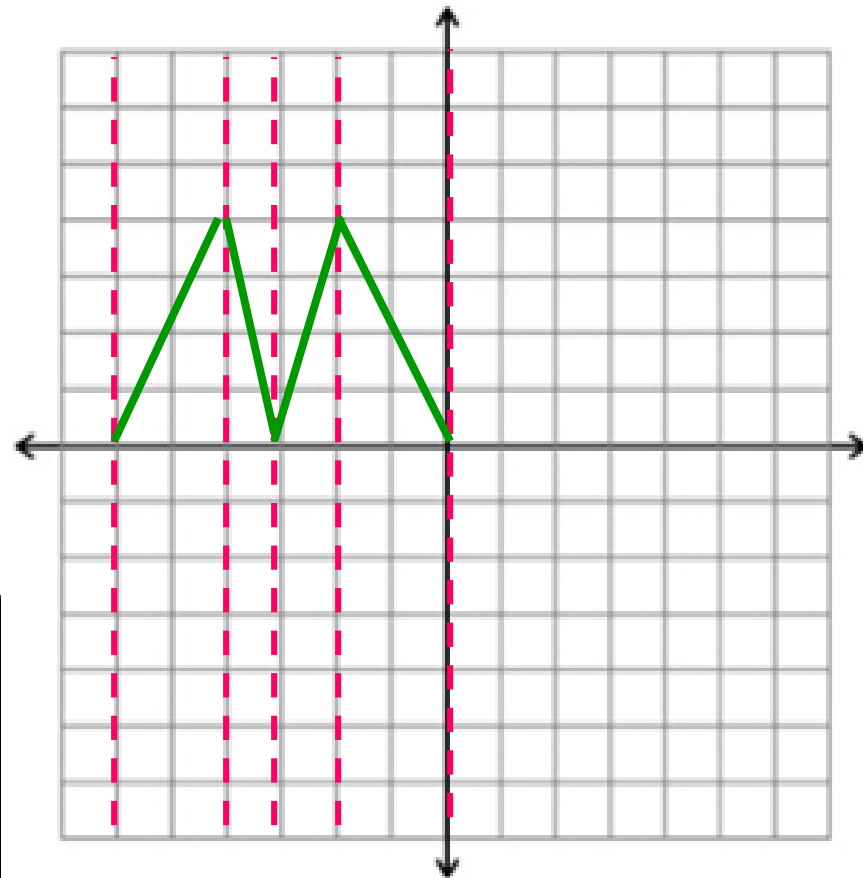
From $x = -4$ to $x = -3$, use the equation $y = -3x - 8$

From $x = -3$ to $x = -2$, use the equation $y = 3x + 10$

From $x = -2$ to $x = 0$, use the equation $y = -2x$

In mathematics, we write this set of directions as a piece-wise function:

$$f(x) = \begin{cases} 2x + 12, & \text{if } -6 \leq x < -4 \\ -3x - 8, & \text{if } -4 \leq x < -3 \\ 3x + 10, & \text{if } -3 \leq x < -2 \\ -2x, & \text{if } -2 \leq x < 0 \end{cases}$$



You Try! NOTES p. 23

4. A wholesaler charges \$3.00 per pound for an order of less than 20 pounds of candy and \$2.50 per pound for 20 or more pounds. Write a piecewise function for this situation. Then graph the function.

$$f(x) = \begin{cases} 3x & \text{if } 0 \leq x < 20 \\ 2.50x & \text{if } x \geq 20 \end{cases}$$

What is the total charge for an order of 15 pounds of candy?

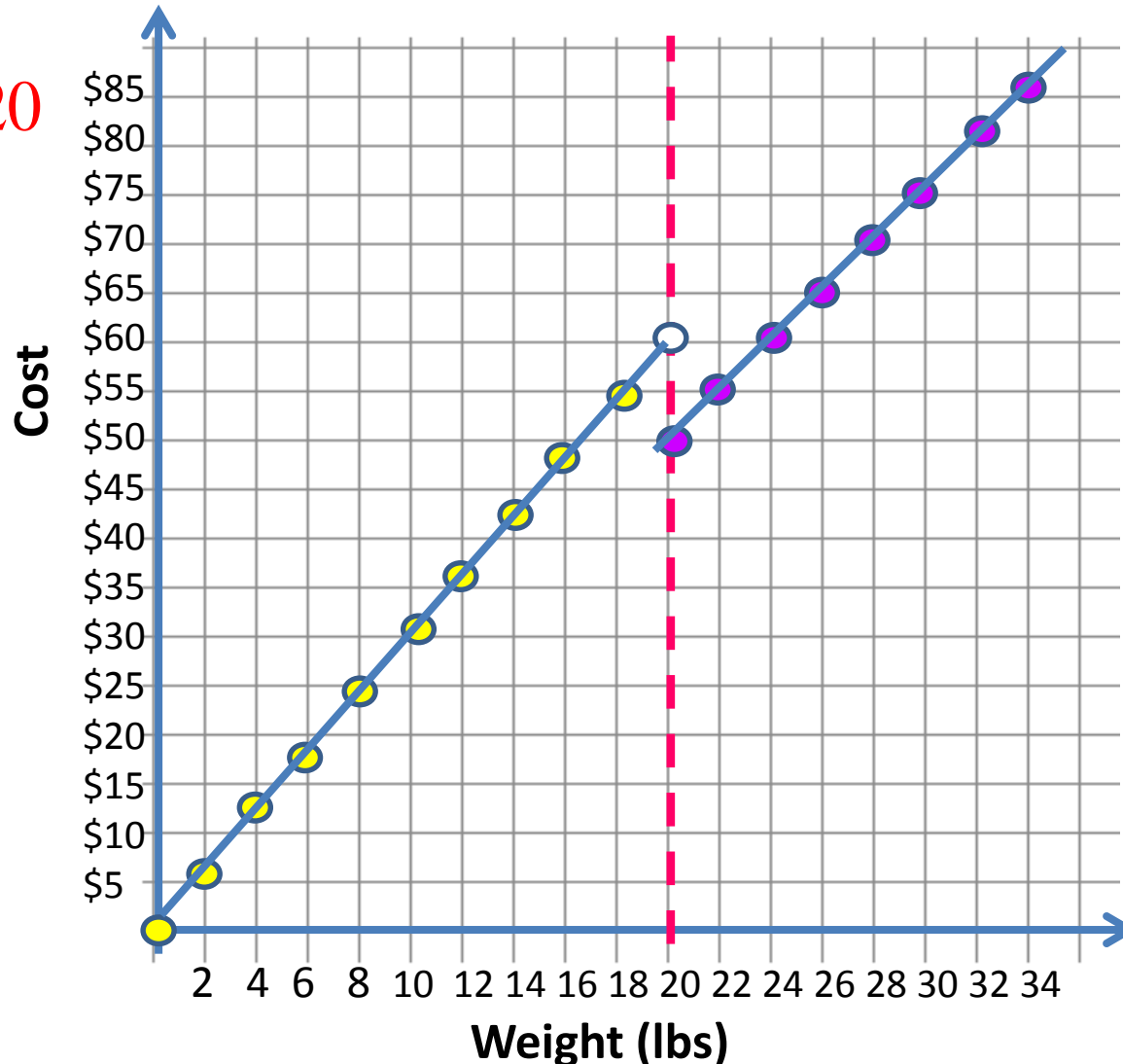
$$3(15) = \$45.00$$

For 20 pounds?

$$2.50(20) = \$50.00$$

For 30 pounds?

$$2.50(30) = \$75.00$$



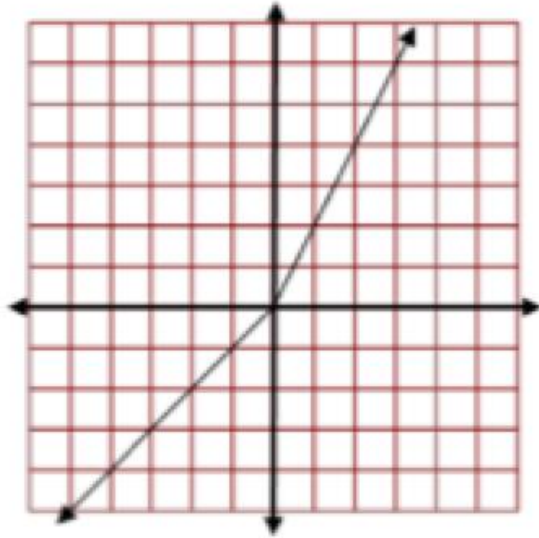
Together:
**Notes on Writing Equations,
given a Piecewise Graph**

NOTES p. 23



Let's do a couple together:

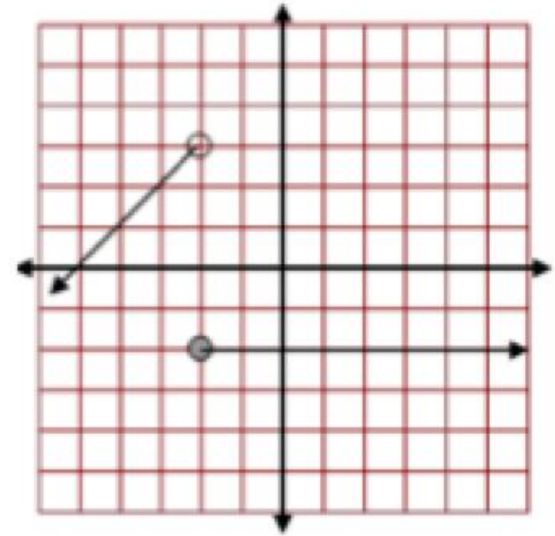
1.



$$f(x) = \left\{ \right.$$

Domain: _____ Range: _____

2.



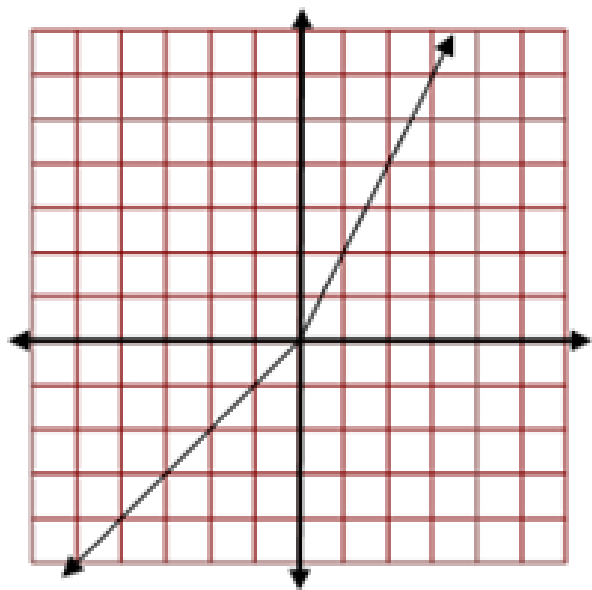
$$f(x) = \left\{ \right.$$

Domain: _____ Range: _____

Answers

Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tick mark.

1.

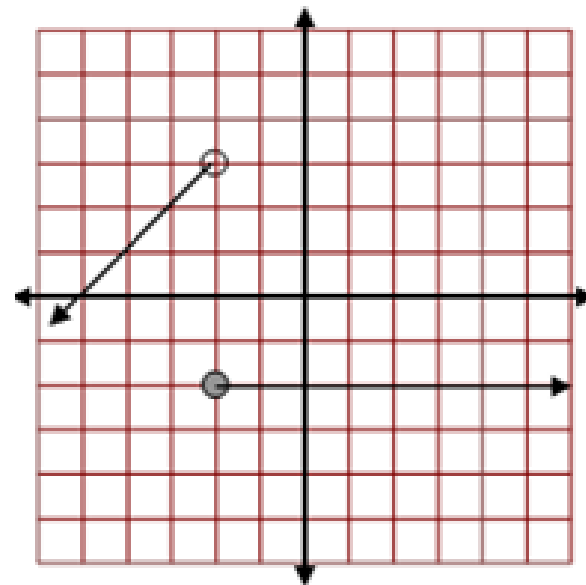


$$f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

2.



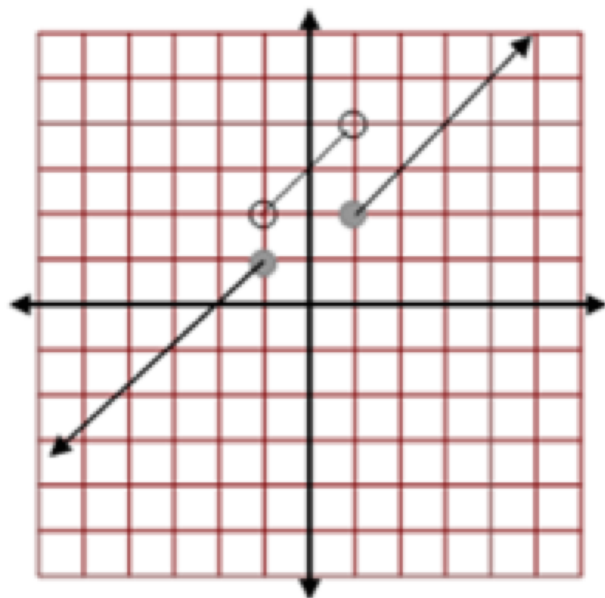
$$f(x) = \begin{cases} x + 5 & \text{if } x < -2 \\ -2 & \text{if } x \geq -2 \end{cases}$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

You Try:

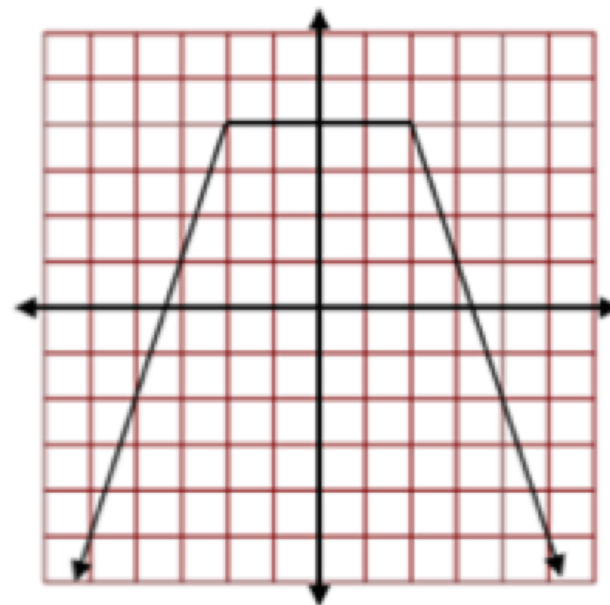
3.



$$f(x) = \left\{ \right.$$

Domain: _____ Range: _____

4.

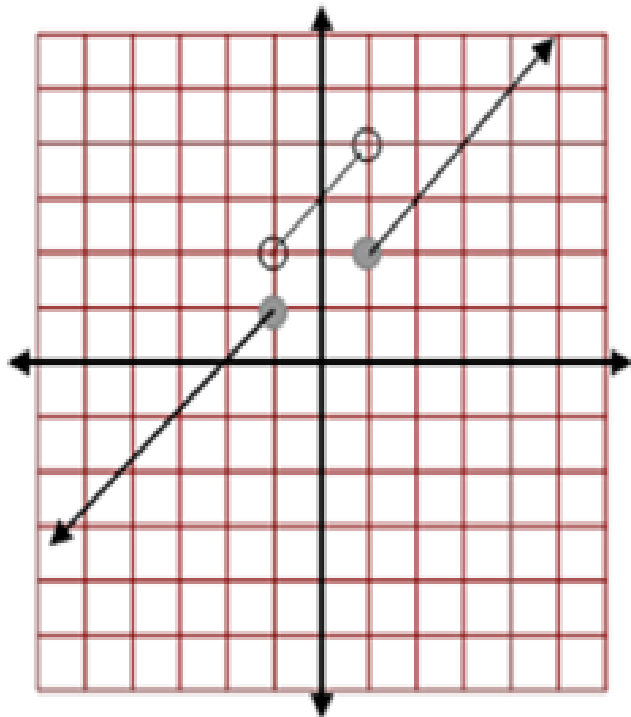


$$f(x) = \left\{ \right.$$

Domain: _____ Range: _____

You Try ~ Answers

3.



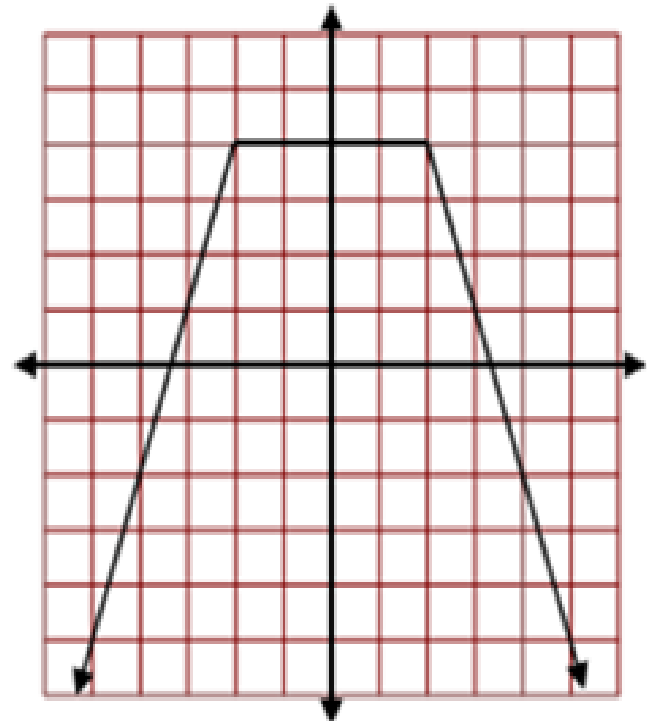
$$f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x+3 & \text{if } -1 < x < 1 \\ x+1 & \text{if } x \geq 1 \end{cases}$$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 1] \cup [2, \infty)$

All real #s

$y \leq 1$ or $y \geq 2$

4.



$$f(x) = \begin{cases} 3x+10 & \text{if } x < -2 \\ 4 & \text{if } -2 \leq x \leq 2 \\ -3x+10 & \text{if } x > 2 \end{cases}$$

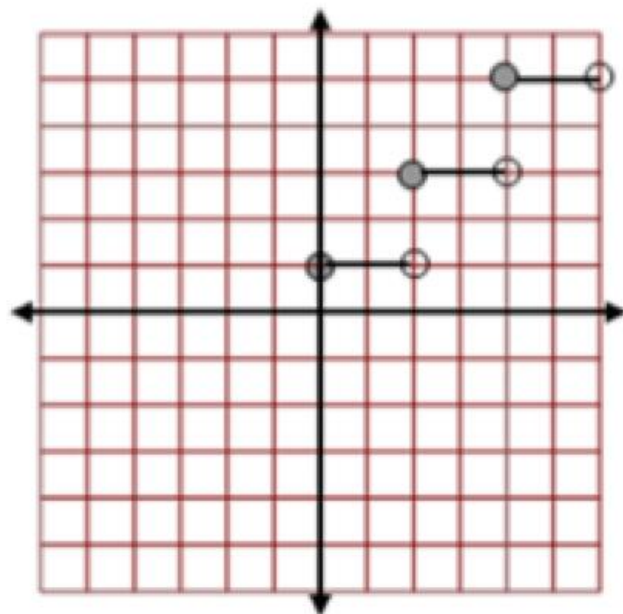
Domain: $(-\infty, \infty)$ Range: $(-\infty, 4]$

All real #s

$y \leq 4$

You Try:

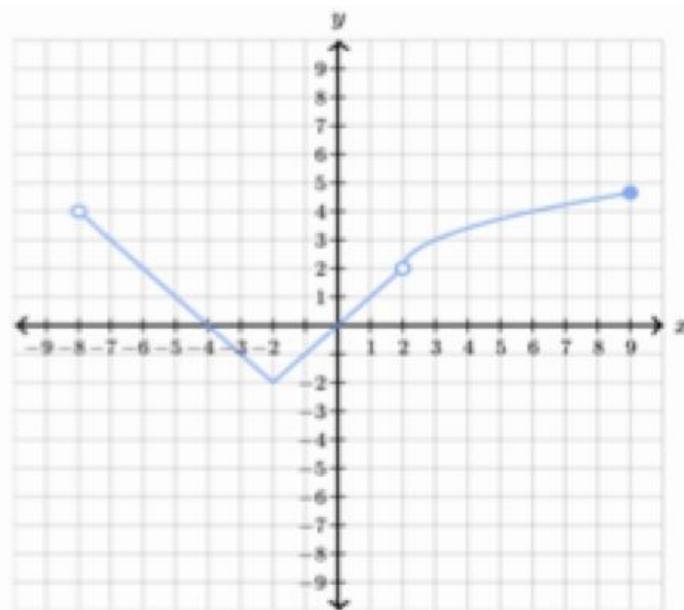
5.



$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$

Domain: _____ Range: _____

6.

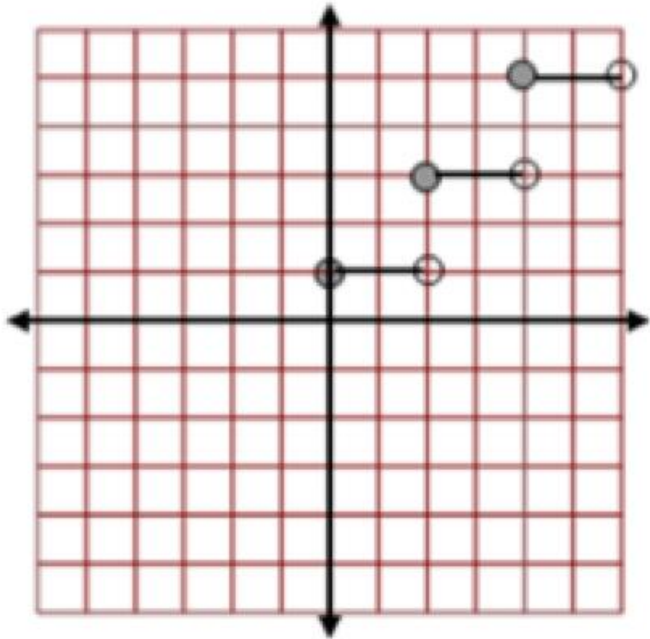


$$f(x) = \left\{ \begin{array}{l} \end{array} \right.$$

Domain: _____ Range: _____

You Try ~ Answers

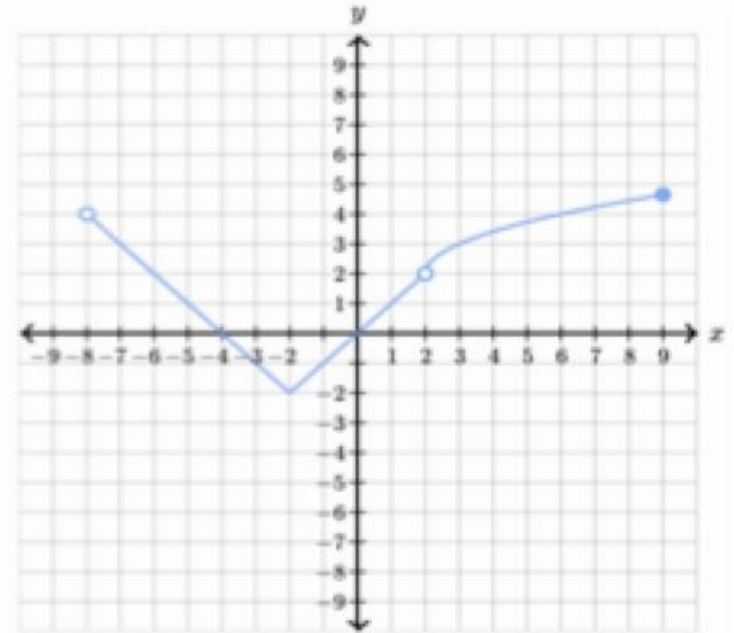
5.



$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 2 \\ 3 & \text{if } 2 \leq x < 4 \\ 5 & \text{if } 4 \leq x < 6 \end{cases}$$

Domain: $\underline{[0, 6)}$ Range: $\underline{\{1, 3, 5\}}$
 $\underline{0 \leq x < 6}$ $\underline{y = \{1, 3, 5\}}$

6.



$$f(x) = \begin{cases} |x+2| - 2 & \text{if } -8 < x < 2 \\ \sqrt{x-2} + 2 & \text{if } 2 < x \leq 9 \end{cases}$$

Domain: $\underline{(-8, 2) \cup (2, 9]}$ Range: $\underline{[-2, 4.65]}$

Another Application

Another Application: We also see piece-wise functions in our tax structure:

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq 15,000 \\ 0.15(x - 15,000), & \text{if } 15,000 < x \leq 40,000 \\ 6000 + 0.25(x - 40,000), & \text{if } 40,000 < x \leq 250,000 \\ 37,500 + 0.40(x - 250,000), & \text{if } 250,000 \leq x \end{cases}$$

For income at or below \$15,000, no tax is charged.

Above \$15,000 and at or below \$40,000, the rate is 15% for all monies earned over \$15,000.

Above \$40,000, the rate increases to 25% on all monies earned over \$40,000 (where did the \$6000 come from?), until income is \$250,000.

Above that level, the rate is 40%. (Where did the \$37,500 come from?)

How much would I owe in taxes if I made:

- a. \$12,000 **0**
- b. \$17,000 **\$300**
- c. \$47,000 **\$7,750**
- d. \$470,000 **\$125,500**

Kahoot! - Piecewise

1. <https://play.kahoot.it/#/?quizId=5640c5ec-e2b1-4bc2-bf7b-d805e31214f3> (easy)

2. <https://play.kahoot.it/#/k/9e900165-87e2-4ca1-b5d3-48def2e47912> (harder)

3. <https://play.kahoot.it/#/?quizId=95c8f843-99cd-416f-8059-84ec66972438>
(basic functions)

Matching Activity or Quiz Review

Then finish **Practice** from
the other day's notes

Practice: Notes p. 18 - 20



Homework

Packet p. 11-12

Write Domain & Range
using Interval Notation