Day 6: Graphing Piece-Wise Functions

Warm-Up: Use the following function to answer #1-4

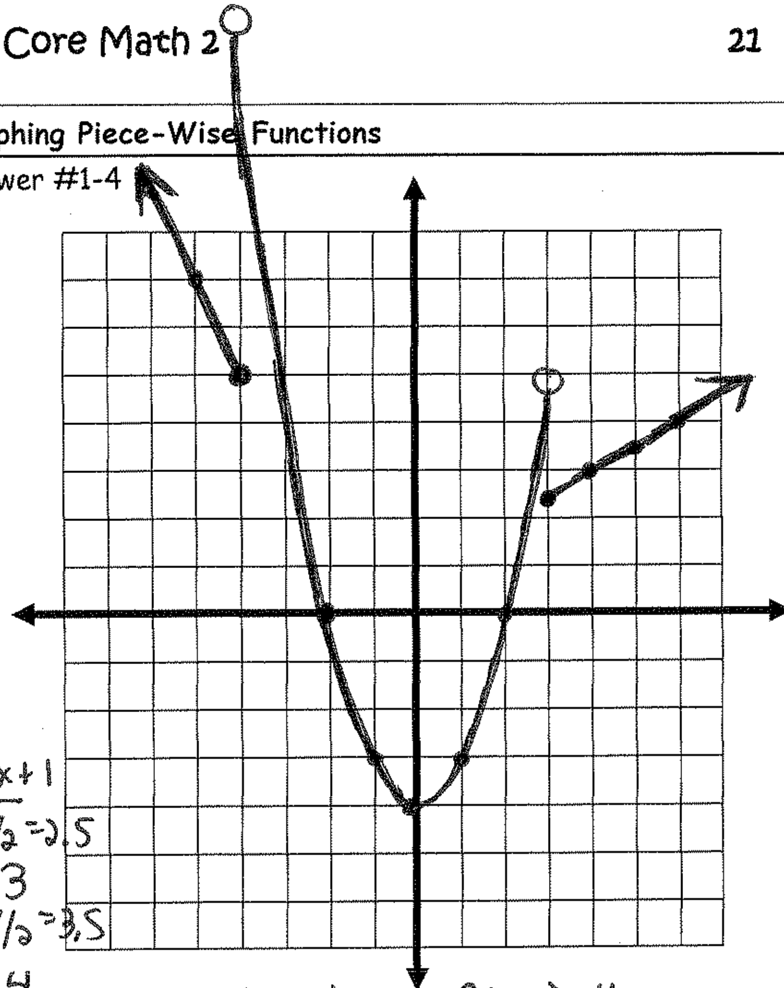
$$f(x) = \begin{cases} -2x-3, & x \le -4 \\ x^2-4, & -4 < x < 3 \\ \frac{1}{2}x+1, & x \ge 3 \end{cases}$$

1. f(2) = 0 — middle rule 2^2-4
2. f(-4) = 5 — top rule $-2(-4)-3$
3. f(8) = 5 — bottom rule $\frac{1}{2}(8)+1$
4. Graph the function

x	$-2x-3$	
-4 closed	5	$-2(-4)-3$
-5	7	$-2(-5)-3$
-6	9	$-2(-6)-3$

x	x^2-4
-4 open	12
-2	0
0	-4
3 open	5
1	-3

x	$\frac{1}{2}x+1$
3 closed	$5/2 = 2.5$
4	3
5	$7/2 = 3.5$
6	4



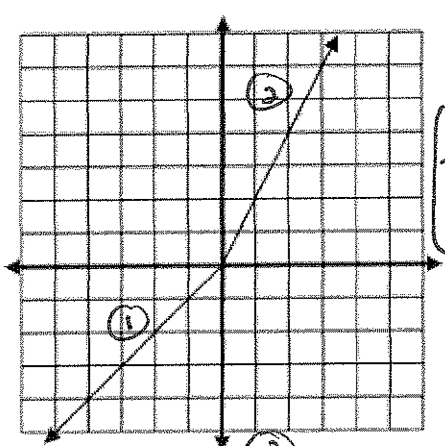
D: all reals R: $y \ge -4$

PIECE-WISE FUNCTIONS CONTINUED

Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tick mark.

Remember linear equation $y = mx+b$ where $m = \text{slope} = \frac{y_2-y_1}{x_2-x_1}$ and $(0, b) = y$ intercept

1.



$$f(x) = \begin{cases} x, & x \le 0 \\ 2x, & x > 0 \end{cases}$$

OR

$$f(x) = \begin{cases} -x, & x < 0 \\ 2x, & x \ge 0 \end{cases}$$

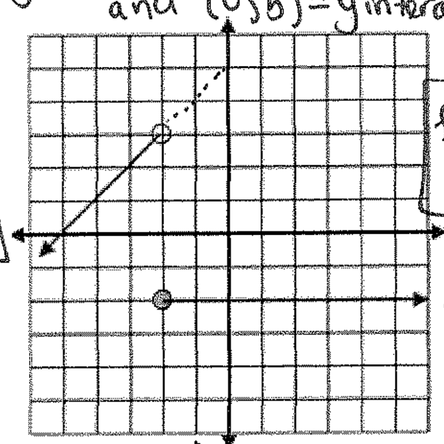
location of = part doesn't matter because (0,0) fits both parts

① $y = mx + b$
m = 1, 0 b/c (0,0) y-int
$y = 1x + 0$
$y = x$

② $y = mx + b$
$m = \frac{2}{1}$ ← find with counting OR with (1,2) (0,0) $m = \frac{2-0}{1-0}$, b = 0
$m = 2$
$y = 2x + 0$
$y = 2x$

D: all reals or $(-\infty, \infty)$
R: all reals or $(-\infty, \infty)$

2.



$$f(x) = \begin{cases} x+5, & x < -2 \\ -2, & x \ge -2 \end{cases}$$

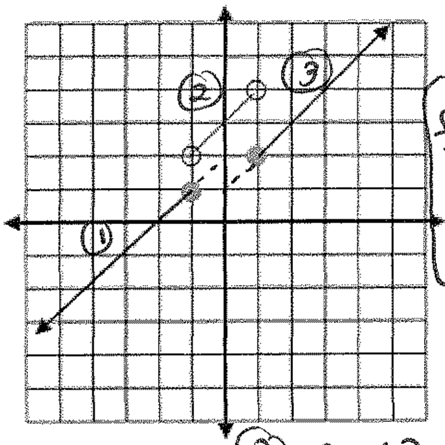
② ← $y = -2$ horizontal line

① if continue left part to y axis, y-intercept is (0,5) so b = 5
m = 1 by counting or formula slope
$y = mx + b$
$y = 1x + 5$
$y = x + 5$

D: all reals $(-\infty, \infty)$
R: $y < 3$ or $(-\infty, 3)$

You Try #3-6

3.

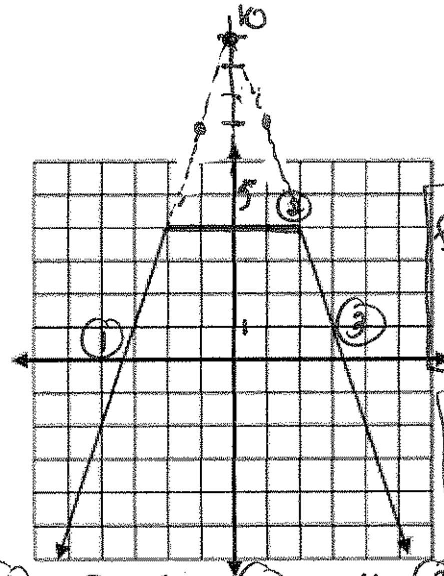


$f(x)=\begin{cases} x+2, & x\le -1 \\ x+3, & -1<x<1 \\ x+1, & x\ge 1 \end{cases}$

D: all reals
R: $y\le 1$ or $y\ge 2$

① $y=1x+2$ — m = 1 by counting or formula; y-int (0,2) if continue graph
② $y=x+3$ — m = 1; y-int (0,3)
③ $y=x+1$ — m = 1; y-int (0,1)

4.

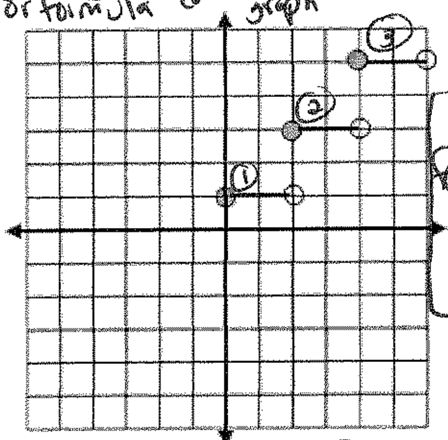


$f(x)=\begin{cases} 3x+10, & x\le -2 \\ 4, & -2<x<2 \\ -3x+10, & x\ge 2 \end{cases}$

D: all reals
R: $y\le 4$

① $y=3x+10$ — m = 3 by counting or formula; y-int (0,10) if continue graph
② $y=4$ — horiz line
③ $y=-3x+10$ — m = -3 by counting or formula; y-int (0,10)

5.



$f(x)=\begin{cases} 1, & 0\le x<2 \\ 3, & 2\le x<4 \\ 5, & 4\le x<6 \end{cases}$

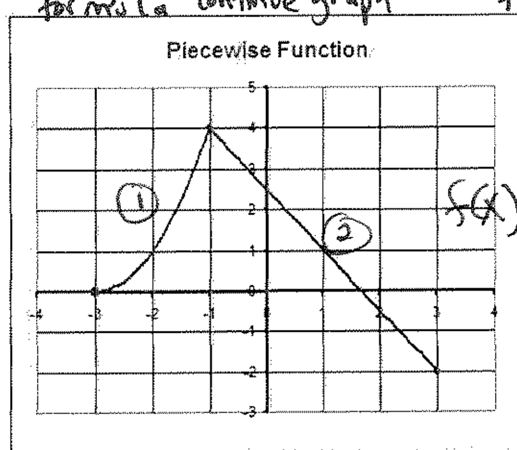
OR

$y=2[\![\tfrac{1}{2}x]\!]+1$

① $y=1$ horiz. line ② $y=3$ horiz. line ③ $y=5$ horiz. line

D: $0\le x<6$
R: $\{1,3,5\}$

6.



$f(x)=\begin{cases} (x+3)^2 \text{ OR } x^2+6x+9, & -3\le x<-1 \\ -\frac{3}{2}x+\frac{5}{2}, & -1\le x\le 3 \end{cases}$

D: $-3\le x\le 3$
R: $-2\le y\le 4$

① $y=(x+3)^2$ OR $y=x^2+6x+9$ — it's a parabola moved left 3 and goes up 1 rt 1, up 3 rt 1
② $y=\frac{-3}{2}x+\frac{5}{2}$ or $y=-1.5x+2.5$ — $m=-\frac{3}{2}$; y-int $(0,\frac{5}{2})$

Another Application: We also see piece-wise functions in our tax structure:

$$f(x)=\begin{cases} 0, \text{ if } 0\le x\le 15{,}000 \\ 0.15(x-15{,}000), \text{ if } 15{,}000<x\le 40{,}000 \\ 6000+0.25(x-40{,}000), \text{ if } 40000<x\le 250{,}000 \\ 37{,}500+0.40(x-250{,}000), \text{ if } 250{,}000\le x \end{cases}$$

*For income at or below $15,000, no tax is charged.
*Above $15,000 and at or below $40,000, the rate is 15% for all monies earned over $15,000.
*Above $40,000, the rate increases to 25% on all monies earned over $40,000 (where did the $6000 come from?), until income is $250,000.
*Above that level, the rate is 40%. (Where did the $37,500 come from?)

How much would I owe in taxes if I made

a. $12,000 $\boxed{0}$ top rule
b. $17,000 $0.15(17000-15000)=\boxed{300}$ 2nd rule
c. $47,000 $6000+0.25(47000-40000)=\boxed{7750}$ 3rd rule
d. $470,000 $37500+.40(470000-250000)$ $=\boxed{125{,}500}$ bottom rule

37500 is 15% of 250000

6000 is .15(40000) → pay 6000 plus 25% on part earned over 40000