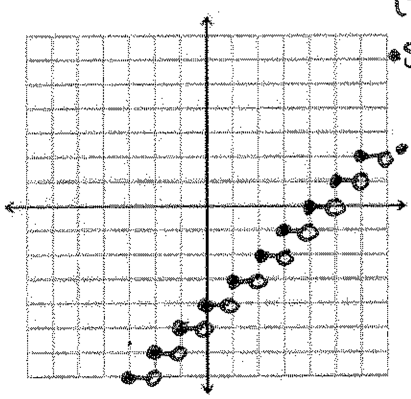
## Day 5: Graphing Piece-Wise Functions

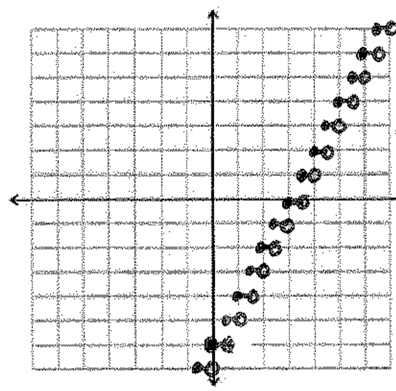
**Warm-Up:**

1) Why do we need to use open and closed dots at the ends of our "steps"?
   - So that we still have a function
   - because each integer x-value can only be on 1 step

2) Graph $y = [\![x-4]\!]$
   - Steps start $(4,0)$
   - step length $1/b = 1/1 = 1$
   - step height $= a = 1$



3) Graph $y = [\![2x]\!] - 6$
   - steps start $(0,-6)$
   - step length $1/b = 1/2$
   - step height $a = 1$



**Notes: Graphing Piece-Wise Functions**

Up to now, we've been looking at functions represented by a single equation. In real life, however, functions are represented by a combination of equations, each corresponding to a part of the domain. These are called **piecewise functions.**

Example 1:

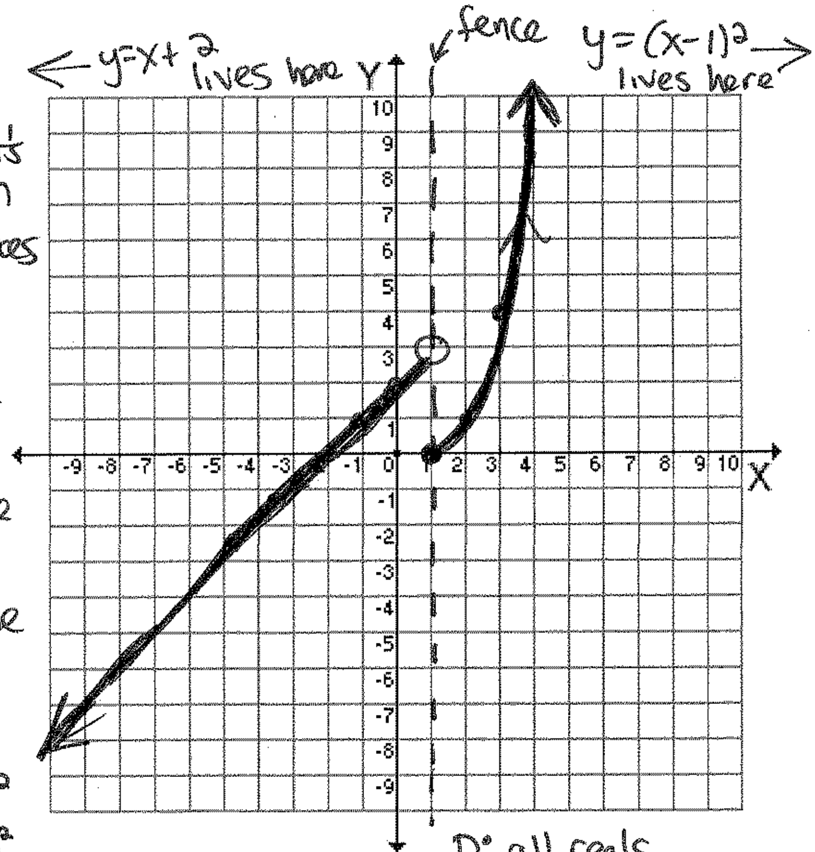
$$Y = \begin{cases} x+2 & ; x < 1 \\ (x-1)^2 & ; x \geq 1 \end{cases}$$

← it's in 2 pieces

$f(-2) = -2+2 = \boxed{0}$ use top rule

$f(3) = (3-1)^2 = 2^2 = \boxed{4}$ use bottom rule

$f(1) = (1-1)^2 = 0^2 = \boxed{0}$ use bottom rule




top rule

| | |
|---|---|
| open circle 1 | 3 = 1+2 |
| 0 | 2 = 0+2 |
| -1 | 1 = -1+2 |

bottom rule

| | |
|---|---|
| closed circle 1 | 0 = (1-1)² |
| 2 | 1 = (2-1)² |
| 3 | 4 = (3-1)² |
| 4 | 9 = (4-1)² |

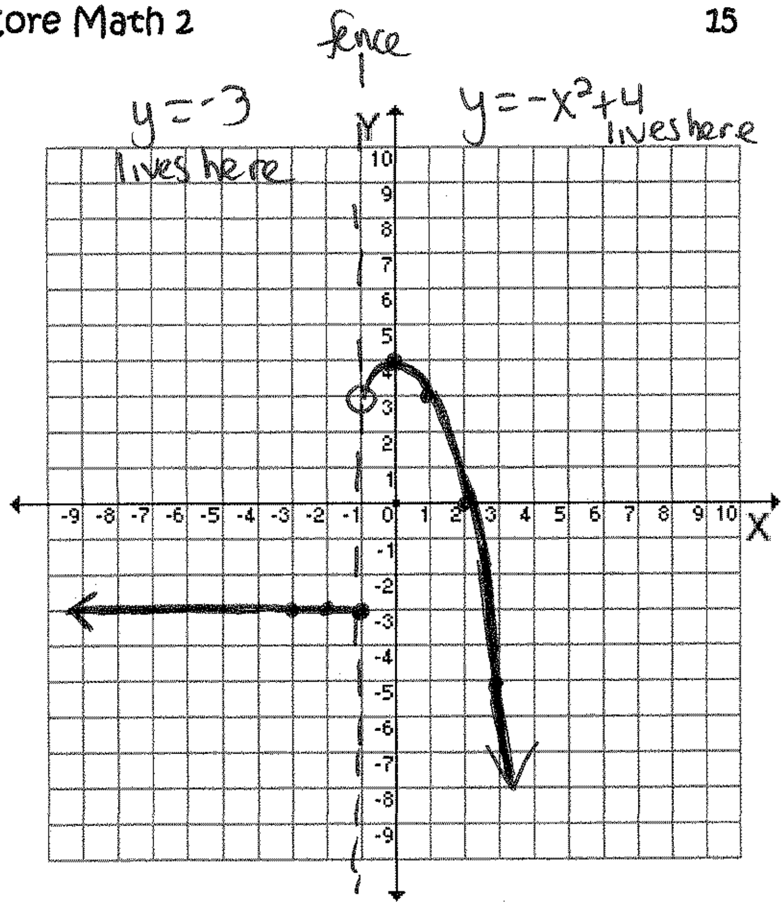
D: all reals
R: all reals

Example 2: You Try!

$$y = \begin{cases} -3 & ; x \le -1 \\ -x^2+4 & ; x > -1 \end{cases}$$

| | x | y |
|---|---|---|
| closed circle | -1 | -3 |
| | -2 | -3 |
| | -3 | -3 |

| | x | y | |
|---|---|---|---|
| open circle | -1 | 3 | $-(-1)^2+4$ |
| | 0 | 4 | $-(0)^2+4$ |
| | 1 | 3 | $-(1)^2+4$ |
| | 2 | 0 | $-(2)^2+4$ |
| | 3 | -5 | $-(3)^2+4$ |




Domain: all reals
Range: $y \le 4$

Example 3:

$$y = \begin{cases} |x+3| & ; x \le -3 \\ 4 & ; -3 < x \le 2 \\ 5-x & ; x > 2 \end{cases}$$

top rule

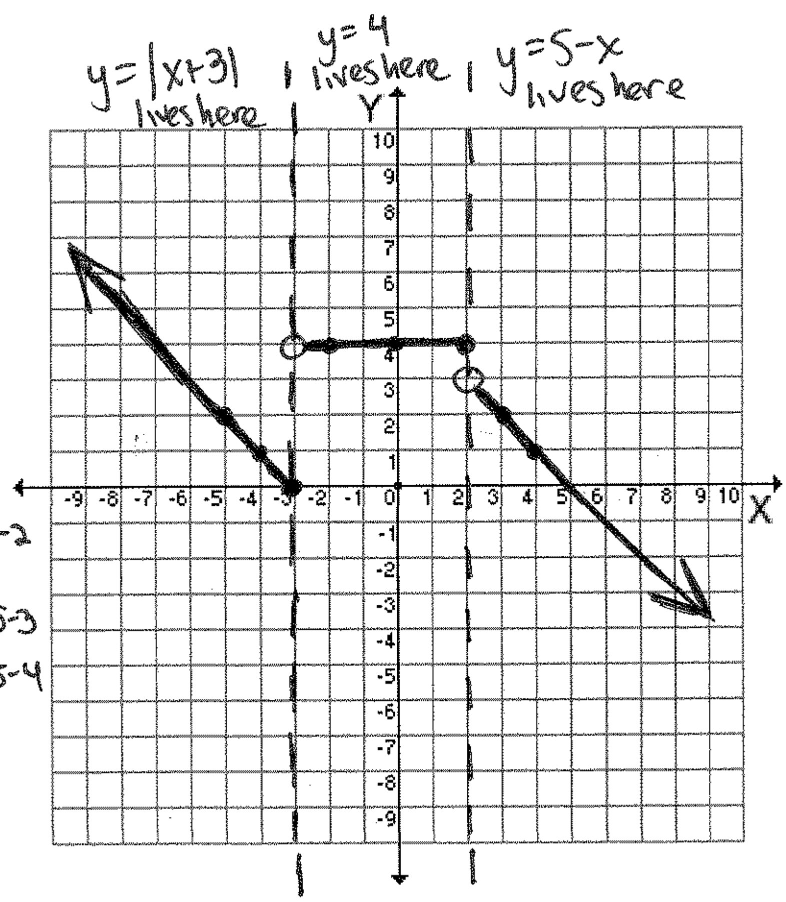
| | x | y |
|---|---|---|
| closed circle | -3 | $0 = \lvert -3+3 \rvert$ |
| | -4 | $1 = \lvert -4+3 \rvert$ |
| | -5 | $2 = \lvert -5+3 \rvert$ |

middle rule

| | x | y |
|---|---|---|
| open circle | -3 | 4 |
| | -2 | 4 |
| | 0 | 4 |
| closed circle | 2 | 4 |

bottom rule

| | x | y |
|---|---|---|
| open circle | 2 | $3 = 5-2$ |
| | 3 | $2 = 5-3$ |
| | 4 | $1 = 5-4$ |




Domain: all reals
Range: all reals

Example 4: You Try!

$$y = \begin{cases} 3 & ; x < -1 \\ (x+1)^2 - 2 & ; -1 \le x \le 1 \\ x - 4 & ; x > 1 \end{cases}$$

top rule

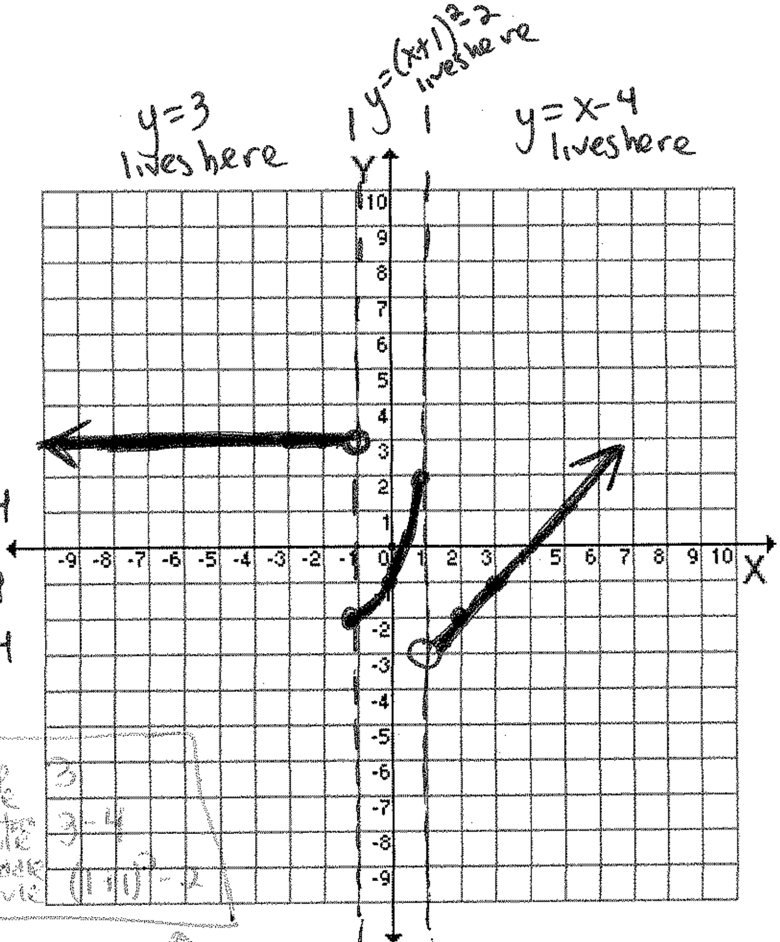
| | |
|---|---|
| open circle -1 | 3 |
| -2 | 3 |
| -3 | 3 |

middle rule

| | |
|---|---|
| -1 closed circle | $-2 = (-1+1)^2-2$ |
| 0 | $-1 = (0+1)^2-2$ |
| 1 closed circle | $2 = (1+1)^2-2$ |

bottom rule

| | |
|---|---|
| 1 open circle | $-3 = 1-4$ |
| 2 | $-2 = 2-4$ |
| 3 | $-1 = 3-4$ |




$f(-2) = 3$ top rule 3
$f(3) = -1$ bottom rule $3-4$
$f(1) = 2$ middle rule $(1+1)^2-2$
Evaluate

Domain: all reals
Range: $y > -3$

Slide Inserted about domain and range

APPLICATIONS

1. When a diabetic takes long-acting insulin, the insulin reaches its peak effect on the blood sugar level in about three hours. This effect remains fairly constant for 5 hours, then declines, and is very low until the next injection. In a typical patient, the level of insulin might be modeled by the following function.

$$f(t) = \begin{cases} 40t + 100 & \text{if } 0 \le t \le 3 \\ 220 & \text{if } 3 < t \le 8 \\ -80t + 860 & \text{if } 8 < t \le 10 \\ 60 & \text{if } 10 < t \le 24 \end{cases}$$

Here, f (t) represents the blood sugar level at time t hours after the time of the injection. If a patient takes insulin at 6 am, find the blood sugar level at each of the following times.

a. 7 am -6
1 hour
$0 \le 1 \le 3 \to 40(1) + 100$
$\boxed{140}$

b. 11 am -6
5 hours
$3 < 5 \le 8$
$\boxed{220}$

c. 3 pm
15:00-6:00
9 hours
$8 < 9 \le 10$
$-80(9) + 860$
$\boxed{140}$

d. 5 pm
17:00 - 6:00
11 hours
$10 < 11 \le 24$
$\boxed{60}$