Day 8: Point Ratio Form

Warm Up: Exponential Regression



When handed to you at the drive-thru window, a cup of coffee was 200°F. Some data has been collected about how the coffee cools down on your drive to school.

- Would you say that the data has a positive, negative, or no correlation? Explain.
- 2. Without using your calculator, would you say that the data appears to be more linear or exponential? Explain.
- 3. Explain the steps you take to enter the data in the calculator and find a regression equation. Consult a neighbor if you need some help.
- 4. Calculate the linear regression and use it to predict how cool the coffee will be in 15 minutes.
- 5. Calculate the exponential regression and use it to predict how cool the coffee will be in 15 minutes.
- 6. Which regression do you think provides a more accurate prediction? Explain your reasoning.

Point Ratio Form

When you ring a bell or clang a pair of cymbals, the sound seems loud at first, but the sound becomes quiet very quickly, indicating exponential decay. This can be measured in many different ways. The most famous, the decibel, was named after Alexander Graham Bell, inventor of the telephone. We can also use pressure to measure sound.

While conducting an experiment with a clock tower bell it was discovered that the sound from that bell had an intensity of 40 lb/in² four seconds after it rang and 4.7 lb/in² seven seconds after it rang.

- 1. Using this information, what would you consider to be the independent and dependent values in the experiment?
- 2. What two points would be a part of the exponential curve of this data?
- 3. What was the initial intensity of the sound?
- 4. Find an equation that would fit this exponential curve.

Time	Temperature
(minutes)	(°F)
0	200
1	190
2	180.5
3	171.48
4	162.9
5	154.76
6	147.02

Honors Math 2

You can find this equation using exponential regression on your graphing calculator, but what if you don't have a graphing calculator?

You may recall learning how to write equations for linear information using *point-slope form*. This is a little bit different, obviously, because we're working with an exponential curve, but when we use *point-ratio form*, you should see some similarities.

Point-Ratio Form

$$\mathbf{y} = \mathbf{y}_1 \cdot \mathbf{b}^{\mathbf{x} - \mathbf{x}_1}$$

Where (x, y) and (x_1, y_1) are ordered pairs that can be used to find the value of b, the common ratio. OR There (x_1, y_1) is an ordered pair and the common ratio b, can be used to write an explicit equation for the see

Where (x_1, y_1) is an ordered pair and the common ratio, b, can be used to write an explicit equation for the scenario or identify other possible values for the scenario.

Let's look back at the Bell problem to see how to solve for an exponential formula by hand using the following steps!

To find the value of b

Step 1: Identify two ordered pairs.

To find the initial value, a

Step 3: Simplify to solve for a.

Step 1: Identify one ordered pair and the b value.

Step 2: Substitute the values into exponential form.

Step 2: Substitute the values into Point-Ratio Form

Step 3: Simplify the exponents.

Step 4: Divide to get the power alone.

Step 5: Use the properties of exponents to isolate b.

The above steps can be used to find any value of x or y if you have one ordered pair and the b value.

Guided Practice: Point – Ratio Form

- The intensity of light also decays exponentially with each additional colored gel that is added over a spotlight. With three gels over the light, the intensity of the light was 900 watts per square centimeter. After two more gels were added, the intensity dropped to 600 watts per square centimeter.
 - a. Write an equation for the situation. Show your work.

b. Use your equation to determine the intensity of the light with 7 colored gels over it.

- 2) Using the points (2, 12.6) and (5, 42.525).
 - a. Find an exponential function that contains the points. Show your work.

- b. Rewrite the function so that the exponent of the function is simply x.
- 3) When the brown tree snake was introduced to Guam during World War II by the US military it devastated the local ecosystem when its population grew exponentially. If 1 snake was accidentally brought to Guam in 1945 and it was estimated that there were 625 snakes in 1949, write an equation for the situation and use it to predict how many snakes there were in 1955. In your equation, let x = number of years after 1945. Show your work.

4) The water hyacinth that was introduced to North Carolina from Brazil ended up clogging our waterways and altering the chemistry of the water. We're not certain exactly when the water hyacinth was introduced, but there were 76.9 square miles of water hyacinths in 1984. Ten years later, there were 80793.6 square miles of watch hyacinth. Using this information, calculate when there was less than 0.1 square miles of this invasive plant in our waterways.

- 5) You recently discovered that your great aunt Sue set up a bank account for you several years ago. You've never received bank statements for it until recently. The first month you received a statement for the account, you had \$1234.56 in the account. One month later, the account had \$1296.29. What is the approximate percentage of monthly interest that this account is earning each month?
 - a. What is the approximate percentage of monthly interest that this account is earning each month?
 - b. In what month will your account have doubled?

- 6) When the wolf population of the Midwest was first counted in 1980, there were 100 wolves. In 1993, that population had grown to 3100 wolves! Assuming the growth of this population is exponential, complete each of the following.
 - a. Write an explicit equation to model the data. Show your work!

- b. Write a recursive (NOW-NEXT) equation for the data.
- c. Use one of your equations to complete the table below.

Time Since	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1980 (in years)														
Estimated	100													3,100
Wolf														
Population														

- d. Identify the theoretical and practical domain of this situation. Explain how you decided upon your answers.
- 7) A 2008 Ford Focus cost \$15452 when purchased. In 2013 it is worth \$8983 if it's in excellent condition.
 - a. Assuming the value of the Focus depreciates by the same percentage each year, what is that percentage?

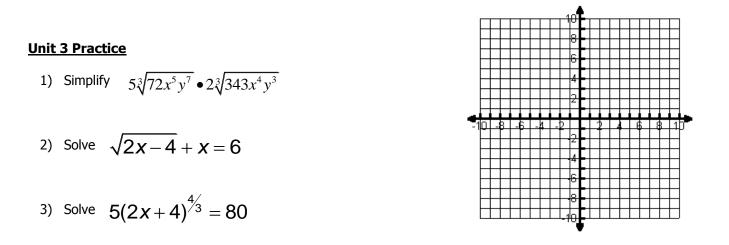
- b. Write an explicit function for the value of the Ford Focus.
- c. If the owner of the car plans on selling it in 2018, how much should she expect to get for it?

An application of point-ratio form: finding the monthly rate of depreciation.

Together: The value, V, of a car can be modeled by the function $V(t) = 13,000(0.82)^t$, where t is the number of
years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?A.1.5%B.1.6%C.9.2%D.18.0%

You Try!

The value of a truck can be modeled by the function $V(t) = 16,000(0.76)^t$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?



- 4) Sr-85 is used in bone scans. It has a half-life of 64.9 hours. Write the exponential function for a 12 mg sample. Find the amount remaining after 72 hours. Show work algebraically.
- 5) Write exponential function given (2, 18.25) and (5, 60.75). Show your work algebraically.
- 6) A plant 4 feet tall grows 3 percent each year. How tall will the tree be at the end of 12 years? Round your answer to the hundredths place.
- 7) A BMX bike purchased in 1990 and valued at \$6000 depreciates 7% each year.
 - a) Write a next-now formula for the situation
 - b) Write an explicit formula for the situation
 - c) How much will the BMX bike be worth in 2006?
 - d) In what year will the BMX bike be worth \$3000?

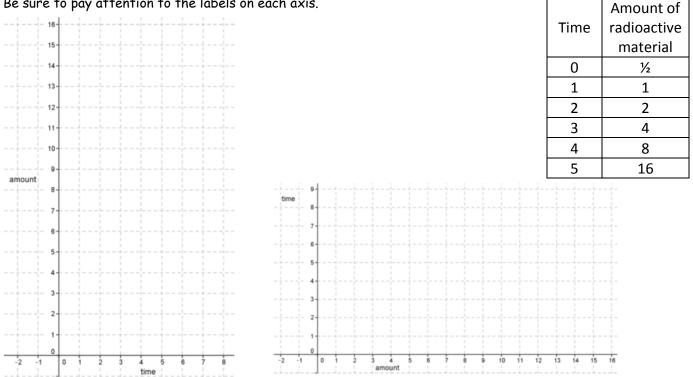
Day 9: Transformations of Graphs...kf(x) and f(kx)

Warm-Up Day 9: Transformations

How are the following graphs changed from their parent graph, $F(x) = x^2$? (If you don't remember some of the transformations, graph the transformed equation and the parent equation in the calculator \odot)

1) $F(x) = (x - 4)^2$	5) $F(x) = 4x^2$
2) $F(x) = x^2 - 4$	6) $F(x) = \frac{1}{2} x^2$
3) $F(x) = x^2 + 1$	7) F(x) = 3(x - 4) ²
4) $F(x) = (x + 3)^2$	8) F(x) = 1/4 x ² + 3

9) Use the following data to graph on each of the following coordinate planes. Be sure to pay attention to the labels on each axis.



Explain how the graphs are alike and different.

11) Solve $2(4x-5)^{2/3} = 18$

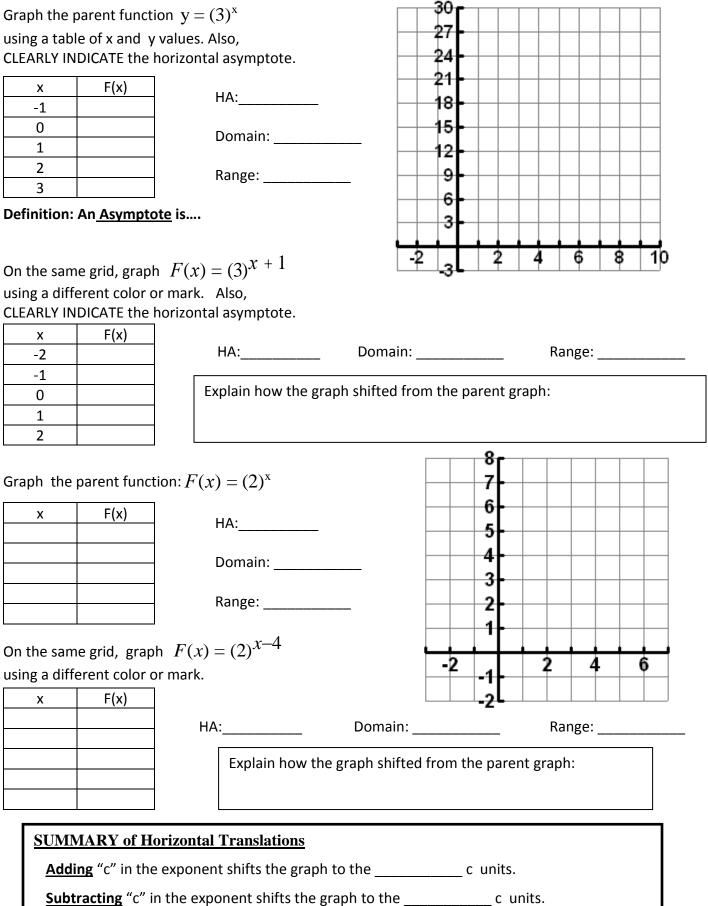
13) How long will it take\$500 to triple if it is invested in a stock that increases 6% a year?

12) Solve $(3x-11)^{1/2} - x = -3$

14) Write an exponential function using the points (4, 10) and (12, 15.6). Show work algebraically.

15) A car driving 102 mph decreases speed 15% per second. Write a recursive AND explicit equation to represent the car speed over time in seconds.

Today's Material ~



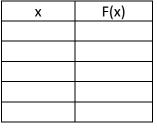
Honors Math 2

Graph the parent function again: $F(x) = (3)^x$ HA:

Domain:

Range: _____

On the same grid, graph $F(x) = (3)^{x} + 1$ using a different color or mark.

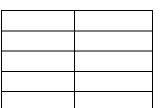


HA:_____

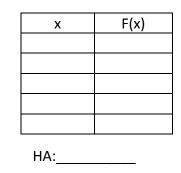
Domain: _____

Range: _____

Explain how the graph shifted from the parent graph:



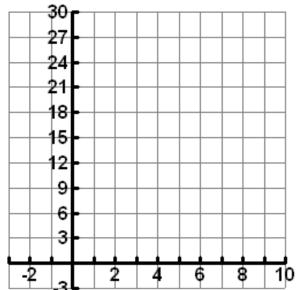
On the same grid, graph $F(x) = (3)^x - 2 \operatorname{using} a$ different color or mark.



Domain: _____

Range: _____

Explain how the graph shifted from the parent graph:



SUMMARY of Vertical Translations

Adding "c" to the whole equation shifts

the graph c units.

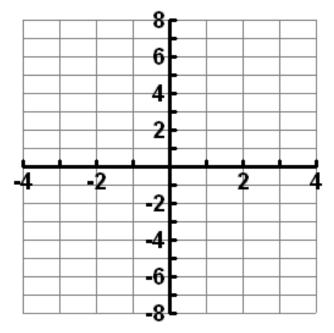
<u>Subtracting</u> "c" to the whole equation

shifts the graph ______ c units.

Use a different color or mark to graph each of the following on the same grid. Be sure to LABEL your curves.

x	$F(x) = (2)^x$	$F(x) = (2)^{-x}$	$F(x) = -(2)^x$
-3			
-2			
-1			
0			
1			
2			
3			

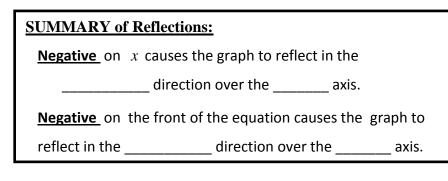
HA		
D		
R		



Honors Math 2

Explain how the parent graph changed to get the graph of $F(x) = (2)^{-x}$.

Explain how the parent graph changed to get the graph of $F(x) = -(2)^x$.



Day 9 Part 2: Inverses of Functions

Investigation: Inverses

Consider the following functions:

1)
$$f(x) = 6 + 3x$$

2) $f(x) = \frac{1}{3}(x-6)$

The notation for the inverse of f(x) is _____

Part 1: Graphs of Inverse Functions

For each of the functions above, follow steps 1 - 4.

- 1) Make a table of 5 values and graph the function on a separate sheet of graph paper.
- 2) Make another table by switching the x and y values and graph the inverse on the same coordinate plane.
- 3) What do you notice about the two graphs?
- 4) What line are the inverses reflected over?

Part 2: Equations of Inverse Functions

We saw in part 1 of the investigation that functions 1 and 2 are inverse functions. We also know that we can find inverses of tables by switching the x and y values in a table. So the question we want to explore now is how to find the equation of an inverse function.

For each of the functions above, follow steps 5 - 6.

- 5) Take the function and switch the x and y values.
- 6) Then solve for y.

The equation for the function 1's inverse should be function 2 and the inverse for function 2 should be the equation for function 1.

This process will work for any function which has an inverse. So, let's try some different types in the problems below.

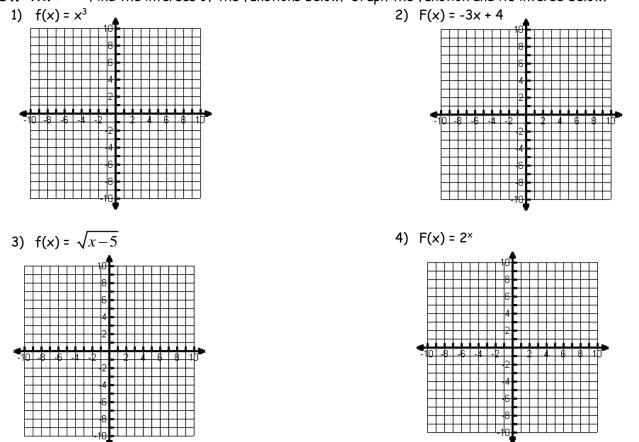
Let's try a few algebraically together.

Find the inverse of the functions below.

1.
$$y = 2x + 5$$
 2. $y = \frac{1}{3}x - 4$

Honors Math 2

NOW TRY: Find the inverses of the functions below. Graph the function and its inverse below.



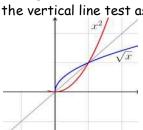


Graph $y = x^2 + 3$ and find the inverse by interchanging the x and y values of several ordered pairs. Is the inverse a function? Check by graphing both $y = x^2 + 3$ and the inverse on the graph on the right.

However, if you consider half of the parabola, then...

Let's look at a simpler function $y = x^2$. If you only consider the positive values of x in the original function, the graph is half of a parabola. When you reflect the "half" over the y = x line, it will pass the vertical line test as shown on the right.

Algebraically, let's switch the x and y values and solve for y. Function: y = x², where x ≥ 0 Switch the x and y values:



Solve for y:

As shown, both graphs pass the vertical line tests and are inverse functions of each other. The above example means that if you restrict the domains of some functions, then you will be able to find inverse functions of them.

Refresher problem!

Write an exponential function for a graph that includes the points (4, 36) and (6, 15). Show your work algebraically using the point ratio formula. Round your "b" value to 4 decimal places.

31

Day 10: Quiz Day

Warm-Up Day 10:

Show work on a separate piece of paper, if needed.

- Solve the following equations.
 - 1. $2b^{5/3} 7 = 479$

2.
$$-46 = -2(33 - 2m)^{74} + 8$$

- 3. The half-life of Zn-71 is 2.4 minutes. If one had 100.0 g at the beginning, how many grams would be left after 7.2 minutes has elapsed?
- 4. $-5(b-20)^{5/4} = -1215$
 - 5. Write an exponential function for a graph that includes the points (2, 17) and (5, 22.5).

Day 11: Inverses of Functions (logs and exponentials)

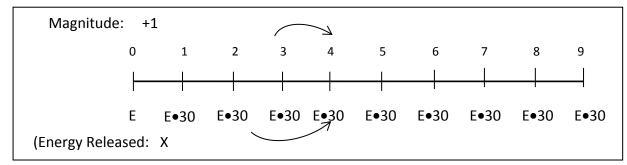
Day 1	1 Warm-up: InversesFind the algebraic inverse.	Sho	w work on a separate piece of paper, if needed.
1.	f(x) = 15x - 1	2.	$f(x)=\frac{4}{7}x$
3.	$f(x)=\frac{1}{3}x+7$	4.	f(x) = -5x - 11
5.	$f(x) = (x-2)^2$	6.	$f(x) = \sqrt{x-4}$

7. I-123 is used in thyroid scans. 13.2 hours after the formation of a 45 mg sample, only 22.5 mg remain. How much of the I-123 will remain after 66 hours? Show your work algebraically.

Day 11 Logarithmic Functions: Inverse of an Exponential Function

The Richter Scale: The magnitude of an earthquake is a measure of the amount of energy released at its source. The Richter scale is an exponential measure of earthquake magnitude. A simple way to examine the Richter Scale is shown below. An earthquake of magnitude 5 releases about 30 times as much energy as an earthquake of

magnitude 4.



In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington State. Let's compare the amounts of energy released in the two earthquakes. For the earthquake in Mexico at 8.0 on the Richter Scale, the energy released is E•30⁸ and for the earthquake in Washington state, the energy released is E•30^{6.8}. A ratio of the two quakes and using the properties of exponents yields the following:

This means the earthquake in Mexico released about _____ times as much energy as the one in Washington.

Honors Math 2

You TRY!

In 1812, an earthquake of magnitude 7.9 shook New Madrid, Missouri. Compare the amount of energy released by that earthquake to the amount of energy released by each earthquake below.

1) Magnitude 7.7 in San Francisco, California, in 1906.

2) Magnitude 3.2 in Charlottesville, Virginia, in 2001.

The exponents used by the Richter scale in the example are called **logarithms** or logs. A logarithm is defined as follows:

The logarithm base b of a positive number y is defined as follows: If $y = b^x$, then $\log_b y = x$. $b \neq 1$ and b > 0

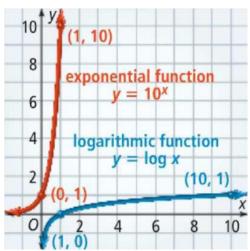
The exponent x in the exponential expression b^x is the logarithm in the equation $\log_b y = x$. The base b in b^x is the same as the base b in the logarithm. In both cases, $b \neq 1$ and b > 0. So what this means is that you use logarithms to undo exponential expressions or equations and you use exponents to undo logarithms, which means that the operations are inverses of each other. Thus, an exponential function is the inverse of a logarithmic function and vice versa.

Key Features of Logarithmic Graphs

A logarithmic function is the inverse of an exponential function. The graph show $y = 10^{x}$ and $y = \log x$. Note that (0, 1) and (1, 10) lie on the graph of $y = 10^{x}$ and that (1, 0) and (10, 1) lie on the graph of $y = \log x$, which demonstrates the reflection over the line y = x.

Since an exponential function $y = b^x$ has an asymptote at y = 0, the inverse function $y = \log_b x$ has an asymptote at x = 0.

The other key features of exponential and logarithmic functions are summarized in the box below.



Key Features of Exponential and Logarithmic Functions					
Characteristic	Exponential Function	Logarithmic Function			
	$y = b^x$	$y = \log_b x$			
Asymptote	<i>y</i> = 0	<i>x</i> = 0			
Domain	All real numbers	<i>x</i> >0			
Range	<i>y</i> > 0	All real numbers			
Intercept	(0,1)	(1,0)			

Honors Math 2

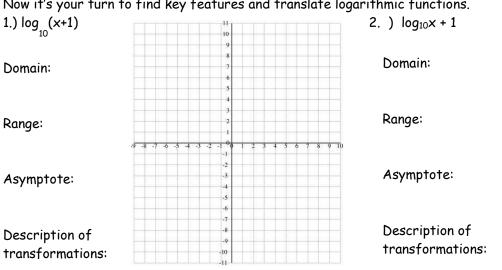
Translations of logarithmic functions are very similar to those for other functions and are summarized in the table below.

Parent Function	y = log₀x
Shift up	$y = log_b x + k$
Shift down	y = log _b x - k
Shift left	$y = log_b(x + h)$
Shift right	y = log _b (x - h)
Combination Shift	$y = log_b(x \pm h) \pm k$
Reflect over the x-axis	y = -log _b x
Stretch vertically	y = a log₀x
Stretch horizontally	y = logbax

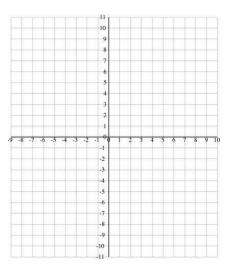
Let's look at the following example.

The graph on the right represents a transformation of the graph of $f(x) = 3 \log_{10} x + 1.$

- $|\mathbf{x}| = 3$: Stretches the graph vertically. •
- h = 0: There is no horizontal shift. •
- k = 1: The graph is translated 1 unit up.



Now it's your turn to find key features and translate logarithmic functions.



f(x)

0

For equations that have transformations, remembering a key feature of logs can help with finding the domain and asymptote.

Remember the domain of logs MUST be positive!

Set the part in the parentheses > 0 and solve to find the domain AND to find the value of the asymptote.

Let's do this one again: $\log_{10}(x+1)$

Domain:

Asymptote:

Honors Math 2

TRY NOW...one a little tougher!

Graph the following function on the graph at right. Describe each transformation, give the domain and range, and identify any asymptotes.

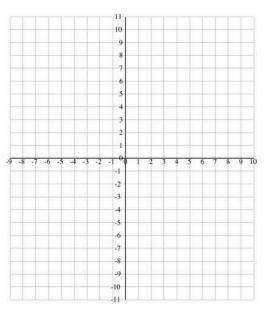
 $F(x) = -2\log_{10}(x+2) - 4$

Domain:

Range:

Asymptote:

Description of transformations:



Graph the following transformations of the function $y = \log_{10} x$ on the coordinate planes. Determine the domain, range, and asymptotes of each transformation. Describe the transformations.

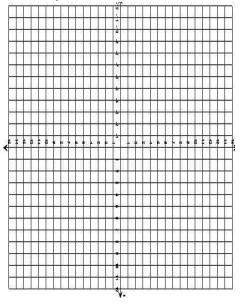
3) $F(x) = \log_{10} x - 6$

Domain:

Range:

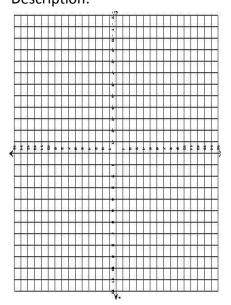
Asymptotes:

Description:



6) F(x) = -log₁₀ (x + 2) Domain: Range: Asymptotes: Description:

7) F(x) = log₁₀ 2x
Domain:
Range:
Asymptotes:
Description:



Day 11 Notes Part 2 Common Logs: Solving Equations and Word Problems

Express each of the numbers below as accurately as possible as a power of 10. You can find exact values for some of the required exponents by thinking about the meanings of positive and negative exponents. Others might require some calculator exploration of ordered pairs that satisfy the exponential equation $y = 10^{\times}$.

a. 100 10 [×] = 100 , ×=2	b. 10,000	c. 1,000,000
d. 0.01	e0.001	f. 3.45
g34.5	h. 345	i. 0.0023

Common Logarithms

We have already discussed logarithms in the previous lesson, but in this lesson we will solve exponent problems which focus on a base of 10. When we solve logarithmic problems in base 10, we call them **common logarithms**.

The definition of common logarithms is usually expressed as:

 $\log_{10} a = b$ if and only if $10^{b} = a$

Log₁₀ a is pronounced "log base 10 of a". Because base 10 logarithms are so commonly used, log₁₀ a is often written as log a. Most calculators have a built-in log function that automatically finds the required exponent value.

1) Use your calculator to find the following logarithms. Then compare the results with your work on Problem 1.

a. log 100	b. log 10,000	c. log 1,000,000
5	5 /	5
d. log 0.01	e. log -0.001	f. log 3.45
a: 10g 0:01	e. log 0.001	1. log 0. lo
g. log -34.5	h. log 345	i. log 0.0023
g. log = 5 +.5	n. log 5 +5	1. 109 0.0020

2) What do your results from Problem 2 (especially Parts e and g) suggest about the kinds of numbers that have logarithms? See if you can explain your answer by using the connection between logarithms and the exponential function y = 10[×].

Day 12: Common Logs: Introduction to Solving Equations and Word Problems

Warm-up: Key Features of Logarithmic Functions

1. Graph the function. Then give each part requested below.

Domain:

Range:

Asymptote:

Description of transformations from the parent graph $F(x) = \log_{10} (x)$:

2. Convert to exponential form then evaluate.

a) log 216

b) log 10000

b) 10^{x-2} = 10000

3. Solve.

a) 10[×] = 1000

Day 12 Notes

Investigation 2: Solving for Exponents

Logarithms can be used to find exponents that solve equations like 10^{\times} = 9.5. For this reason, they are an invaluable tool in answering questions about exponential growth and decay. For example, the world population is currently about 6.2 billion and growing exponentially at a rate of about 1.14% per year. To find the time when this population is likely to double, you need to solve the equation

As you work on the problems of this investigation, look for ways to answer this question:

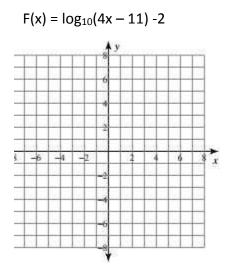
How can common logarithms help in finding solutions of exponential equations?

1) Use number sense and what you already know about logarithms to solve these equations.

a.
$$10^{\times} = 1,000$$
 b. $10^{\times + 2} = 1,000$

c.
$$10^{3x+2} = 1,000$$
 d. $2(10)^{x} = 200$

e.
$$3(10)^{x+4} = 3,000$$
 f. $10^{2x} = 50$



Honors Math 2

g. 10^{3x + 2} = 43

h. 12(10)^{3x + 2} = 120

i. 3(10)^{x + 4} + 7 = 28

Unfortunately, many of the functions that you have used to model exponential growth and decay have not used 10 as the base. On the other hand, it is not too hard to transform any exponential expression in the form b[×] into an equivalent expression with base 10. You will learn how to do this after future work with logarithms. The next three problems ask you to use what you already know about solving exponential equations with base 10 to solve several exponential growth problems.

2) If a scientist counts 50 bacteria in an experimental culture and observes that one hour later the count is up to 100 bacteria, the function $P(t) = 50(10^{0.3t})$ provides an exponential growth model that matches these data points.

- a. Explain how you can be sure that P(0) = 50.
- b. Show that $P(1) \approx 100$.
- c. Use the given function to estimate the time when the bacteria population would be expected to reach 1,000,000. Explain how to find this time in two ways one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning.
- 3) The world population in 2005 was 6.2 billion and growing exponentially at a rate of 1.14% per year. The function $P(t) = 6.2(10^{0.005t})$ provides a good model for the population growth pattern.
 - a. Explain how you can be sure that P(0) = 6.2
 - b. Show that P(1) = 6.2 + 1.14%(6.2)
 - c. Find the time when world population would be expected to reach 10 billion if growth continues at the same exponential rate. Explain how to find this time in two ways one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning.

CHECK YOUR UNDERSTANDING:

Use logarithms and other algebraic methods as needed to complete the following tasks.

- a. Solve these equations.
 - I. $5(10)^{\times} = 450$ II. $4(10)^{2\times} = 40$
 - III. 5(10)^{4x 2} = 500

Honors Math 2

IV. $8x^2 + 3 = 35$

b. The population of the United States in 2006 was about 300 million and growing exponentially at a rate of about 0.7% per year. If that growth rate continues, the population of the country in year 2006 + t will be given by the function P(t) = 300(10^{0.003t}). According to that population model, when is the U.S. population predicted to reach 400 million? Check the reasonableness of your answer with a table or a graph of P(t).

Review: Find the solution to each equation algebraically.

1) $\sqrt{20x-6} = \sqrt{5x+39}$ 2) $2(x-2)^{\frac{2}{3}}-8=192$

3)
$$(x+7)^{\frac{1}{2}} - x = 5$$

4) Your new painting is valued at \$2400. It's value depreciates 7% each year. The value is a function of time.

- a) Write a recursive (next-now) equation for the situation
- b) Write an explicit function for the situation
- c) When will the painting be worth \$1000?

5) Graph y = 2^{x+4} - 3. Identify the domain, range, asymptotes, and transformations of the parent function y = 2^{x} .

Day 13: Wrap-Up Review

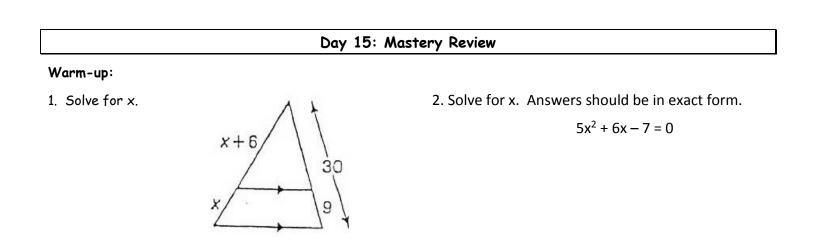
Warm-up:

1. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% each year. Write a function describing the value of the land as a function of time (let time be years after 1960).

- a.) Write an explicit equation to model the relationship:
- b.) Write a recursive (Now-Next) equation to model the relationship:
- c.) What was the value of the land in 2011?

Honors Math 2

- d.) In what year will the land be worth \$50,000?
- 2. The value of an SUV can be modeled by the function $V(t) = 30,000(0.84)^{t}$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?



3. H is in the interior of \angle IGF. You are given that m \angle IGH = (2x+5), m \angle FGH = 47, m \angle IGF = (18x-12). Find m \angle IGH and m \angle FGI.