Unit 4 Day 4 & 5

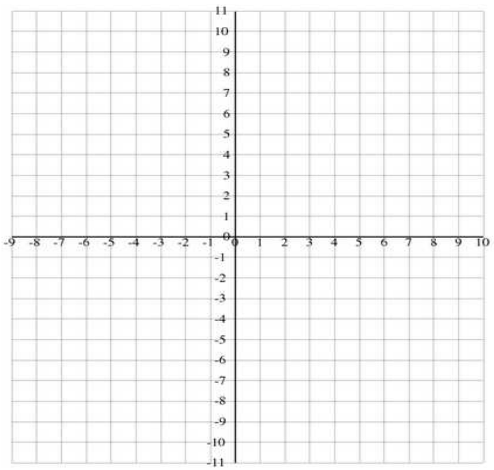
Piecewise Functions

Warm Up

1. Why does the inverse variation have a vertical asymptote?

2. Graph. Find the asymptotes. Write the domain and range using interval notation.

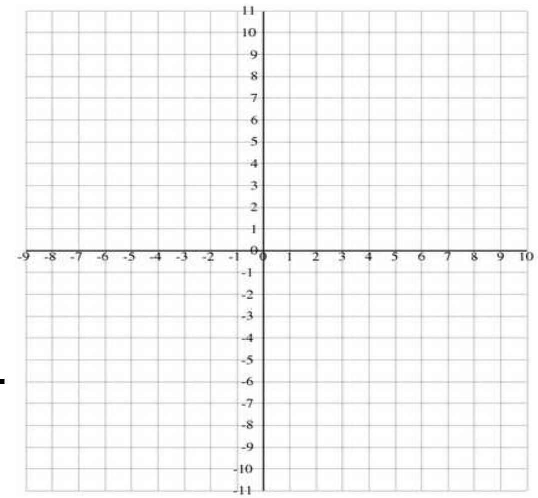
a. $f(x) = \dfrac{-9}{x - 4} + 2$



VA:_____
HA:_____

Domain: ______
Range: ______

b. $f(x) = \dfrac{3}{x+2}$



VA:_____
HA:_____

Domain:____
Range: ____

3. In #2, where is the graph discontinuous? Why? (do for a & b)
4. Given $f(x) = 1 - 4x^2$ and $g(x) = 1 - 2x$
   a. Find $f(1 - 2x)$. Give your answer in standard form.
   b. Evaluate $f(x)/g(x)$. (What value is excluded from the domain?)

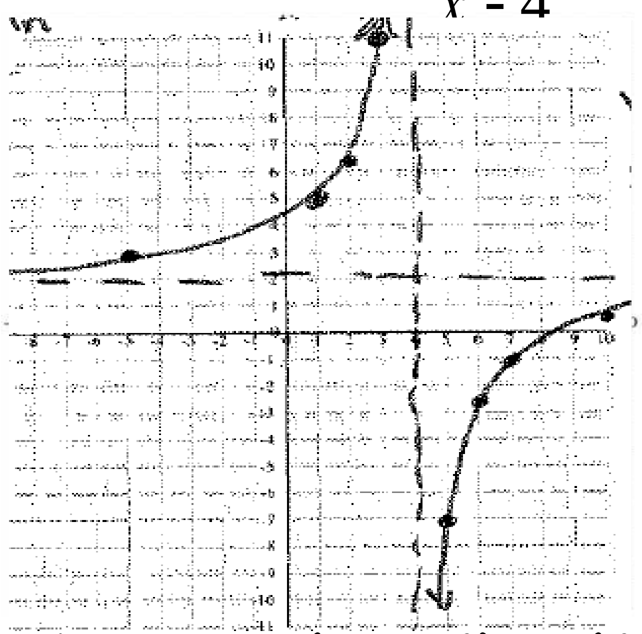
Warm Up ANSWERS

1. Why does the inverse variation have a vertical asymptote?

   y = k/x has a vertical asymptote because we can't divide by zero so the domain must skip the x that would create division by zero

2. Graph. Find the asymptotes. Write the domain and range using set notation.

a. $f(x) = \dfrac{-9}{x-4} + 2$



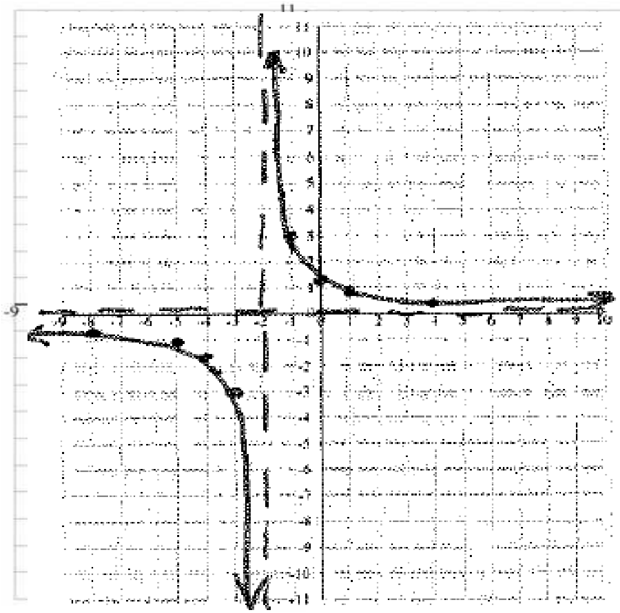
VA: x = 4

HA: y = 2

Domain: (-∞, 4) U (4, ∞)

Range: (-∞, 2) U (2, ∞)

b. $f(x) = \dfrac{3}{x+2}$



VA: x = -2

HA: y = 0

Domain: (-∞, -2) U (-2, ∞)

Range: (-∞, 0) U (0, ∞)

Warm Up ANSWERS

3. In #2, at what value is the graph discontinuous? Why?

What do you think the word **"discontinuous"** means?

-> Where the graph does not continue – where it has a "break" or "skip".

$$f(x) = \frac{-9}{x - 4} + 2$$

a. The graph is discontinuous at **x = 4**

(because VA is x = 4, because x = 4 would give division by 0)

$$f(x) = \frac{3}{x+2}$$

b. The graph is discontinuous at **x = -2**

(because VA is x = -2, because x = -2 would give division by 0)

Warm Up ANSWERS

4. Given $f(x) = 1 - 4x^2$ and $g(x) = 1 - 2x$

   a. Find $f(1 - 2x)$. Give your answer in standard form.

   $= 1 - 4(1 - 2x)^2 \quad = 1 - 4(1 - 2x)(1 - 2x)$

   $= 1 - 4(1 - 4x + 4x^2) = 1 - 4 + 16x - 16x^2$

   $= \mathbf{-16x^2 + 16x - 3}$

   NOTE: $f(1 - 2x)$ could have also been expressed as $f(\,g(x)\,)$. You'll learn more about this later! ☺

   b. Evaluate $f(x)/g(x)$. (What value is excluded from the domain?)

   $$\frac{f(x)}{g(x)} = \frac{1 - 4x^2}{1 - 2x} = \frac{\cancel{(1 - 2x)}(1 + 2x)}{\cancel{1 - 2x}}$$

   Do you remember difference of squares?

   $f(x)/g(x) = 1 + 2x$ but what must we consider?

   $f(x)/g(x) = \mathbf{1 + 2x,\ x \neq ½}$

   (because x = ½ gives division by zero)

Homework Answers Packet p. 5 odd

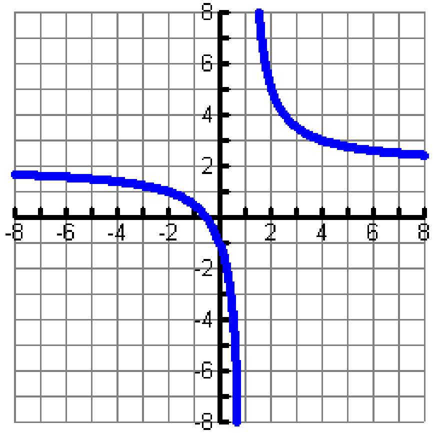
1) $y = \dfrac{-3}{x-2} + 1$

3) $y = \dfrac{-3}{x-4} - 2$

5) $y = \dfrac{-3}{x-3}$

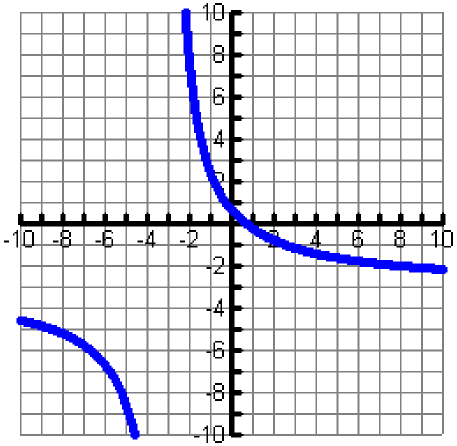
7) $y = \dfrac{-3}{x+3} - 1$

9)



HA: y = 2
VA: x = 1

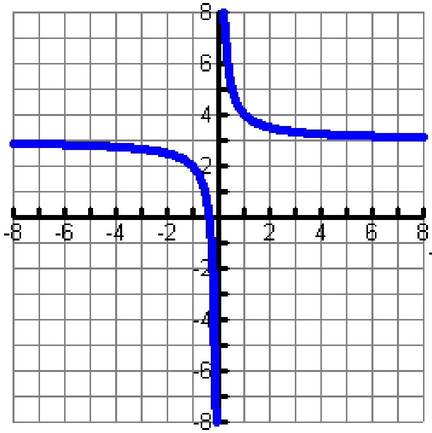
11)



HA: y = -3
VA: x = -3

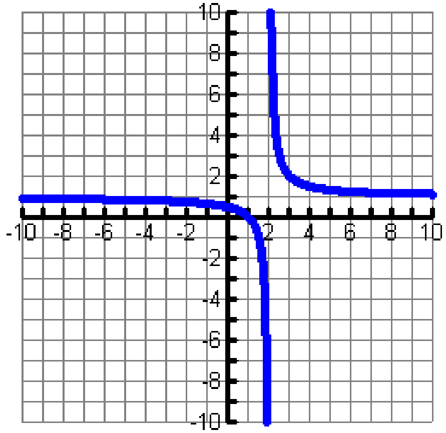
Homework Answers Packet p. 5 odds

13)



HA: y = 3
VA: x = 0

15)



HA: y = 1
VA: x = 2

17) Let x = dependent variable = # of awards
Let a = # of awards, c = cost of awards

$$c = \frac{750}{a}$$

H.A. c = 0 (like y = 0) V.A. a = 0 (like x = 0)

Theoretical Domain: a < 0, a > 0
Theoretical Range: c < 0, c > 0

Practical Domain: a > 0
Practical Range: c > 0

As the school buys more awards, they have less money to spend on each award so c = 0 (y = 0), the horizontal asymptote.
As the school buys less awards, they have more money to spend on each award so a = 0 (x = 0), the vertical asymptote.

Notes p. 10 Sometimes the function isn't in a nice graphing form.

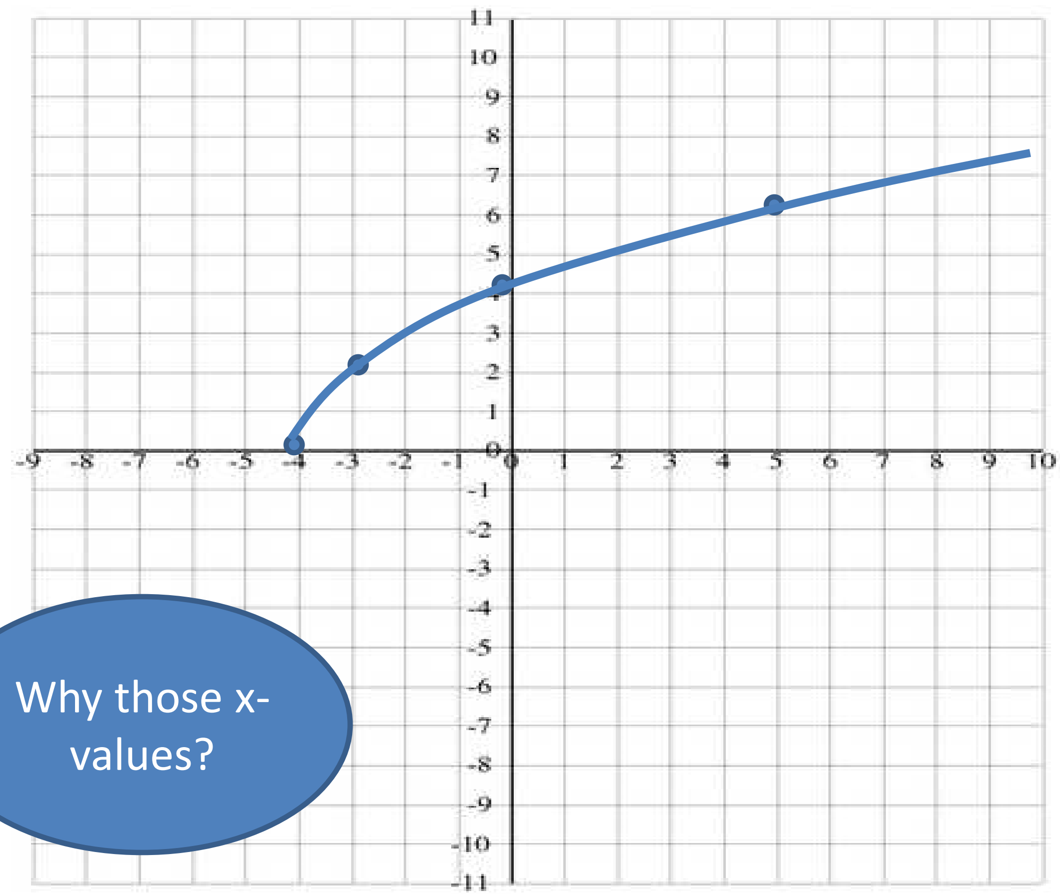
Hint: First change the following into the $y = a\sqrt{x-h}$ form.

11. $y = \sqrt{4x+16}$

$$y = \sqrt{4(x+4)}$$

$$y = 2\sqrt{x+4}$$

x	y
-4	0
-3	2
0	4
5	6



Why those x-values?

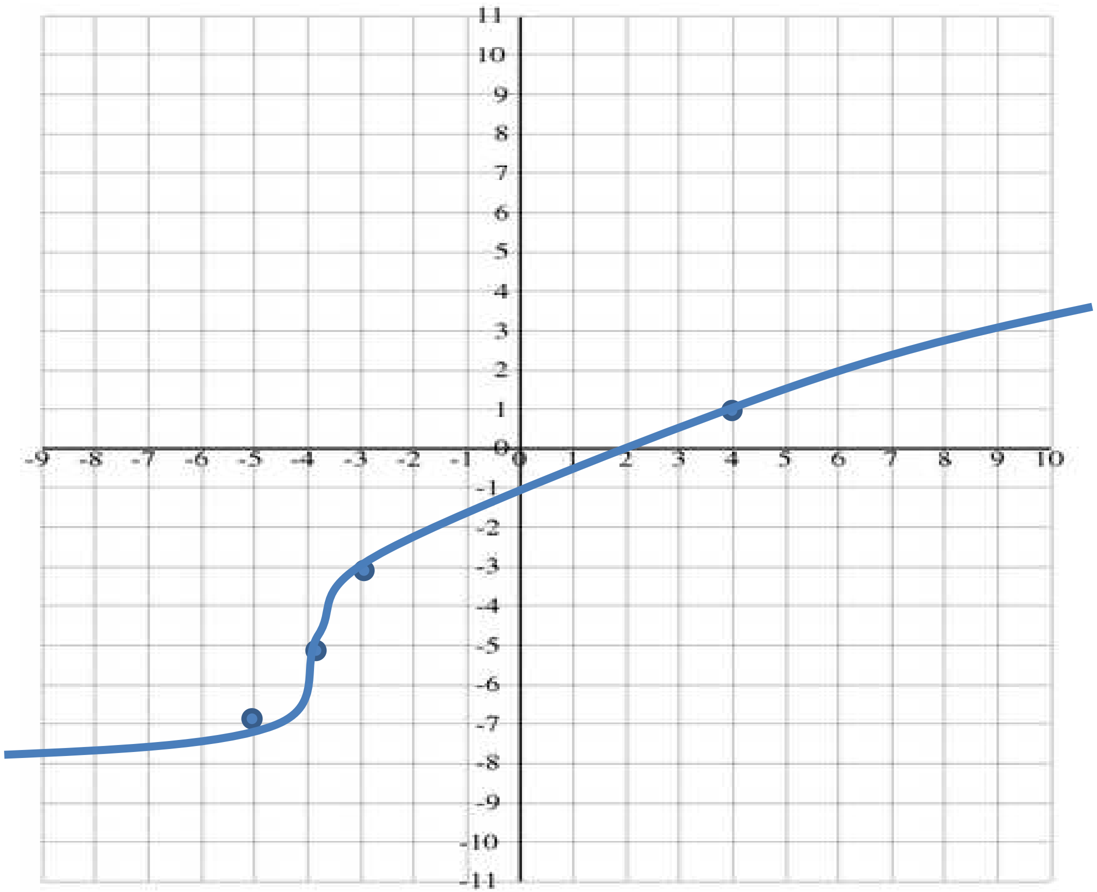
Notes p. 10 Put the following in graphing form. Then graph it.

12. $f(x) = \sqrt[3]{8x+32} - 5$ (This is not graphing form)

$$y = \sqrt[3]{8(x+4)} - 5$$

$$y = 2\sqrt[3]{x+4} - 5$$

x	y
-5	-7
-4	-5
-3	-3
4	-1



Notes p. 10

1) Given $f(x) = 3x – 2x^2$
Evaluate $f(2x + 2) – f(x)$

$– 6x^2 – 13x – 2$

2) Given $g(x) = 2x^2 + 4$
Evaluate $g(x - 1) + g(3)$

$2x^2 – 4x + 28$

Tonight's Homework

Packet Pages. 6 – 7

It's also a good idea to start studying for our quiz coming soon! ☺

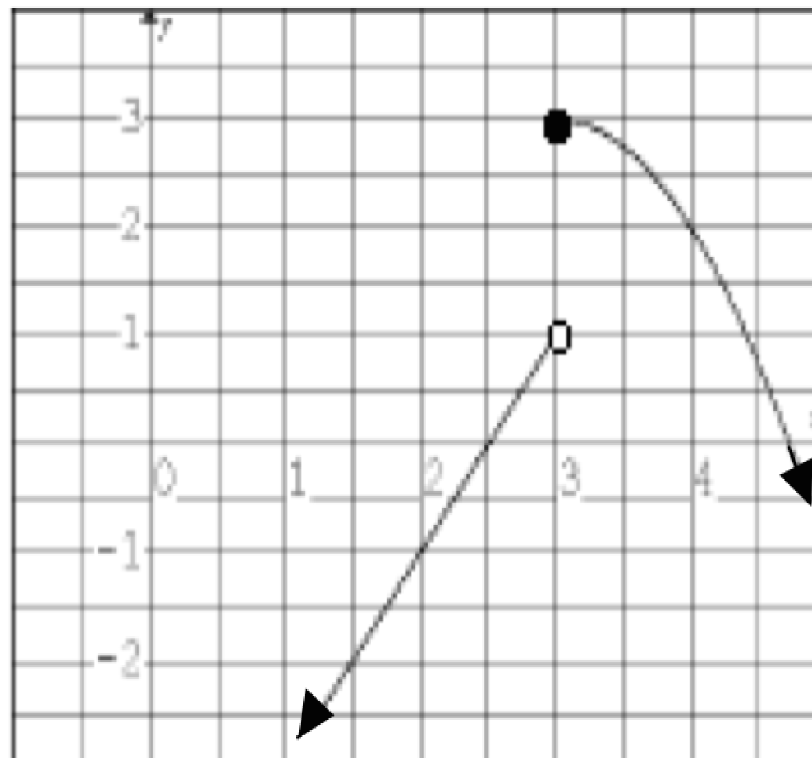
Piece-Wise Function Notes

A function can be in pieces…
You can create functions that behave differently depending on the input (x) value.

<u>Example</u>:

$$\text{Let } f(x) = \begin{cases} 2x - 5 & \text{for } x < 3 \\ -(x-3)^2 + 3 & \text{for } x \geq 3 \end{cases}$$

Note: We will refer to the *x*-value where the function changes as the "transition point."



Piecewise Functions

$$y = \begin{cases} x + 2 \; ; & x < 1 \leftarrow ① \\ (x-1)^2 \; ; & x \geq 1 \leftarrow ② \end{cases}$$

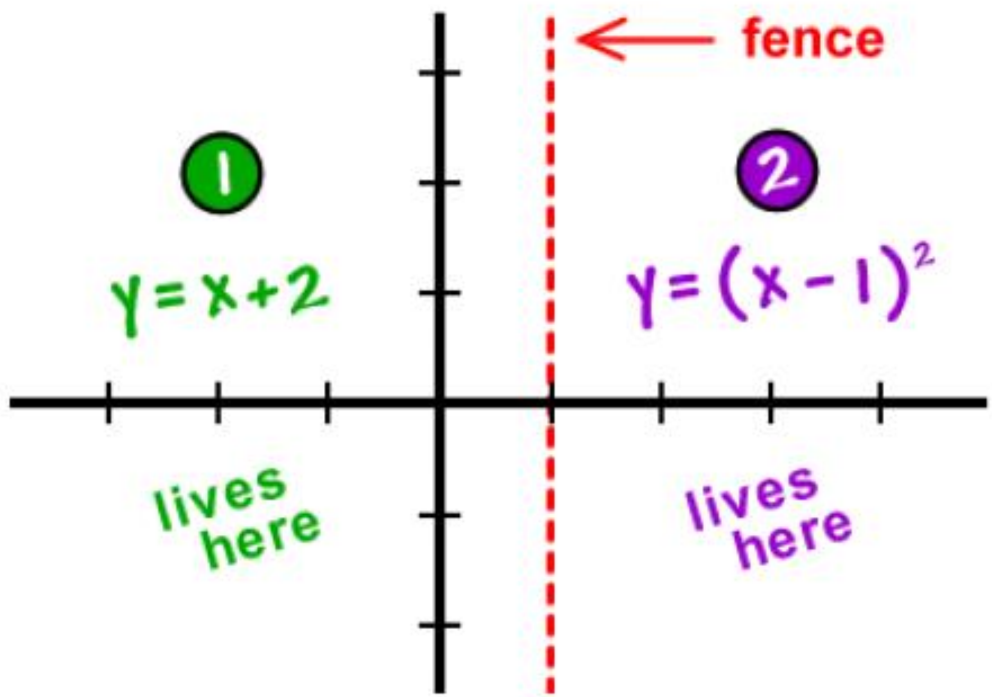
It's in two pieces!

You will be graphing pieces of $y = x + 2$ and $y = (x - 1)^2$

$$y = \begin{cases} x+2 & ; x < 1 \\ (x-1)^2 & ; x \geq 1 \end{cases}$$

Each piece must live ONLY in its own neighborhood.

Let's put up a fence, so we don't make any mistakes:



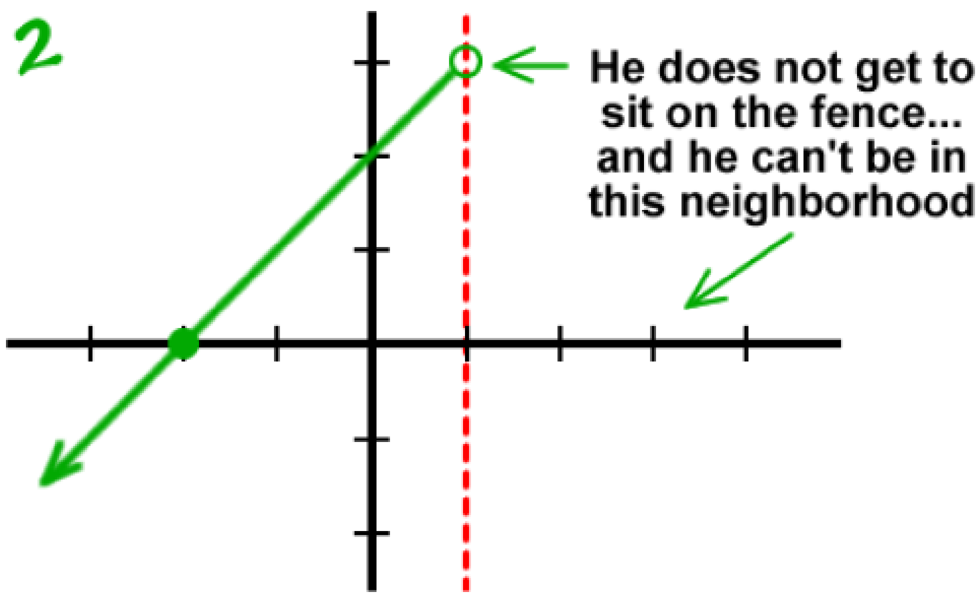
Now, we just need to figure out who the fence owner is...

$$y = \begin{cases} x+2 \ ; & x < 1 \\ (x-1)^2 \ ; & x \geq 1 \end{cases}$$

← This guy has the "=", so he gets to live ON the fence.

Let's graph part ① :

$y = x + 2$



Top Rule

1	3	open
0	2	
-1	1	
-2	0	

<u>Tip to help with graphing</u>: Make a table of values.

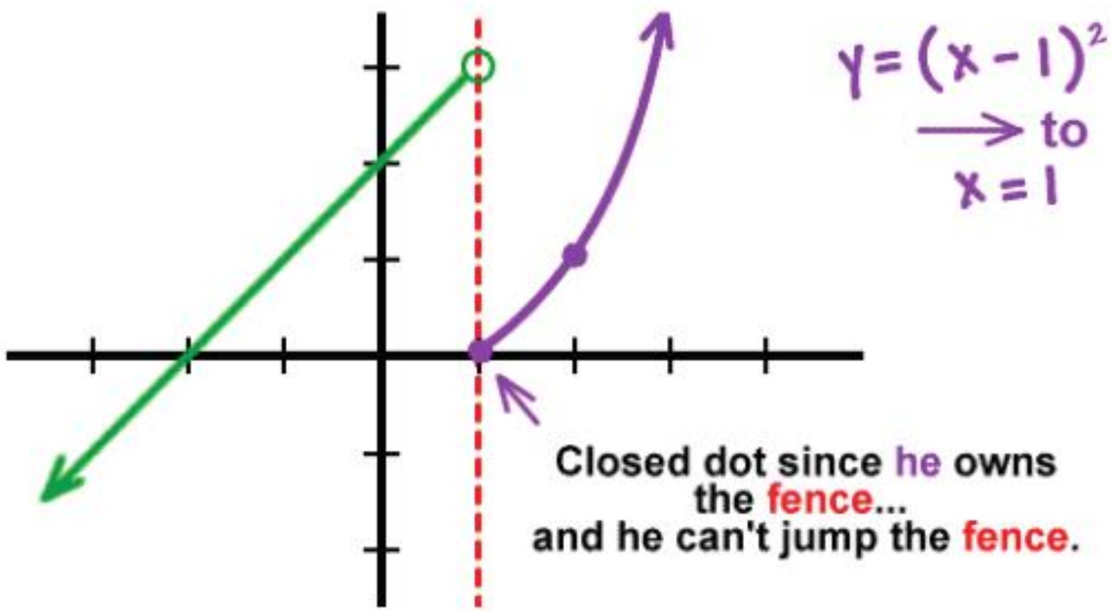
Your table MUST start with the break point, x = 1 in this case. In some cases (like this one) this first point in the table will be an open circle.

Then, pick other x-values that fit in the rule, x < 1 in this case.

Example 1:

$$y = \begin{cases} x+2 & ; x < 1 \\ (x-1)^2 & ; x \geq 1 \end{cases}$$

Now, let's graph part 2 :



Bottom Rule

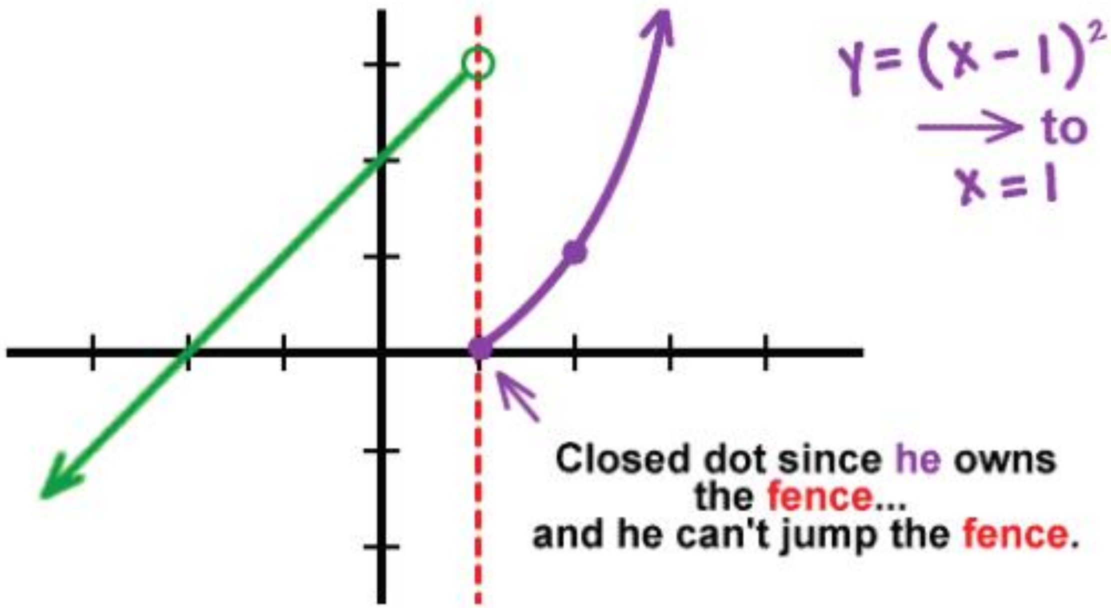
1	0	closed
2	1	
3	4	

Done!

<u>Tip to help with graphing</u>: Make a table of values.
Your table MUST start with the break point, x = 1 in this case. Here the break point is a closed circle because the rule is x ≥ 1 (NOT x >1).
Then, pick other x-values that fit in the rule, x ≥ 1 in this case.

Example 1:

$$y = \begin{cases} x + 2 & ; \; x < 1 \\ (x-1)^2 & ; \; x \geq 1 \end{cases}$$



Find:

Top rule

f(-2) = **-2 + 2 = 0**

Bottom rule

f(3) = **$(3 - 1)^2 = 4$**

Bottom rule

f(1) = **$(1-1)^2 = 0$**

Domain: (-∞ , ∞)

Range: (-∞ , ∞)

Example 2:

$$y = \begin{cases} -3 & ; x \le -1 \\ -x^2 + 4 & ; x > -1 \end{cases}$$

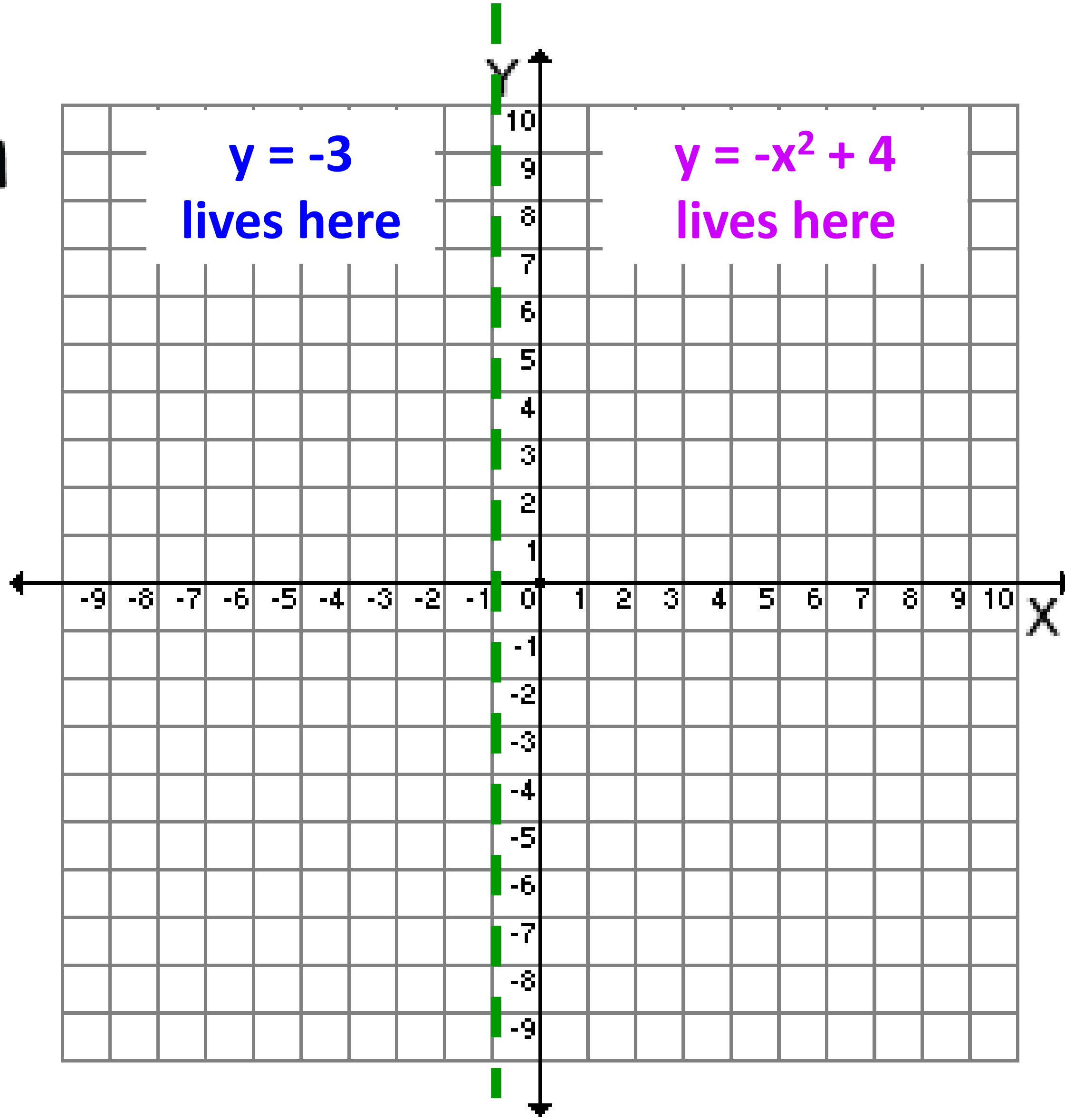
Set up your fence first!! ☺

Top Rule

-1	-3
-2	-3
-3	-3

Bottom Rule

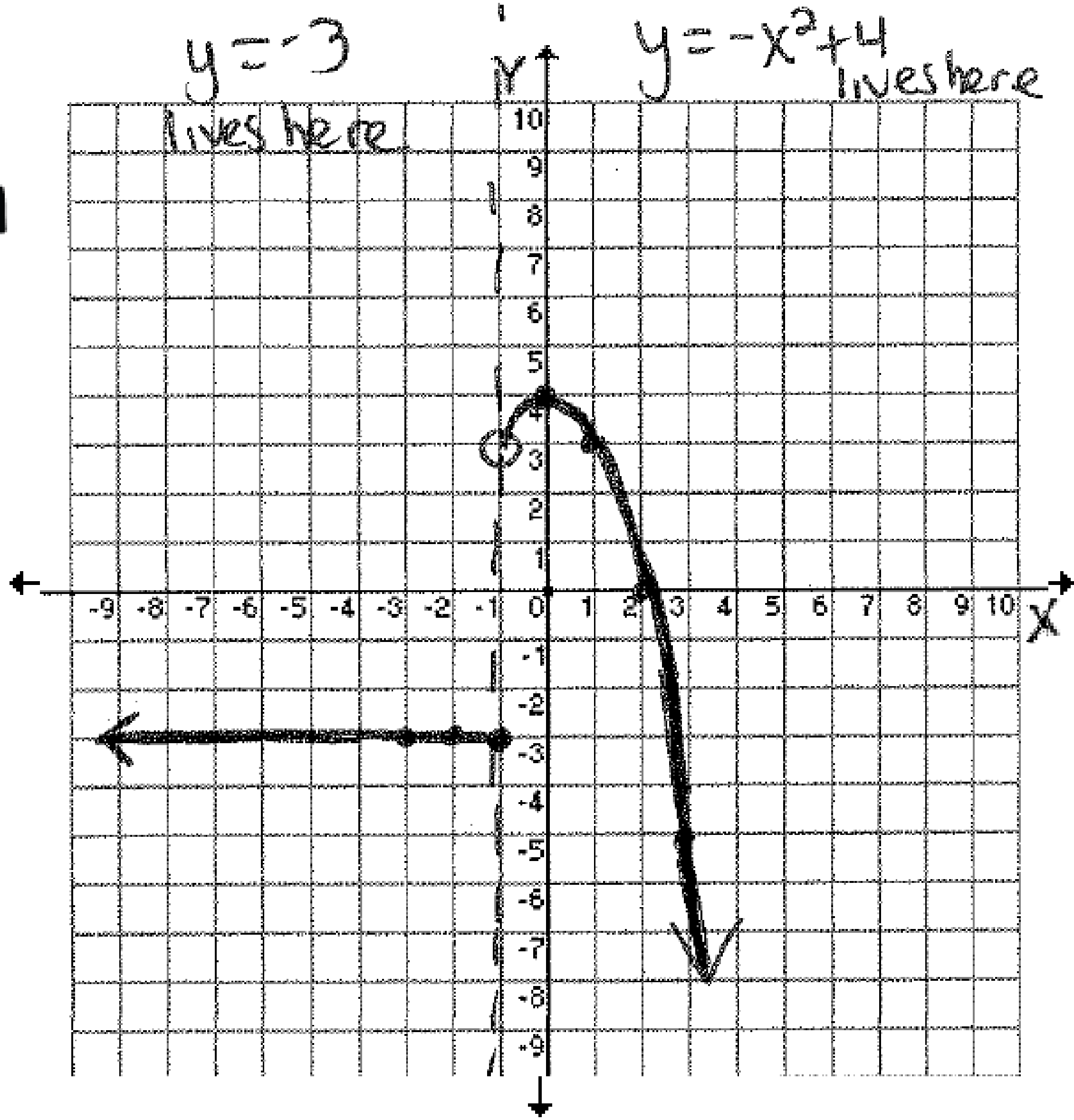
-1	3	open
0	4	
1	3	



<u>Tip to help with graphing</u>: Make a table of values. Remember, each table MUST start with the break point, x = -1. Be sure to check if your break point is an open or closed circle.

Example 2:

$$y = \begin{cases} -3 & ; x \le -1 \\ -x^2 + 4 & ; x > -1 \end{cases}$$



Find:

Top rule

f(-2) = **$x \le -1$ so $y = -3$**

Bottom rule

f(3) = **$-(3)^2 + 4 = -5$**

Bottom rule

f(1) = **$-(1)^2 + 4 = 3$**

$Domain: (-\infty, \infty)$

$Range: (-\infty, 4]$

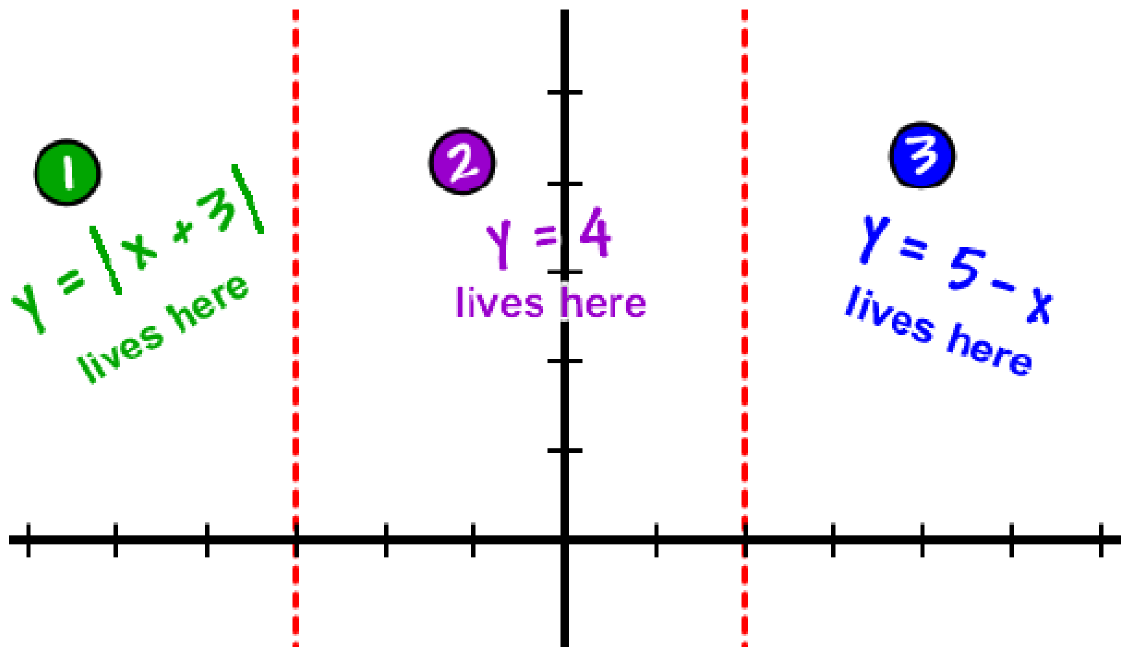
Example 3

Let's do one with three pieces...

Graph

$$y = \begin{cases} |x+3| \;; & x \le -3 \quad \leftarrow (1) \\ 4 \;; & -3 < x \le 2 \quad \leftarrow (2) \\ 5 - x \;; & x > 2 \quad \leftarrow (3) \end{cases}$$

Let's put up the fencing:



Example 3:

$$y = \begin{cases} |x+3| & ; x \le -3 \\ 4 & ; -3 < x \le 2 \\ 5 - x & ; x > 2 \end{cases}$$

Top Rule

-3	0	closed
-4	1	
-5	2	

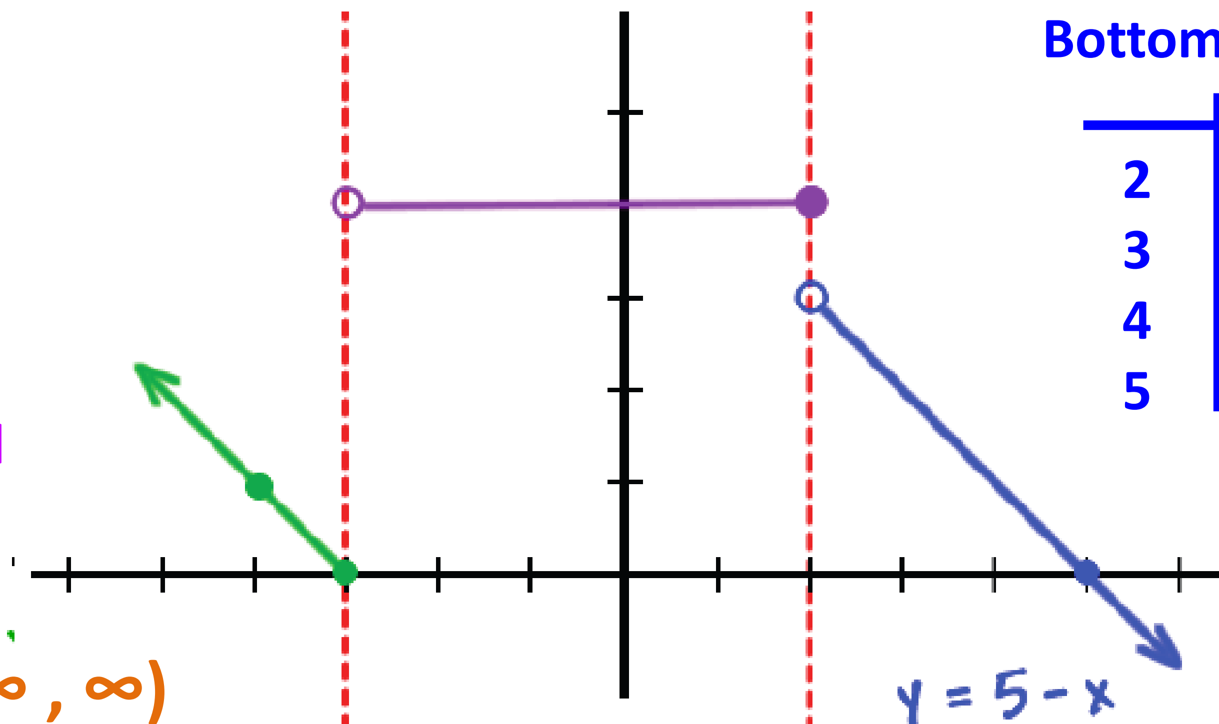
Remember that they can't cross over into the other neighborhoods!

Middle Rule

-3	4	open
0	4	
2	4	closed

Bottom Rule

2	3	open
3	2	
4	1	
5	0	



$y = 5 - x$

really

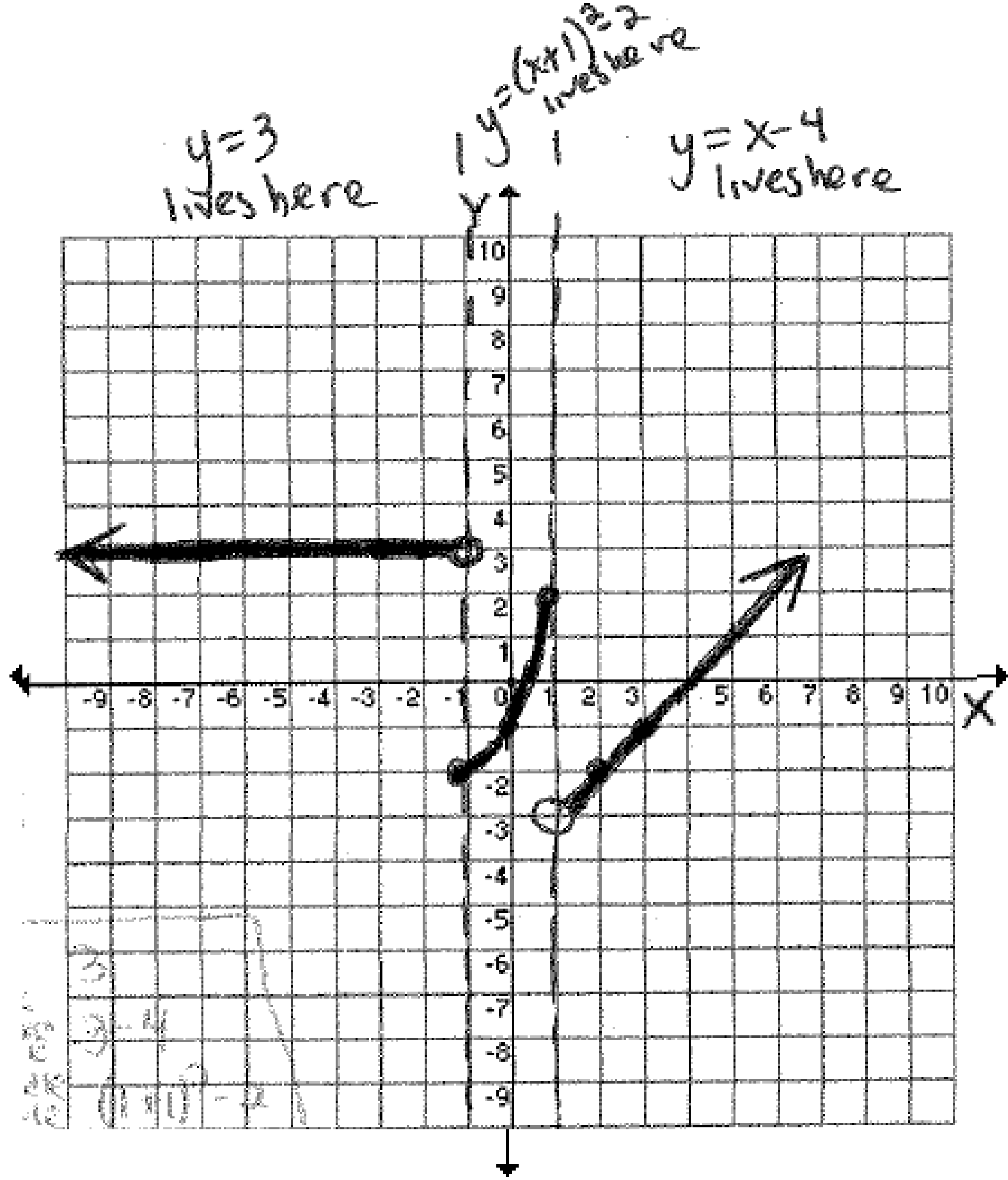
$y = -x + 5$

Domain: (-∞ , ∞)

Range: (-∞ , ∞)

Example 4: YOU TRY!

$$y = \begin{cases} 3 & ; x < -1 \\ (x+1)^2 - 2 & ; -1 \leq x \leq 1 \\ x - 4 & ; x > 1 \end{cases}$$



Find:

Top rule

f(-2) = **x < -1 so y = 3**

Bottom rule

f(3) = **3 – 4 = -1**

Middle rule

f(1) = **$(1 + 1)^2$ - 2 = 2**

$Domain: (-\infty, \infty)$

$Range: (-3, \infty)$

Day 5 Lesson

Starts next

Day 5 Warm-up

On Bottom of Notes p. 14

Given $f(x) = x^2 - 5x - 2$, evaluate:

1) $f(-3)$

2) $f(x - 4)$

3) $f(x - 3) - 4f(x)$

Done Early? Complete Notes p. 17 #4 ☺

Warm-up ANSWERS

Given $f(x) = x^2 - 5x - 2$, evaluate:

1) $f(-3)$

$= (-3)^2 - 5(-3) - 2$

$= 9 + 15 - 2$

$= \mathbf{22}$

2) $f(x - 4)$

$= (x - 4)^2 - 5(x - 4) - 2$

$= (x - 4)(x - 4) - 5x + 20 - 2$

$= x^2 - 8x + 16 - 5x + 18$

$= \mathbf{x^2 - 13x + 34}$

3) $f(x - 3) - 4f(x)$

$= (x - 3)^2 - 5(x - 3) - 2 - 4(x^2 - 5x - 2)$

$= (x - 3)(x - 3) - 5x + 15 - 2 - 4x^2 + 20x + 8$

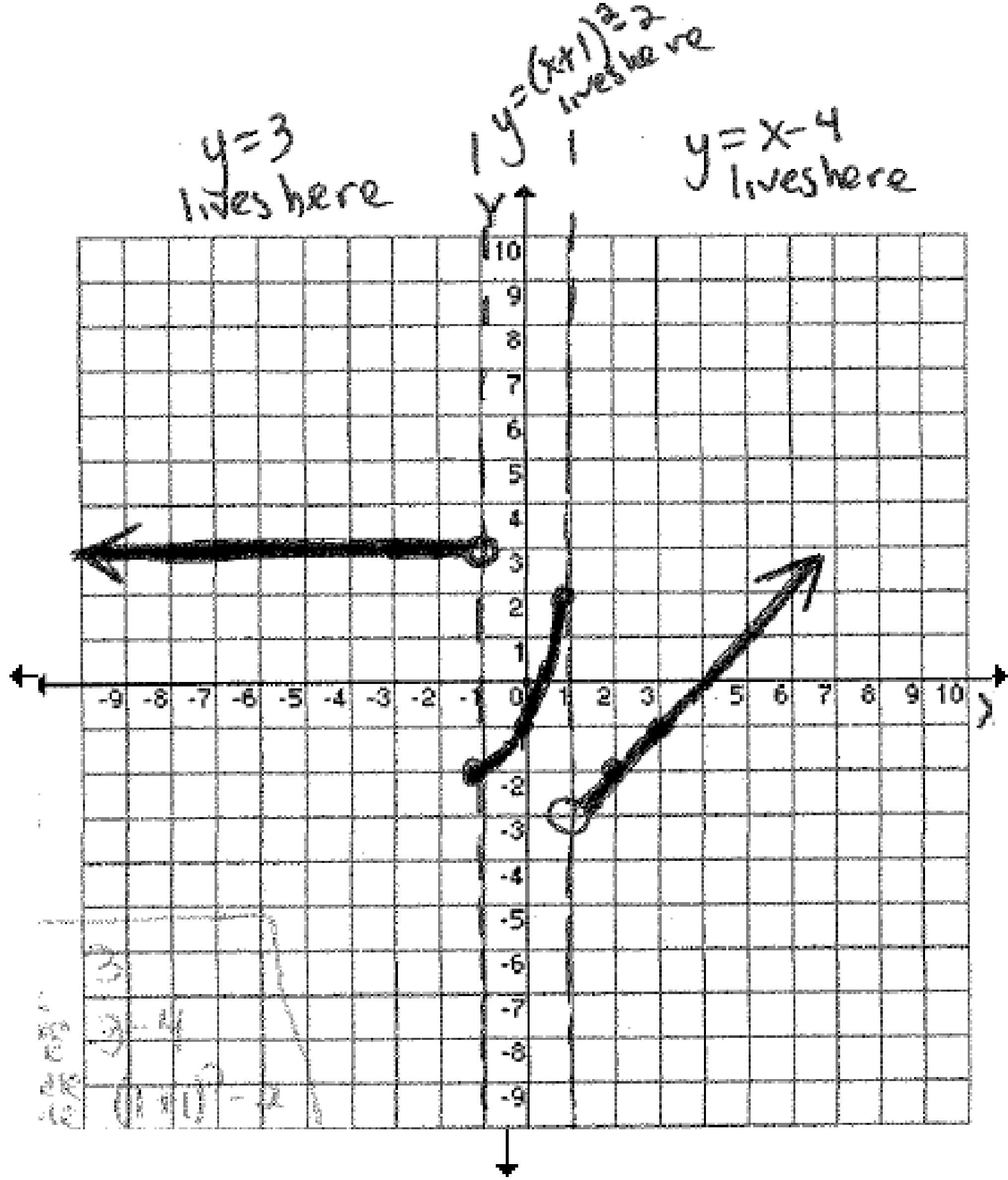
$= x^2 - 6x + 9 - 5x + 13 - 4x^2 + 20x + 8$

$= \mathbf{-3x^2 + 9x + 30}$

Notes p. 17 #4

Example 4: YOU TRY!

$$y = \begin{cases} 3 & ; x < -1 \\ (x+1)^2 - 2 & ; -1 \le x \le 1 \\ x - 4 & ; x > 1 \end{cases}$$



Find:

Top rule

f(-2) = **x < -1 so y = 3**

Bottom rule

f(3) = **3 – 4 = -1**

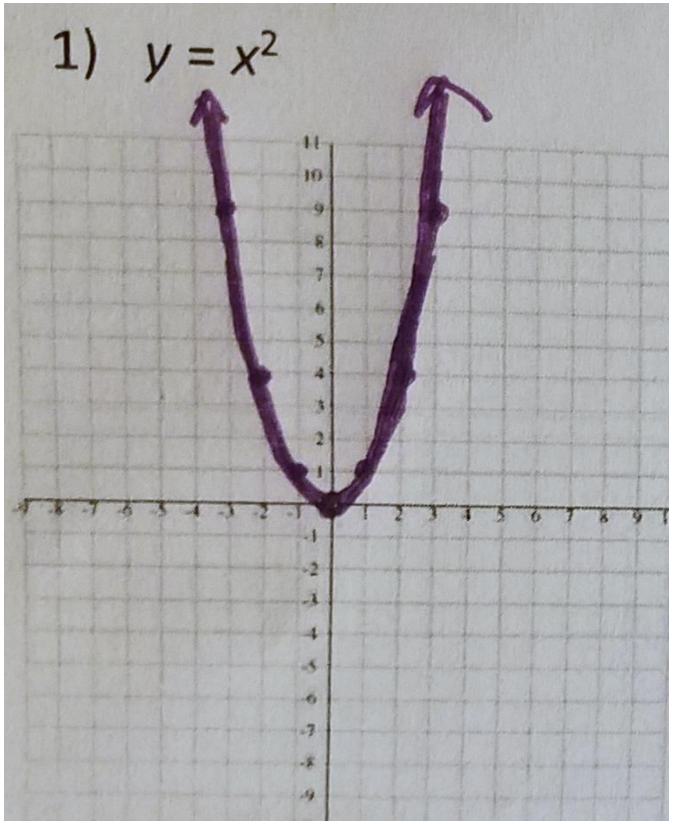
Middle rule

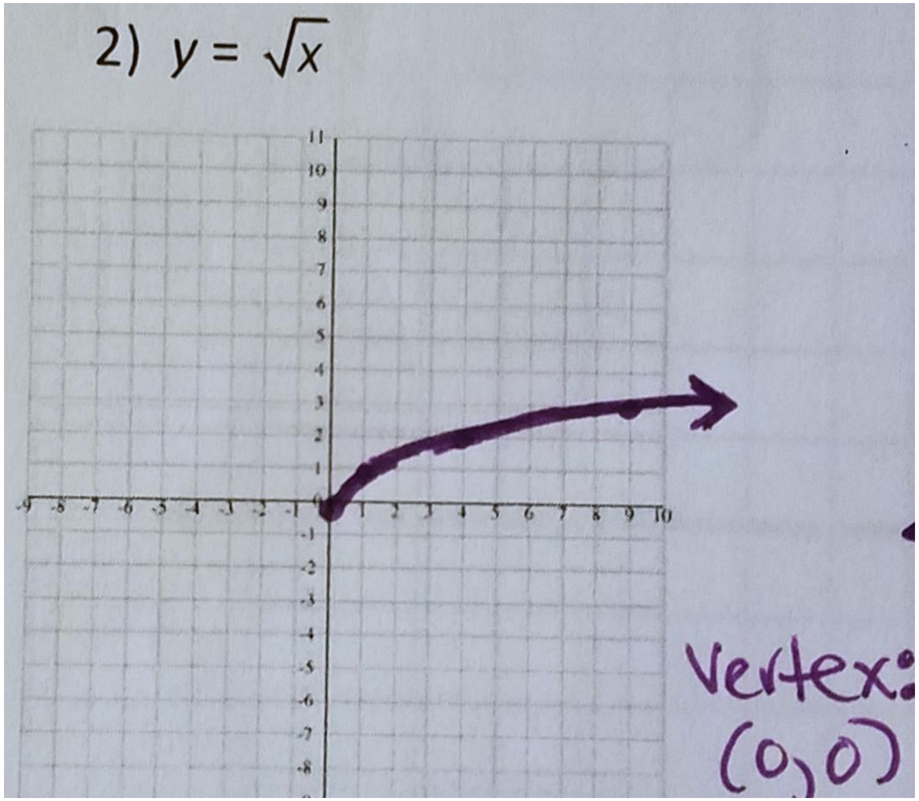
f(1) = $\mathbf{(1 + 1)^2 - 2 = 2}$

$$Domain: (-\infty, \infty)$$

$$Range: (-3, \infty)$$

Homework Answers p. 6 top





Key Features:

Vertex (0, 0)

Axis of Symmetry x = 0

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

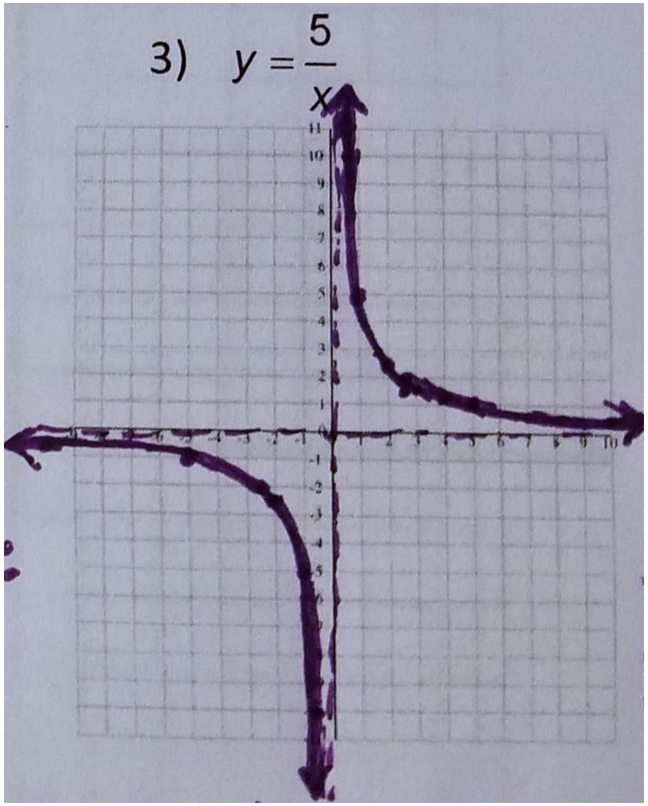
Key Features:

Vertex (0, 0)

Domain: $[0, \infty)$

Range: $[0, \infty)$

Homework Answers p. 6 top



Key Features: **Vertical Asymptote: x = 0**

Horizontal Asymptote: y = 0

Domain: **$(-\infty, 0) \cup (0, \infty)$**

Range: **$(-\infty, 0) \cup (0, \infty)$**

Homework Answers p. 6 bottom

1. $f(x) = \begin{cases} x+5 & x < -2 \\ -2x-1 & x \geq -2 \end{cases}$

Function? (Yes) or No

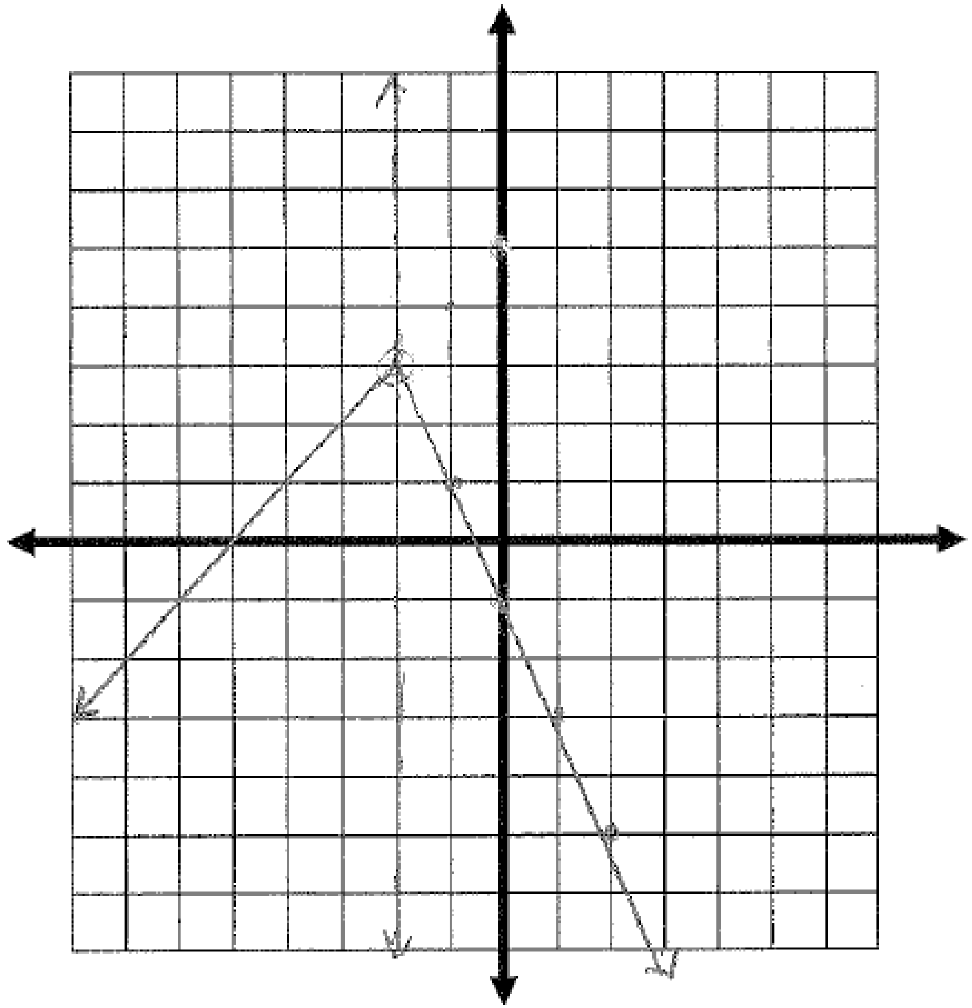
$f(3) = -2(3)-1 = \boxed{-7}$

$f(-4) = -4+5 = \boxed{1}$

$f(-2) = -2(-2)-1 = 3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$



Homework Answers p. 7

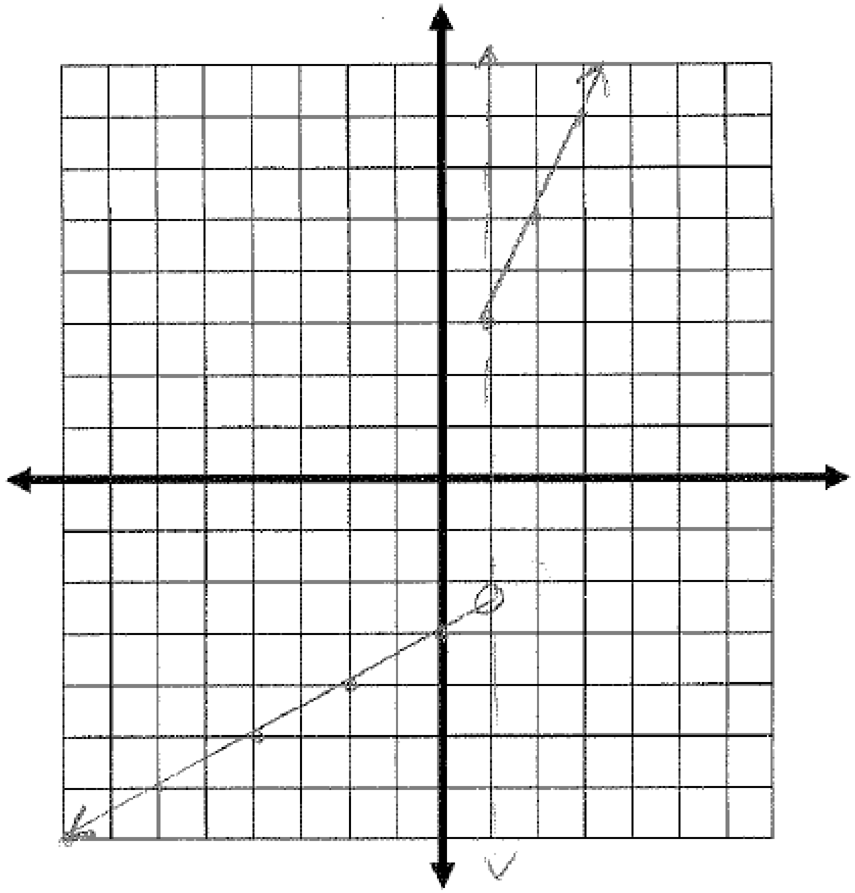
2. $f(x)=\begin{cases} 2x+1 & x\geq 1 \\ \frac{x}{2}-3 & x<1 \end{cases}$

Function? (Yes) or No

$f(-2)= \frac{-2}{2}-3=\boxed{-4}$

$f(6)=2(6)+1=\boxed{13}$

$f(1)=2(1)+1=\boxed{3}$



Domain: **(-∞, ∞)**

Range: **(-∞, -2.5) U [3, ∞)**

Homework Answers p. 7

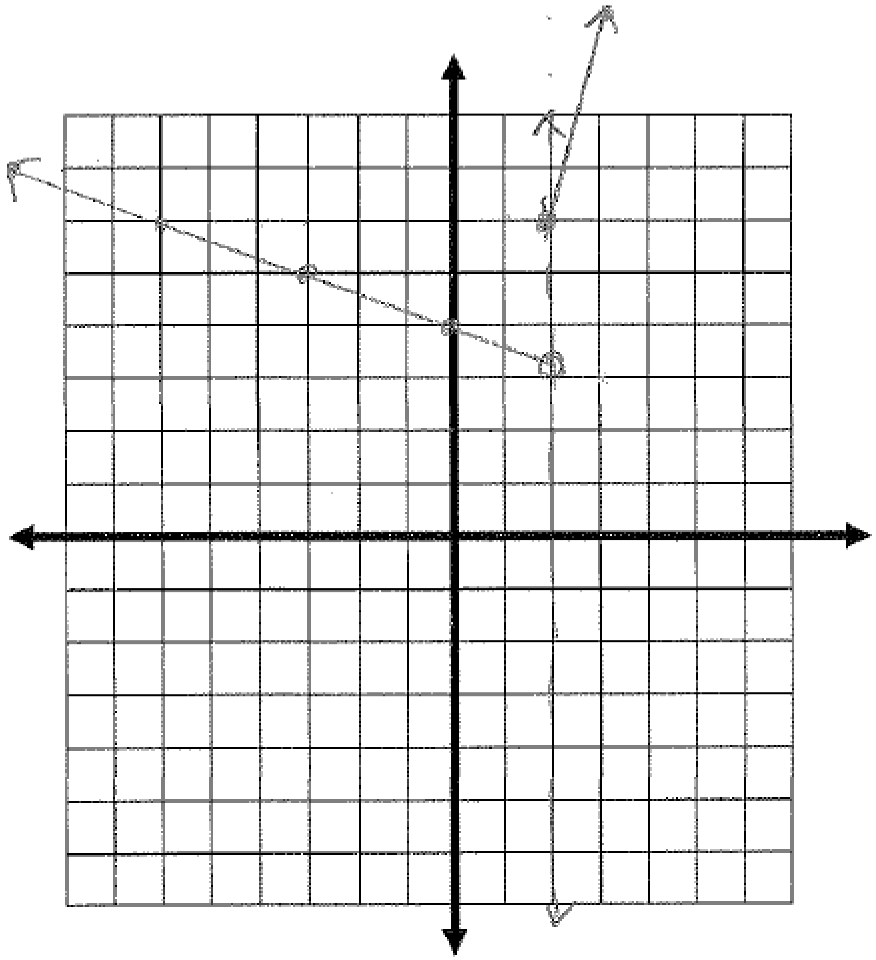
3. $f(x) = \begin{cases} 4x - 2 & x \geq 2 \\ -\dfrac{x}{3} + 4 & x < 2 \end{cases}$

Function? (Yes) or No

$f(-4) = -\dfrac{(-4)}{3} + 4 = \boxed{5\frac{1}{3}}$

$f(8) = 4(8) - 2 = \boxed{30}$

$f(2) = 4(2) - 2 = \boxed{6}$



Domain: **(-∞ , ∞)**

Range: **(10/3 , ∞)**

Tonight's Homework

Packet Pages: 8 - 9

*remember to do Domain & Range in Interval Notation

Day 5 Notes

For today's notes you need Notebook Paper AND the printed notes ☺

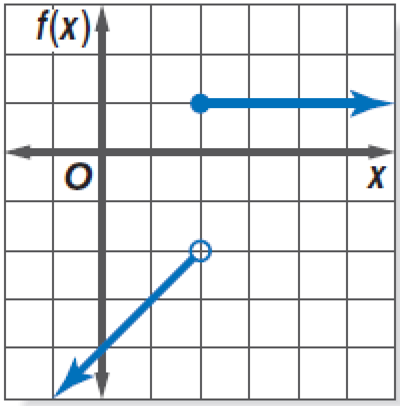
Not in printed notes – take your own notes ☺

With Piecewise Functions, sometimes the
Domain and/or Range can be interesting

Graph $f(x) = \begin{cases} x - 4 \text{ if } x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$. Identify the domain and range.

Step 1 Graph the linear function $f(x) = x - 4$ for $x < 2$. Since 2 does not satisfy this inequality, stop with an open circle at $(2, -2)$.

Step 2 Graph the constant function $f(x) = 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a closed circle at $(2, 1)$ and draw a horizontal ray to the right.



The function is defined for all values of x, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 1 and all real numbers less than -2, so the range is $\{y \mid y < -2 \text{ or } y = 1\}$.

Domain: $(-\infty, \infty)$

Range: $(-\infty, -2) \cup \{1\}$

Notes p. 17 Using Technology to graph piecewise.

Given: $f(x)=\begin{cases} x^2+2 & \text{if } x \le 1 \\ -2x+7 & \text{if } x>1 \end{cases}$

Carefully define each piece in the following way:

Enter in Y1: Y1 = (X^2 + 2) (X ≤ 1) + (-2X + 7) (X > 1)

To get the best view of this function, set your window carefully based on your previous sketch or on the table above. You can use the table to check x=1 (where should the open and closed circle be?)

Verify that what you graphed by hand is the same as the graph on the calculator screen.

Practice: Notes p. 18-20 #1-6

Practice
Graphing Piecewise Functions:
Notes p. 18-20 #1-6

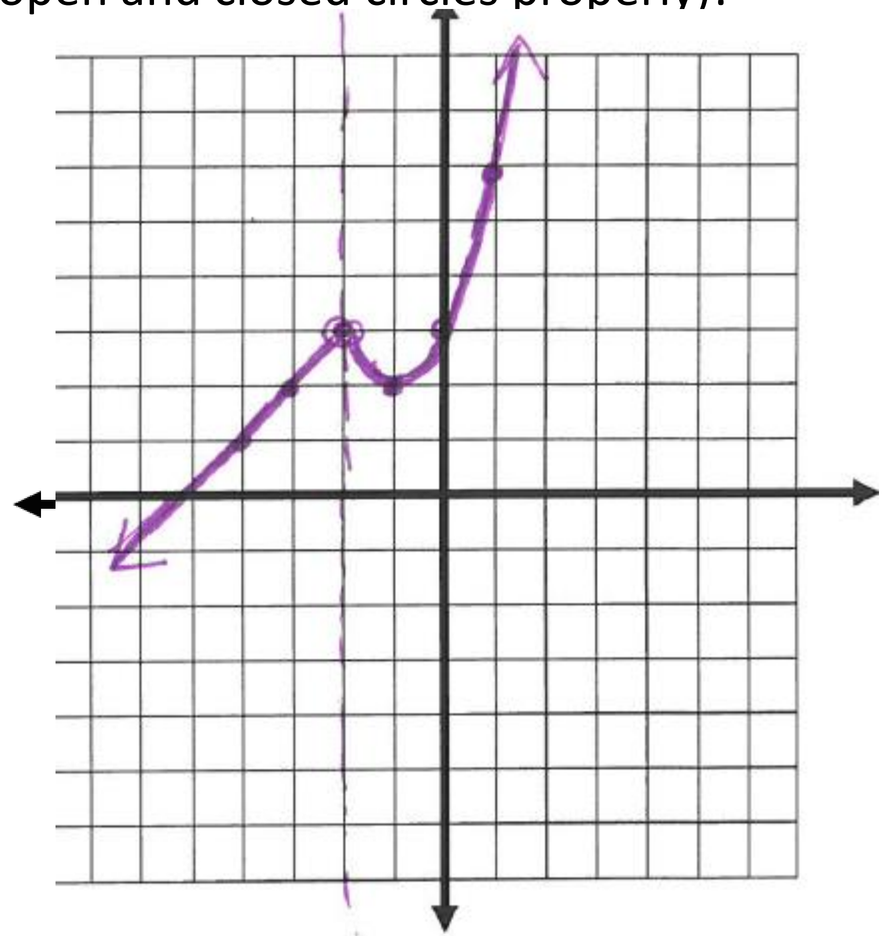
Remember, graphing by hand is good practice!!

Check your work by hand with the calculator.

Practice Time: Piece-Wise Functions

Part I. Carefully graph each of the following. Identify whether or not he graph is a function. Then, evaluate the graph at any specified domain value. You may use your calculators to help you graph, but you must sketch it carefully on the grid (be sure to use open and closed circles properly)!

1. $f(x) = \begin{cases} x+5 & x < -2 \\ x^2 + 2x + 3 & x \geq -2 \end{cases}$



Function? Yes or No

$f(3) =$ 18

$f(-4) =$ 1

$f(-2) =$ 3

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Practice Time: Piece-Wise Functions Continued…

2. $f(x) = \begin{cases} 2x + 1 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$

Function? Yes or No

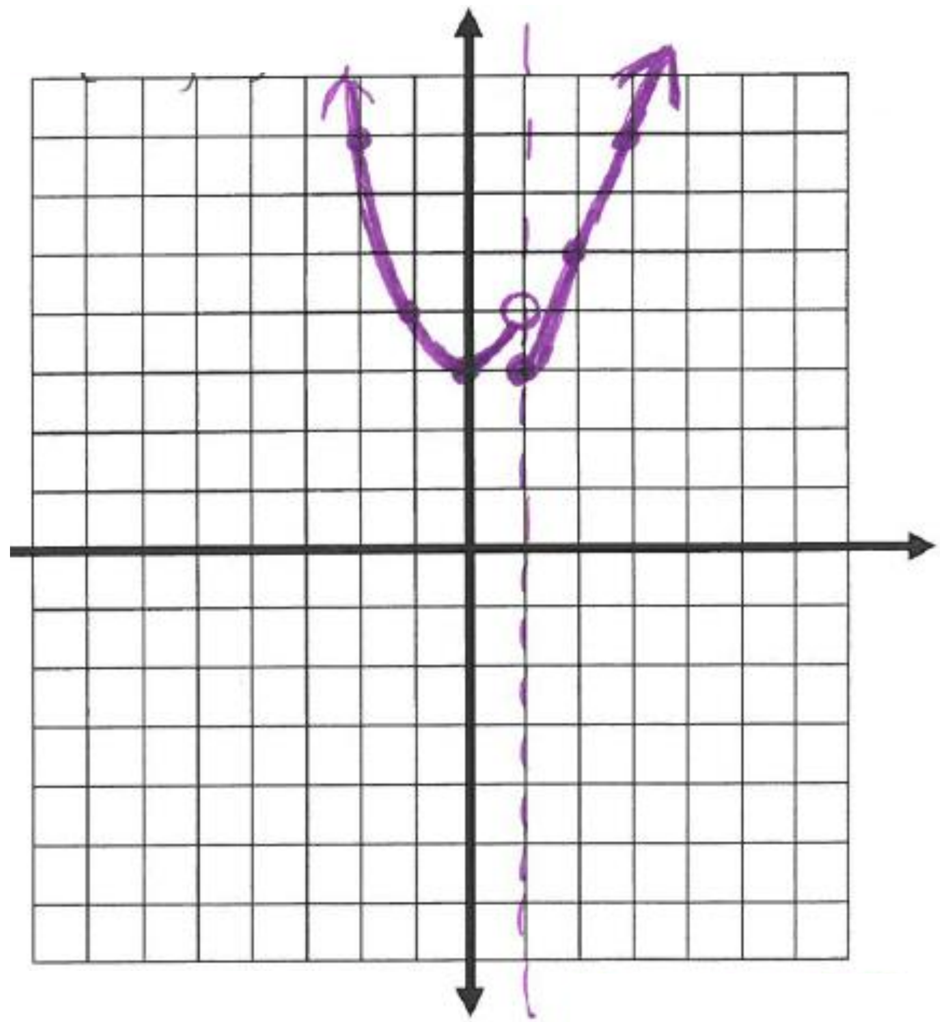
$f(-2) =$ 7

$f(6) =$ 13

$f(1) =$ 3

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$



Practice Time: Piece-Wise Functions Continued…

3. $f(x) = \begin{cases} -2x + 1 & x \le 2 \\ 5x - 4 & x > 2 \end{cases}$

Function? Yes or No

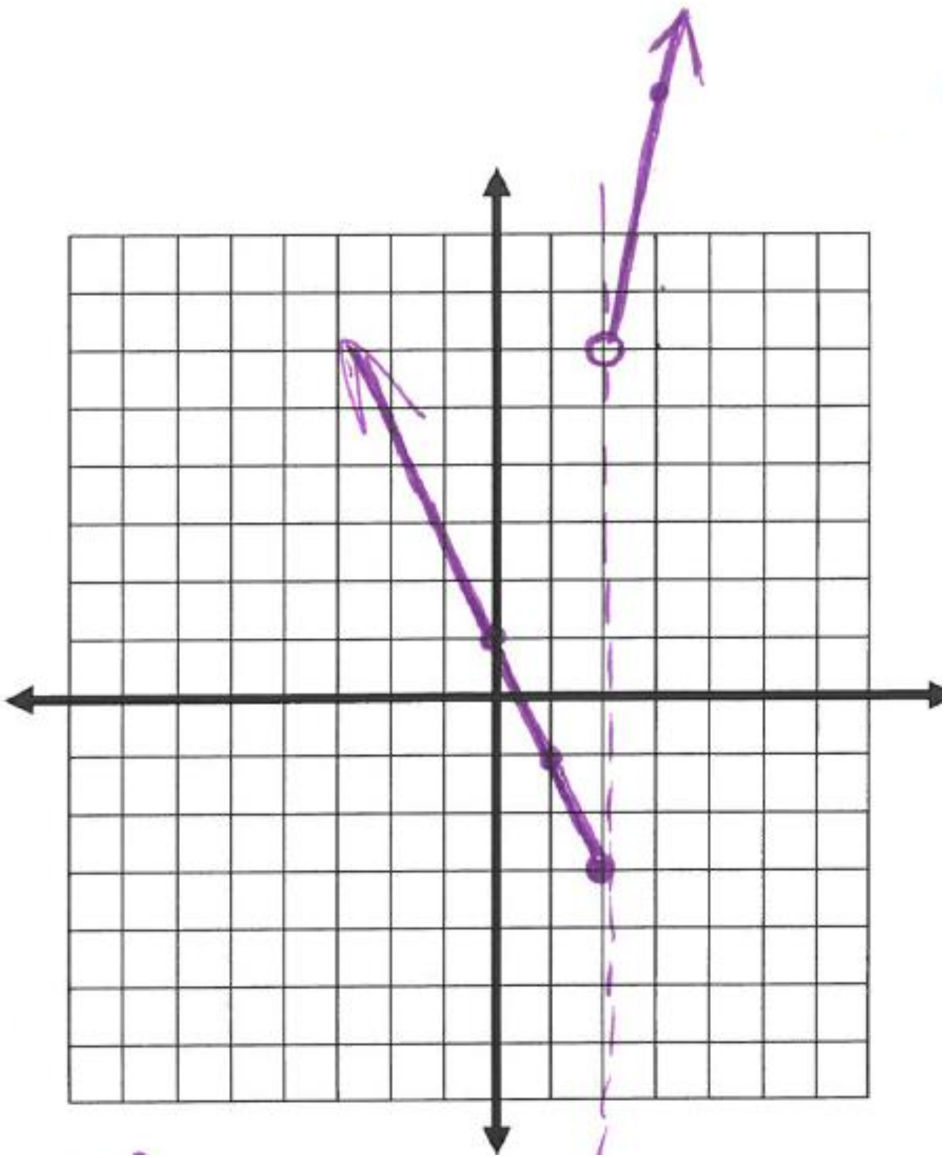
$f(-4) =$ 9

$f(8) =$ 36

$f(2) =$ -3

Domain: $(-\infty, \infty)$

Range: $[-3, \infty)$



Practice Time: Piece-Wise Functions Continued…

4. $f(x) = \begin{cases} x^2 - 1 & x \le 0 \\ 2x - 1 & 0 < x \le 5 \\ 3 & x > 5 \end{cases}$

Function? Yes or No

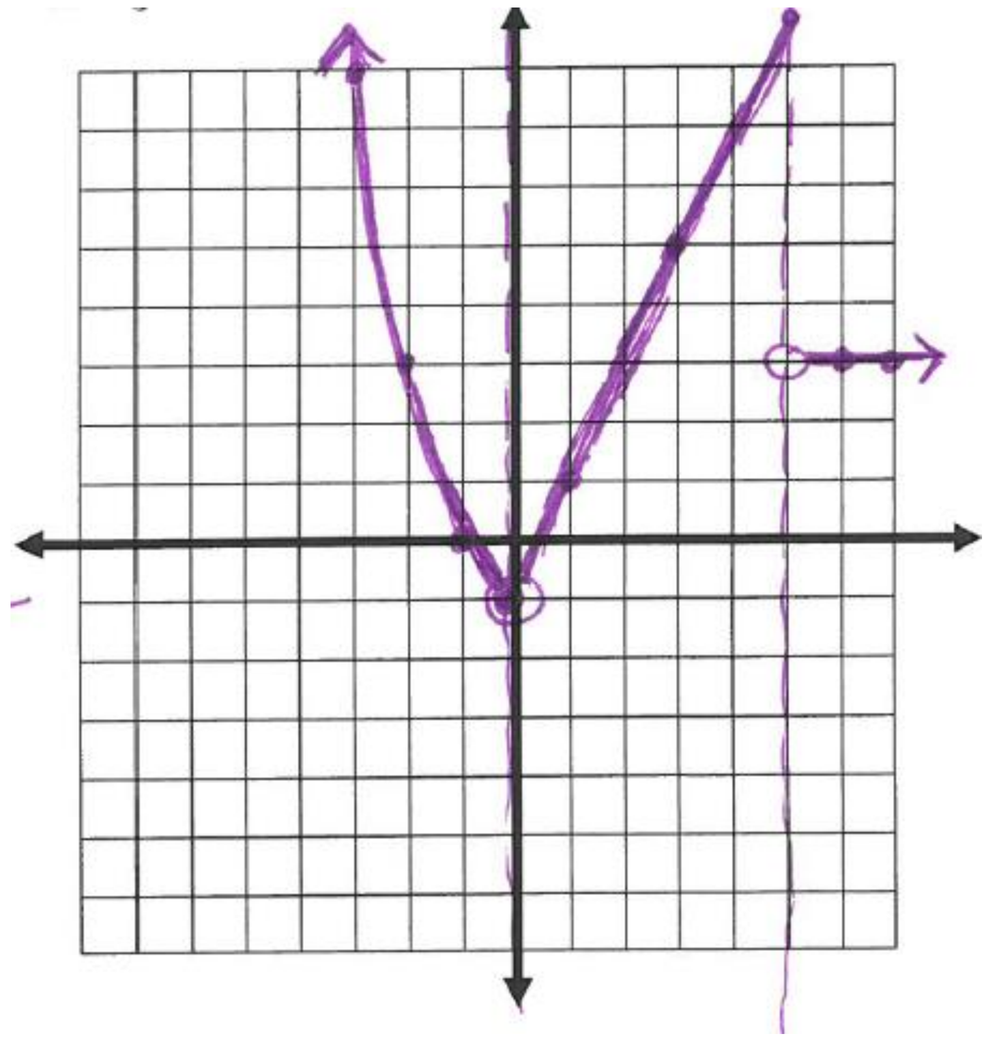
$f(-2) =$ 3

$f(0) =$ -1

$f(5) =$ 9

Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$



Practice Time: Piece-Wise Functions Continued…

5. $f(x) = \begin{cases} x^2 & x \le 0 \\ -x^2 + 4 & x > 0 \end{cases}$

Function? Yes or No

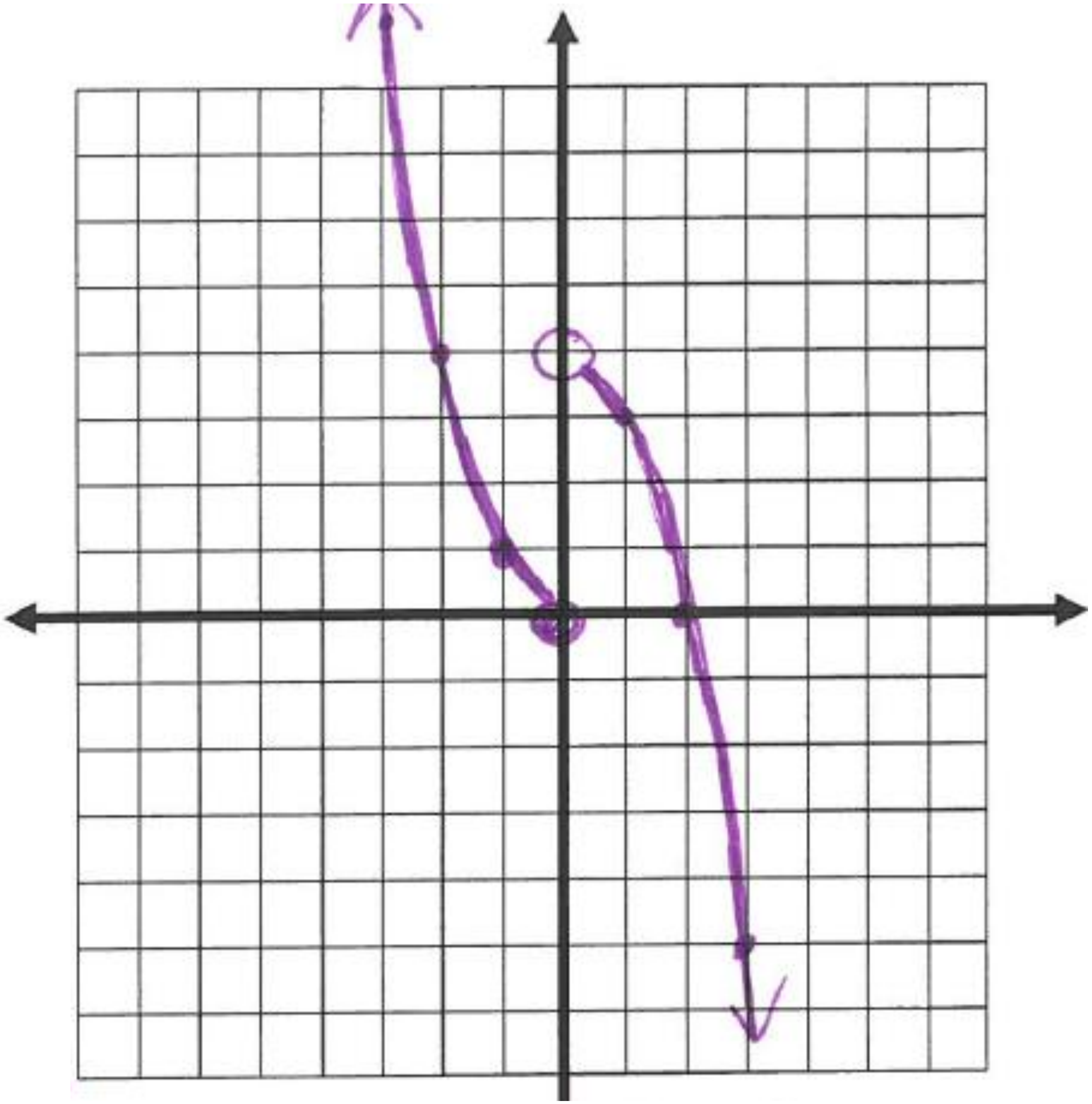
$f(-4) =$ 16

$f(0) =$ 0

$f(3) =$ -5

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$



Practice Time: Piece-Wise Functions Continued…

6. $f(x) = \begin{cases} 5 & x \le -3 \\ -2x - 3 & x > -3 \end{cases}$

Function? Yes or No

$f(-4) =$ 5

$f(0) =$ -3

$f(3) =$ -9

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3) \cup \{5\}$