Solving Simple Rational Equations Practice

Solve the rational equation. Do not forget to determine the excluded values.

1. $\dfrac{3}{x} = \dfrac{2}{x+4}$ EV: _____

2. $\dfrac{x+1}{2x+5} = \dfrac{2}{x}$ EV: _____

3. $\dfrac{3}{x+2} + 5 = \dfrac{4}{x+2}$ EV: _____

4. $\dfrac{6}{x-3} = \dfrac{x}{18}$ EV: _____

5. $\dfrac{2x}{x+4} - 3 = \dfrac{-12}{x+4}$ EV: _____

6. $\dfrac{14}{2-x} = \dfrac{2}{x}$ EV: _____

End of Day 9 Lesson

⇓ DAY 10 LESSON ⇓

Day 10: Solving Harder Rational Equations

Warm-up

Fraction = Fraction so cross multiply

1. $\dfrac{x+2}{x+1} - x = \dfrac{-6}{x+1}$ EV: $x \neq -1$ LCD: $x+1$

$(x+2) - x(x+1) = -6$
$x+2-x^2-x = -6$
$x^2 = 8$
$\sqrt{x^2} = \sqrt{8}$
$x = \pm 2\sqrt{2}$

2. $\dfrac{4}{x-5} = \dfrac{2}{x+8}$ EV: $x \neq 5, 8$

$2(x-5) = 4(x+8)$
$2x-10 = 4x+32$
$2x = -42$
$x = -21$

3. $\dfrac{2}{x-4} + 2 = \dfrac{6}{x-4}$ EV: $x \neq 4$ LCD: $x-4$

$2 + 2(x-4) = 6$
$2 + 2x - 8 = 6$
$2x = 12$
$x = 6$

4. $\dfrac{x}{x+24} = \dfrac{2}{x}$ EV: $x \neq 0, -24$

$x^2 = 2(x+24)$
$x^2 = 2x+48$
$x^2 - 2x - 48 = 0$
$(x-8)(x+6)$
$x = 8, -6$

5. The volume, V, of a certain gas varies inversely with the amount of pressure, P, placed on it. The volume of this gas is 175 cm³ when 3.2 kg/cm² of pressure is placed on it. What amount of pressure must be placed on 400 cm³ of this gas?

$V = \dfrac{k}{P}$ $175 = \dfrac{k}{3.2}$ $k = 175(3.2)$ $k = 560$

$V = \dfrac{560}{P}$ $400 = \dfrac{560}{P}$ $\dfrac{400P}{400} = \dfrac{560}{400}$ $P =$

6. The time, t, in hours, that it takes x people to plant n trees varies directly with the number of trees and inversely with the number of people. Suppose 6 people can plant 12 trees in 3 hours. How many people are needed to plant 28 trees in 5 hours and 15 minutes?

$t = \dfrac{kn}{x}$ $\dfrac{18}{12} = \dfrac{12k}{12}$ $t = \dfrac{3n}{2x}$ $5.25 = \dfrac{3(28)}{2x}$ cross multiply

$3 = \dfrac{k(12)}{6}$ $\dfrac{3}{2} = k$ or $t = \dfrac{1.5n}{x}$ $\dfrac{10.5x}{10.5} = \dfrac{84}{10.5}$ $x = 8$ ppl

Notes Day 10: Solving Harder Rational Equations

Steps:
1. Factor the denominator (if possible).

2. Find the LCD.

~~3~~ Multiply each side by the LCD. *Everything on*

3. Simplify.

~~4~~ Solve for x!

* Be careful with distributing, signs, and parentheses!!

Example 1: $12\dfrac{(x-4)}{4} + \dfrac{12x}{3} = 6 \cdot^{12}$ LCD: 12 $(b/c\ 4\cdot 3)$

$3(x-4) + 4x = 72$ no EV because no variable in denom.

$3x - 12 + 4x = 72$
$+12 +12$

$7x = 84$ $\boxed{x = 12}$

Example 2: $\dfrac{3\,2x(x+1)}{2x} - \dfrac{2x\,2x(x+1)}{x+1} = -2\,(2x)(x+1)$

EV — Note that $x \neq -1$ and $x \neq 0$. The LCD of the fractions is $2x(x+1)$

Multiply each side of the equation by $2x(x+1)$.

$3(x+1) - 2x(2x) = -2(2x)(x+1)$
$ -4x(x+1)$

$3x + 3 - 4x^2 = -4x^2 - 4x$
$-3x \qquad +4x^2 \qquad +4x^2 \qquad -3x$

$\boxed{x = -3/7}$ $\dfrac{3}{-7} = \dfrac{-7x}{-7}$

Example 3: $\dfrac{4}{x+3} - \dfrac{3}{x^2+6x+9} = 1$

LCD: $(x+3)(x+3)$ $(x+3)(x+3)$

Least Common Denominator OR LCD

$\dfrac{x}{3} + \dfrac{2}{5} = 7 \qquad LCD = 15 \rightarrow$

$[(15)\dfrac{x}{3}] + [(15)\dfrac{2}{5}] = 7(15) \rightarrow$

$5(x) + 3(2) = 105 \rightarrow$

$x = 19.8$

YOU TRY — Find LCD 1st — THEN Solve (work shown on next page)

Example 4: $\dfrac{6}{x} - \dfrac{9}{x-1} = \dfrac{1}{4}$

LCD: $4x(x-1)$

Example 5: $\dfrac{2m}{m-1} + \dfrac{m-5}{m^2-1} = 1$

$(m-1)(m+1)$

LCD: $(m-1)(m+1)$

Notes Day 10: Solving Harder Rational Equations

Steps:
1. Factor the denominator (if possible).
2. Find the LCD.
2. Multiply each side by the LCD. ~everything~
3. Simplify.
4. Solve for x!

Example 1: $\dfrac{12(x-4)}{4} + \dfrac{x \cdot 12}{3} = 6 \cdot 12$

LCD: 12

EV: none b/c no variable in denominator

$3(x-4) + 4x = 72$

$3x - 12 + 4x = 72$ $+12$

$7x = 84$ $\boxed{x=12}$

Example 2: $\dfrac{3}{2x} - \dfrac{2x}{x+1} = -2$ (multiplied by $2x(x+1)$)

Note that $x \neq -1$ and $x \neq 0$. The LCD of the fractions is $2x(x+1)$

Multiply each side of the equation by $2x(x+1)$.

EV: $x \neq 0, -1$
LCD: $2x(x+1)$

$3(x+1) - 2x(2x) = -2(2x)(x+1)$
$-4x(x+1)$

$3x + 3 - 4x^2 = -4x^2 - 4x$
$+4x^2 \quad +4x^2$
$-3x$

$3 = -7x$
$\dfrac{-3}{7} = x$

Least Common Denominator OR LCD

$\dfrac{x}{3} + \dfrac{2}{5} = 7 \qquad LCD = 15 \rightarrow$

$[(15)\dfrac{x}{3}] + [(15)\dfrac{2}{5}] = 7(15) \rightarrow$

$5(x) + 3(2) = 105 \rightarrow$

$x = 19.8$

Example 3: $\dfrac{4}{x+3} - \dfrac{3}{x^2+6x+9} = 1$ (multiplied by $(x+3)(x+3)$)

LCD: $(x+3)(x+3)$
EV: $x \neq -3$

$4(x+3) - 3 = (x+3)(x+3)$

$4x + 12 - 3 = x^2 + 6x + 9$

$4x + 9 = x^2 + 6x + 9$

$0 = x^2 + 2x$

$0 = x(x+2)$

$\boxed{x = 0, -2}$

Example 4: $\dfrac{6}{x} - \dfrac{9}{x-1} = \dfrac{1}{4}$ (multiplied by $4x(x-1)$)

LCD: $4x(x-1)$
EV: $x \neq 0, 1$

$6(4)(x-1) - 9(4x) = 1x(x-1)$

$24(x-1) - 36x = x^2 - x$

$24x - 24 - 36x = x^2 - x$

$-12x - 24 = x^2 - x$

$0 = x^2 + 11x + 24$

$0 = (x+8)(x+3)$

$\boxed{x = -8, -3}$

Example 5: $\dfrac{2m}{m-1} + \dfrac{m-5}{m^2-1} = 1$ (multiplied by $(m-1)(m+1)$)

** Factor the denom. first

$(m-1)(m+1)$ EV: $m \neq 1, -1$
LCD: $(m-1)(m+1)$

$2m(m+1) + m - 5 = (m-1)(m+1)$

$2m^2 + 2m + m - 5 = m^2 - 1 + 5$
$-2m^2 \quad -2m \quad -m + 5 \quad -2m - 2m$
$-m$

(but EV $m \neq 1$)

$0 = -m^2 - 3m + 4$

$0 = -1(m^2 + 3m - 4)$

$0 = -1(m+4)(m-1)$

$m = -4, \cancel{1}$

$\boxed{m = -4}$

YOU TRY

<u>Solving Rational Equations Practice</u>
Please complete work on a separate sheet of paper!

Work + answers at bottom of next page

1. $\dfrac{2a-3}{6} = \dfrac{2a}{3} + \dfrac{1}{2}$

6. $\dfrac{4x}{3x-2} + \dfrac{2x}{3x+2} = 2$

2. $\dfrac{2b-3}{7} - \dfrac{b}{2} = \dfrac{b+3}{14}$

7. $\dfrac{5}{5-p} - \dfrac{p^2}{5-p} = -2$

3. $\dfrac{3}{5x} + \dfrac{7}{2x} = 1$

8. $\dfrac{2a-3}{a-3} - 2 = \dfrac{12}{a+3}$

4. $\dfrac{5k}{k+2} + \dfrac{2}{k} = 5$

9. $\dfrac{2b-5}{b-2} - 2 = \dfrac{3}{b+2}$

5. $\dfrac{m}{m+1} + \dfrac{5}{m-1} = 1$

10. $\dfrac{4}{k^2-8k+12} = \dfrac{k}{k-2} + \dfrac{1}{k-6}$

If quadratic denominator FACTOR 1st!! $m^2 - 1 \neq 0$ $\sqrt{m^2 \neq 1}$ $m \neq \pm 1$

Example 4: $\dfrac{6}{x} - \dfrac{9}{x-1} = \dfrac{14}{4}$ EV: $x \neq 0, 1$ LCD: $4x(x-1)$

$4 \cdot 6(x-1) - 9(4x) = x(x-1)$

$24x - 24 - 36x = x^2 - 1x$

$-12x - 24 = x^2 - x$

$+12x + 24 \qquad +12x + 24$

$0 = x^2 + 11x + 24$

$0 = (x+3)(x+8)$

$\boxed{x = -3, -8}$

Example 5: $\dfrac{2m}{m-1} + \dfrac{m-5}{m^2-1} = 1$ EV: $x \neq 1, 1$ LCD: $(m+1)(m-1)$

$2m(m+1) + m - 5 = m^2 - 1$

$2m^2 + 2m + m - 5 = m^2 - 1$

$-m^2 \qquad\qquad +1 \quad -m^2 + 1$

$m^2 + 3m - 4 = 0$

$(m+4)(m-1) = 0$

$\boxed{m = -4, \cancel{1}}$ ← EV!

YOU TRY!   Solving Rational Equations Practice

1. $\dfrac{2a-3}{6} = \dfrac{2a}{3} + \dfrac{1}{2}$ LCD: 6 EV: none

$2a - 3 = (2a)2 + 1 \cdot 3$

$2a - 3 = 4a + 3$

$-2a \qquad -2a$

$-3 \qquad -3$

$-6 = 2a$

$\boxed{a = -3}$

2. $\dfrac{2b-3}{7} - \dfrac{b}{2} = \dfrac{b+3}{14}$ LCD: 14 EV: none

$2(2b-3) - 7b = b+3$

$4b - 6 - 7b = b + 3$

$-3b - 6 = b + 3$

$-b \quad +6 \quad -b \quad +6$

$-4b = 9$

$\boxed{b = -9/4}$

3. $\dfrac{3}{5x} + \dfrac{7}{2x} = 1$ LCD: $10x$ EV: $x \neq 0$

$3 \cdot 2 + 7 \cdot 5 = 10x$

$41 = 10x$

$\boxed{x = \dfrac{41}{10} \text{ or } 4.1}$

4. $\dfrac{5k}{k+2} + \dfrac{2}{k} = 5k(k+2)$ LCD: $k(k+2)$ EV: $k \neq -2, 0$

$5k^2 + 2(k+2) = 5k(k+2)$

$5k^2 + 2k + 4 = 5k^2 + 10k$

$-5k^2 - 2k \qquad -5k^2 - 2k$

$4 = 8k$

$\dfrac{4}{8} = \dfrac{8k}{8}$

$\boxed{1/2 = k}$

5. $\dfrac{m}{m+1} + \dfrac{5}{m-1} = 1$ LCD: $(m+1)(m-1)$ EV: $m \neq 1, -1$

$m(m-1) + 5(m+1) = (m+1)(m-1)$

$m^2 - m + 5m + 5 = m^2 - 1$

$-m^2 \qquad\qquad\qquad -m^2$

$4m + 5 = -1$

$-5 \quad -5$

$4m = -6$

$\boxed{m = -3/2}$

6. $\dfrac{4x}{3x-2} + \dfrac{2x}{3x+2} = 2$ LCD: $(3x-2)(3x+2)$ EV: $x \neq 2/3, -2/3$

$4x(3x+2) + 2x(3x-2) = 2(9x^2-4)$

$12x^2 + 8x + 6x^2 - 4x = 18x^2 - 8$

$-12x^2 \qquad\qquad -6x^2 \qquad -12x^2$

$-6x^2$

$4x = -8$

$\boxed{x = -2}$

7. $\dfrac{5}{5-p} - \dfrac{p^2}{5-p} = -2$ LCD: $5-p$ EV: $x \neq 5$

$5 - p^2 = -10 + 2p$

$-5 \quad +p^2 \quad -5 \quad +p^2$

$0 = p^2 + 2p - 15$

$0 = (p+5)(p-3)$

$\boxed{p = -5, 3}$

8. $\dfrac{2a-3}{a-3} - 2 = \dfrac{12}{a+3}$ LCD: $(a-3)(a+3)$ EV: $a \neq 3, -3$

$(2a-3)(a+3) - 2(a-3)(a+3) = 12(a-3)$

$2a^2 + 3a - 9 - 2a^2 + 18 = 12a - 36$

$3a + 9 = 12a - 36$

$45 = 9a$

$\boxed{a = 5}$

9. $\dfrac{2b-5}{b-2} - 2 = \dfrac{3}{b+2}$ LCD: $(b-2)(b+2)$ EV: $b \neq -2, 2$

$(2b-5)(b+2) - 2(b-2)(b+2) = 3(b-2)$

$2b^2 - b - 10 - 2b^2 + 8 = 3b - 6$

$-b - 2 = 3b - 6$

$-3b \quad +2 \quad -3b \quad +2$

$-4b = 4$

$\boxed{b = 1}$

10. $\dfrac{4}{k^2-8k+12} = \dfrac{k}{k-2} + \dfrac{1}{k-6}$ LCD: $(k-6)(k-2)$ EV: $k \neq 6, 2$

$(k-6)(k-2)$

$4 = k(k-6) + k - 2$

$4 = k^2 - 6k + k - 2$

-4

$0 = k^2 - 5k - 6$

$0 = (k-6)(k+1)$

$\boxed{x = \cancel{6}, -1}$