Day 1: Properties of Exponents

Warm-Up: Before we begin today's lesson, how much do you remember about exponents? Use expanded form to write the rules for the exponents.

OBJECTIVE 1 Multiplying Exponential Expressions

$3^2 \bullet 3^4$	$y^4 \bullet y^{10}$	$12^3 \bullet 12^5$	SUMMARY: $a^m \bullet a^n =$ _______

OBJECTIVE 2 Dividing Exponential Expressions (Remember: $\frac{x}{x} = 1$)

$\frac{3^6}{3^2}$	$\frac{y^{10}}{y^4}$	$\frac{12^5}{12^3}$	SUMMARY: $\frac{a^m}{a^n} =$ _______

OBJECTIVE 3 Negative Exponential Expressions: Simplify 2 WAYS using expanded form AND the rule from OBJECTIVE 2

$\frac{3^2}{3^6}$	$\frac{y^4}{y^{10}}$	$\frac{12^3}{12^5}$	SUMMARY: $\frac{1}{a^n} =$ __________

OBJECTIVE 4 Exponential Expressions Raised to a Power

$(3^6)^2$	$(y^3)^4$	$(12m)^5$	SUMMARY: $(a^m)^n =$ ________ SUMMARY: $(a \bullet b)^n =$ ________

The lesson Part 1…

I. Find the value of x in each of the following expressions.

1. $5^x \bullet 5^2 = 5^7$	5. $3^{-2} \bullet 3^x = 3^2$	9. $4^{2/3} \bullet 4^x = 4$
2. $(5^3)^x = 5^6$	6. $(3^{-2})^x = \frac{1}{3^2}$	10. $(4^x)^{1/2} = 4$
3. $\frac{5^6}{5^x} = 5^4$	7. $\frac{3^x}{3^{12}} = \frac{1}{3^2}$	11. $\frac{4^{3/2}}{4^x} = 4$
4. $\frac{5^2}{5^x} = 1$	8. $(3^x)^2 = 1$	12. $4^6 4^x = 1$

II. Find the values of x and y in each of the following expressions.

$\frac{5^x}{3^y}=\left(\frac{5}{3}\right)^2$	$\left(\frac{2^3}{3^x}\right)^{-2}=\frac{3^6}{2^y}$	$\left(\frac{x^2}{5}\right)^3=\frac{2^6}{5^y}$
$(5\bullet 6)^2=5^x6^y$	$(2^x\bullet 6^3)^4=2^8\bullet 6^y$	$(3^x4^2)^3=4^y$

III. These will have more than 1 correct solution pair for x & y. Find at least 3 solution pairs.

$(5^x)(5^y)=5^{12}$

Possible Solutions

Option 1: _______

Option 2: _______

Option 3: _______

$(3^x)^y=3^{18}$

Possible Solutions

Option 1: _______

Option 2: _______

Option 3: _______

$\frac{4^x}{4^y}=1$

Possible Solutions

Option 1: _______

Option 2: _______

Option 3: ______

IV. When there are so many rules to keep track of, it's very easy to make careless mistakes. To help you guard against that, it helps to become a critical thinker. Take a look at the expanded and simplified examples below. One of them has been simplified correctly and there's an error in the other two. Identify the correctly simplified example with a ☺ . For the incorrectly simplified examples, write the correct answer and provide suggestions so that the same mistake is not made again.

$(4x)(x)=4x^2$

$\frac{x^2}{x^3}=x$

$\frac{50c^2d^2}{5cd^5}=45c^2d^3$

V. You've seen some of the more common mistakes that can happen when simplifying exponential expressions, and you may have made similar mistakes in the past. For each of the next rows of problems, complete one of the problems correctly and two of the problems incorrectly. For the incorrect problems, try to use errors that you think might go unnoticed if someone wasn't paying close attention. When you finish, you'll switch papers with two different neighbors (one for each row) so that they can check your work, find, fix, and write suggestions for how those mistakes can be avoided.

$(2x^2y^3)^5$

$\frac{-3x^2}{y^6}$

$(3x)^{-2}(x^2)$

$\frac{2xy^2}{8x^2y}$

$3^{-2}2^4x^3x$

$(-2xy)^4$

VI. Practice: Simplify and rewrite <u>without</u> negative exponents.

1) $6 \cdot c^3 \cdot d^{-2}$

2) $6x^4x^{-10}$

3) $\left(2^0 \cdot x^{-3}\right)^4$

4) $\dfrac{a^{12}b^{-3}}{a^5b^5}$

5) $\left(\dfrac{5x^{13}y^5z^2}{3 \cdot 5^2}\right)^0$

6) $(g^3 \cdot g^{-2})^4$

7) $\left(\dfrac{4c^{-5}}{8d^0}\right)^3$

8) $\left(\dfrac{x^{-8}}{y^{11}}\right)^{-2}$

9) $\dfrac{(2x^3)\cdot(x^4)^2}{8x^{11}}$

Day 2: Basic Radical Operations and Rational Exponents & Radicals

Warm-up:

Part I: **(**Properties of Exponents…with FRACTIONS!)

Even though they seem more complicated, fractions are numbers too. You can use all the same properties with fraction (rational) exponents as you can with integer exponents. Write down those properties first.

$a^m \bullet a^n =$ ______ $\quad \dfrac{a^m}{a^n} =$ ______ $\quad \dfrac{1}{a^n} =$ ______ $\quad (a^m)^n =$ ______ $\quad (a \bullet b)^n =$ ______

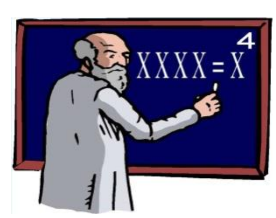
Part II: Match the radical on the left with a radical on the right with the equivalent value. Show your work by hand. Use your calculator only to double- check answers.

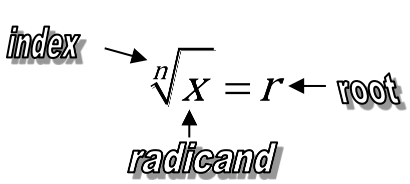
1. $\sqrt{98}$
2. $\sqrt{108}$
3. $3\sqrt{2} \bullet 4\sqrt{3}$
4. $-\sqrt{8} \bullet 3\sqrt{2}$
5. $4\sqrt{6} + 3\sqrt{6}$
6. $\sqrt{8} + \sqrt{18}$
7. $3\sqrt{2} - 2\sqrt{2}$
8. $\sqrt{48} - \sqrt{27}$

A	$7\sqrt{2}$
B	$4\sqrt{2}$
C	$5\sqrt{2}$
D	-12
E	$\sqrt{2}$
F	$12\sqrt{6}$
G	$6\sqrt{3}$
H	$\sqrt{3}$
I	$7\sqrt{6}$

The lesson Part 1:

We are familiar with taking square roots ($\sqrt{}$) or with taking cubed roots ($\sqrt[3]{}$), but you may not be as familiar with the elements of a radical.





An index in a radical tells you how many times you have to multiply the root times itself to get the radicand. For example, in the equation $\sqrt{81}=9$, 81 is the radicand, 9 is the root, and the index is 2 because you have to multiply the root by itself twice to get the radicand $(9 \bullet 9 = 9^2 = 81)$. When a radical is written without an index, there is an understood index of 2.

$\sqrt[3]{64}=?$
Radicand = _____ Index= _____
Root is ______

because ___•___•___ = ___$^3 = 64$

$\sqrt[5]{32x^5}=?$
Radicand= _____ Index= _____
Root is ______

because ___•___•___•___•___ = ___$^5 = 32x^5$

You can use your calculator to do this, but for some of the more simple problems, you should be able to figure them out in your head.

Reminder: To use your calculator:
Step 1: Type in the index.
Step 2: Press MATH
Step 3: Choose 5: $\sqrt[x]{\ }$
Step 4: Type in the radicand.

You Try:

$\sqrt[3]{343}=$ $\sqrt[5]{243y^5}$ $\sqrt[4]{1296m^4n^8}$ $\sqrt{144v^8}$

BUT not every problem will work out that nicely!

Try using your calculator to find an ***exact*** answer for $\sqrt[3]{24}$ = ________________

The calculator will give us an estimation, but we can't write down an irrational number like this exactly – it can't be written as a fraction and the decimal never repeats or terminates. The best we can do for an exact answer is use simplest radical form.

Here are some examples of how to write these in simplest radical form. See if you can come up with a method for doing this. Compare your method with your neighbor's and be prepared to share it with the class. (Hint: do you remember how to make a factor tree?)

$\sqrt{12}=2\sqrt{3}$	$\sqrt[3]{24}=2\sqrt[3]{3}$	$\sqrt[4]{48}=2\sqrt[4]{3}$

Simplifying Radicals: 1) ____

2) ____

3) ____

Negative Inside of the radical?

If you have an even root → The negative means an ____________________

If you have an odd root → The negative CAN stay FROM the radicand TO the coefficient…
Does not mean an imaginary number

Examples:

$\sqrt{16x^2}$	$\sqrt{8x}$	$\sqrt{15x^3}$
$\sqrt[3]{-8}$	$\sqrt[3]{80n^5}$	$\sqrt[4]{96}$
$\sqrt[4]{81}$	$\sqrt[5]{486}$	$\sqrt[3]{-40}$
$\sqrt[3]{18x^4}$	$\sqrt[4]{64x^3}$	$\sqrt[5]{-32x^3y^6}$
$\sqrt[3]{81x^3y^2z^4}$	$\sqrt[3]{192x^5y^7z^2}$	$\sqrt[4]{1875x^4z^2}$

The lesson Part 2:

Multiplying Radicals: When written in radical form, it's only possible to write two multiplied radicals as one if the index is the same. As long as this requirement is met,

1) Multiply the __________________
2) Multiply the __________________
3) Simplify!

$2\sqrt{3}\cdot 5\sqrt{2}$	$-3\sqrt{8}\cdot\sqrt{2}$	$4\sqrt{5}\cdot 3\sqrt{10}$
$\sqrt{3x^2y}\cdot\sqrt{5xy}$	$6\sqrt{8x^3y^2}\cdot\sqrt{10xy^3}$	$-\sqrt{5x^4y^3}\cdot\sqrt{15x^2y^5}$
$\sqrt[3]{4x^2}\cdot 5\sqrt[3]{8xy}$	$\sqrt[4]{2x^5}\cdot\sqrt[4]{40x^3y^3}$	$4\sqrt[5]{27x^3}\cdot\sqrt[5]{9x^3y^5}$
$3\sqrt[3]{5x^3}\cdot 2\sqrt[3]{50y}$	$\sqrt[3]{9}\cdot\sqrt[3]{-24}$	$\sqrt[4]{8}\cdot\sqrt[4]{32}$

Adding and Subtracting Radicals: You've been combining like terms in algebraic expressions for a long time! Show your skills by simplifying the following expressions. Just like Combining "like" Terms.

$2x - x + 4x$ = ______________________ $3y - 2x + y - 6y$ = ____________________

Some tips:

-You are now combining "like" radical expressions instead of variables.

-Add/Subtract only when the radicals have the same ___________ and ______________.

-When you add/subtract, you add the ________________. **The radicands do not change.**

-Always ___________________ **FIRST**.

Examples:

$3\sqrt{3} + 4\sqrt{3}$	$\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$	$4\sqrt{12} - \sqrt{75}$
$\sqrt{45x^3} - \sqrt{20x^3}$	$5\sqrt[3]{32} - 2\sqrt[3]{108}$	$3\sqrt[3]{16} + \sqrt[3]{54}$
$2\sqrt[3]{125a^4} - 5\sqrt[3]{8a}$	$9\sqrt[3]{40a} - 7\sqrt[3]{135a}$	$5\sqrt[3]{16y^4} + 7\sqrt[3]{2y}$
$6\sqrt{18} + 3\sqrt{50}$	$\sqrt[3]{54} + \sqrt[3]{16}$	$\sqrt[4]{32} + \sqrt[4]{48}$

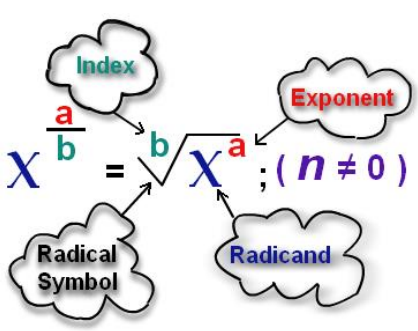
Day 3 Warm-Up: Rational Exponents & Radicals

Write each expression in simplified form (one coefficient, each base used only once with one exponent each, no negative exponents). Check your work with your partner.

$\left(x^{\frac{2}{3}}\right)^{-3}$	$\left(x^{-\frac{4}{7}}\right)^{7}$
$\left(3x^{\frac{2}{3}}\right)^{-1}$	$5\left(x^{\frac{2}{3}}\right)^{-1}$
$\left(-27x^{-9}\right)^{\frac{1}{3}}$	$\left(-32x^{15}\right)^{\frac{1}{5}}$
$\left(\dfrac{x^{3}}{y^{-1}}\right)^{-\frac{1}{4}}$	$\left(\dfrac{x^{2}}{y^{-10}}\right)^{\frac{1}{2}}$
$\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6}$	$\left(x^{\frac{2}{3}}y^{-\frac{1}{6}}\right)^{12}$
$\left(\dfrac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12}$	$\left(\dfrac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$

The Lesson: **Rational Exponents & Radicals….**

An expression containing a rational exponent can be written in an equivalent radical form.



You try: Rewrite each of the following expressions in radical form.

$x^{\frac{3}{2}}$	$(-27)^{\frac{2}{3}}$	$(16x)^{\frac{5}{4}}$	$y^{-9/8}$
$2a^{\frac{1}{4}}$	$4^{\frac{-7}{2}}$	$\left(3^{\frac{2}{5}}\right)^{5}$	$x^{1.2}$

Now, reverse the rule you developed to change radical expressions into rational expressions.

$\sqrt[5]{2}$	$(\sqrt[3]{6})^5$	$(\sqrt{5})^7$
$\sqrt{7}$	$(\sqrt[4]{9^3})$	$(\sqrt[7]{3x})^2$

Earlier in this unit, you learned that when written in radical form, it's only possible to write two multiplied radicals as one if the index is the same. However, if you convert the radical expressions into expressions with rational exponents, you CAN multiply or divide them (as you saw in your warm-up)! Give it a try ☺ Write your final answer as a simplified radical.

1. $\dfrac{12\sqrt[3]{y}}{4\sqrt{y}}$

2. $\left(\dfrac{\sqrt[3]{a^2}}{\sqrt{b}}\right)^{-6}$

3. $(2\sqrt[4]{a})^3 \bullet \sqrt{a^3}$

4. $\sqrt[4]{x^{12}} \bullet \sqrt{y^{-2}}$

5. $\dfrac{\sqrt{64x^3}}{\sqrt[3]{512x^9}}$

6. $\sqrt[4]{625x^8}$

7. $\sqrt[7]{x^2} \bullet \sqrt[14]{x^3}$

8. $\dfrac{1}{\sqrt[3]{-27x^9}}$

9. $(\sqrt{x} \bullet \sqrt[3]{y^2}\ )^{-6}$

Mixed Review: Simplify each expression.

1. $\sqrt{75y} - 2\sqrt{27y} + \sqrt{45y}$

3. $2\sqrt{18a^2b} \bullet 6\sqrt{3b^2}$

2. $\sqrt{108yz^2} + 3\sqrt{98yz^2} + 2\sqrt{55yz^2}$

4. $\dfrac{3x^{-14}y^{11}}{18x^2}$

Day 4: Warm-Up

Modeling with Exponential Functions: Solving Equations with Rational Exponents and Radicals

Warm-up: You know a lot about inverses in mathematics – we use them every time we solve equations. Write down the inverse operation for each of the following (there could be more than one correct answer) and then give a definition for "inverse" in your own words.

If you get stuck, it may be helpful for you to write the expression out and think what you would do to solve an equation that had that expression on one side of the equation.

The phrase…	Is the expression…	And its inverse is…
adding 5 to a number	$x+5$	Subtracting 5 from a number
subtracting 7 from a number		
multiplying a number by $\frac{1}{2}$		
Multiplying a number by $\frac{2}{5}$		
dividing a number by 3		
squaring a number		
Taking the square root of a number		
Raising a number to the 5^{th} power		
Taking the 5^{th} root of a number		
Raising a number to the $\frac{2}{5}$ power		

Use the information from the table above to solve the following equations. Express your answers in simplified radical form.

a. $\sqrt[3]{x^5} = 64$

b. $(y^{\frac{1}{4}})^3 = 16$

An "inverse" is…

Day 4: Class Instruction

Modeling with Exponential Functions: Solving Equations with Rational Exponents and Radicals

With your partner, decide who is Partner A and B. Using the information about inverses from the warm-up, perform the inverse operation to solve the following equations. Then compare your answers with your partner to see what patterns you have found between the two columns. (Hint: on some of these problems, it may help to use exponent properties to simplify the expression first).

Skill	Partner A	Partner B
1	$\sqrt[4]{a} = 14$	$\sqrt[5]{b} = 50$
2	$\sqrt[5]{a^9} = 26$	$\sqrt[4]{b^3} = 27$
3	$\left(\sqrt[7]{a}\right)^3 = 21$	$\left(\sqrt{b}\right)^5 = 12$
4	$a^{\frac{1}{5}} = 50$	$b^{\frac{1}{4}} = 14$
5	$a^{\frac{3}{4}} = 27$	$b^{\frac{9}{5}} = 26$
6	$\left(a^{\frac{1}{2}}\right)^5 = 12$	$\left(b^{\frac{1}{7}}\right)^3 = 21$

Make sure you have all of the answers correct to the table before going on. Now that you're finished, we're going to combine our skills from this unit so far to solve some more challenging equations.

Yesterday's problems only had one step. However, today there are multiple steps, meaning that one cannot just simplify the radical or rational exponent until it is ***isolated*** on one side of the equation.

- You can isolate the radical using ______________________.
- There are also some problems below in which the rational exponent or radical is applied to the entire side of the equation. Only in these situations will you undo the rational exponents or radicals first. Before solving the entire problem, make sure you know what the first step will be!

Steps to solving equations with rational exponents and radicals

1. Isolate the term with the Rational Exponent or Radical (Get that term alone!)
2. Do the Inverse Operation
   *For Rational Exponents, raise both sides to the reciprocal power
   *For Radicals, raise both sides to the index
3. Solve for the variable
4. Check the solution
   - Extraneous solution? Or Actual Solution? Or No Solution?

Examples ~

$x^{1/4} - 2 = 3$ Step 1	$4x^{7} - 6 = -2$ Step 1
$3\left(x^{2/3} + 5\right) = 207$ Step 1	$1450 = 800\left(1 + \frac{x}{12}\right)^{7.8}$ Step 1
$14.2 = 222.1 \cdot x^{3.5}$ Step 1	$3x^{3/4} + 5 = 53$ Step 1
$x^{1/2} - 5 = 0$ Step 1	$(2x + 7)^{1/2} = 3$ Step 1

$\sqrt[3]{x - 2} = 4$ Step 1	$\sqrt{a + 2} - 2 = 12$ Step 1
$\sqrt{2x - 5} = 9$ Step 1	$\sqrt[4]{3x + 1} - 5 = 0$ Step 1

One more example! A tougher one….

Solve $x = \sqrt{110 - x}$

Day 5: Warm-up

Modeling with Exponential Functions: Solving Equations with Rational Exponents and Radicals

Solve for the missing variable

1. $-8+\sqrt{5a-5}=-3$

2. $10+\sqrt{10m-1}=13$

3. $-12=-6\sqrt{b+4}$

4. $-10\sqrt{v-10}=-60$

Part 2 – Solving equations with radicals:

The next few problems are…different. We're going to come across some equations that have no solution and some that have two solutions. Remember, you can always check your answers by substituting your solution into the equation to make sure it works. In fact, you really ***need*** to check your answers to these problems! When we solve an equation correctly, but the answer doesn't work when we check it, we call the solution extraneous.

$\sqrt{a+2}-2=a$

$\sqrt{3x-2}=-5$

$(2x+7)^{1/2}-x=2$

$3x^{4/3}+5=53$

You're going to come across some tougher problems that involve multiple steps. Let's try a couple. ☺

$\sqrt{x-5}-\sqrt{x}=-2$

$\sqrt{3x+7}=x-1$

Applications of Equations with Rational Exponents or Radicals.

1. The distance between two points is $5\sqrt{2}$. If one of the points is located at (4, 2) and the other point has an x-value of -1, what are the possible y-values of the other point?

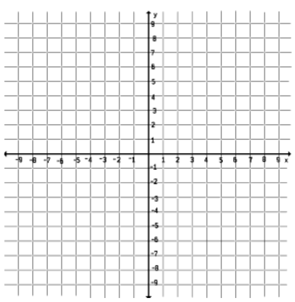
2. The volume of a sphere is 2145. If the formula $V = (4/3)\pi r^3$ is used to calculate the volume of a sphere, what is the radius of the sphere?

3. The equation $v = \sqrt{2.5r}$ allows you to calculate the maximum velocity, v, that a car can safely travel around a curve with a radius of r feet. This is used by the Department of Transportation to determine the best speed limit for a given stretch of road. If a road has a speed limit of 45 mph, what is the tightest turn on that road?

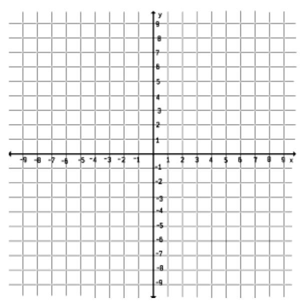
Graphical Investigation

Solve, then graph on your calculator to check solutions. Sketch what you see in the graph provided.

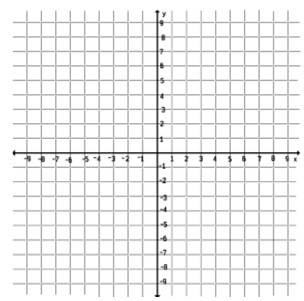
1. $\sqrt{x-1} = 2$



2. $\sqrt{2x-1} + 5 = 2$



3. $x - 1 = \sqrt{5x-9}$



4. What does a graph look like for equations that have an extraneous solution?

Day 6: Warm-Up

Quiz Day

Simplify the following:

1. $9\sqrt[3]{16} + \sqrt[3]{54}$

2. $6\sqrt{8x^3y^2} \cdot \sqrt{10xy^3}$

3. $\sqrt[4]{625x^5}$

Solve the following:

4. $\left(\frac{5^3}{3^x}\right)^{-2} = \frac{3^8}{5^y}$

5. $(x^{\frac{1}{2}})^3 = 27$

6. $\sqrt{a+4} - 4 = a$

Day 6: Quiz Review
Quiz Day

Days 1 Through 5, Concept Review:

Multiplying Exponential Functions: $a^m \bullet a^n =$ ________

Dividing Exponential Functions: $\frac{a^m}{a^n} =$ ________

Negative Exponential Functions: $\frac{1}{a^n} =$ ____________

$\frac{1}{a^{-n}} =$ ____________

Exponential Functions Raised to a Power:

$(a^m)^n =$ __________

$(a \bullet b)^n =$ __________

Label The Following:

$$\sqrt[n]{x} = r$$

Multiplying Radicals ~

Characteristics And Properties For Doing This: Example(s)

Adding And Subtracting Radicals ~

Characteristics And Properties For Doing This: Example(s)

Reminder: To use your calculator:
Step 1: Type in the radicand in the base of the exponent.
Step 2: Raise the base (Using this following symbol " **^** ") to the power of the 1/(index).

OR

Reminder: To use your calculator:
Step 1: Type in the index.
Step 2: Press MATH
Step 3: Choose 5: $\sqrt[x]{\ }$
Step 4: Type in the radicand.

Circle the correct answer:
Can you *Sometimes/Always/Never* check your answers by substituting your solution into the equation to make sure it works? How often should you check?

"Radical"

1. Isolate the radical/Get the radical Alone
2. Do the inverse operation: ____________________
3. Solve for the variable
4. Check the solution

- Extraneous solution? Or Actual Solution? Or No Solution?

"Rational Exponent"

1. Isolate the term with the Rational Exponent/Get the term Alone
2. Do the inverse operation: ______________________
3. Solve for the variable
4. Check the solution

- Extraneous solution? Or Actual Solution? Or No Solution?

Day 7: Exponential Growth and Decay Warm-Up

SCENARIO 1

A pack of zombies is growing exponentially! After 1 hour, the original zombie infected 5 people, and those 5 zombies went on to infect 5 people, etc. After a zombie bite, it takes 1 hour to infect others. How many newly infected zombies will be created after 4 hours?

If possible, draw a diagram, create a table, a graph, and an equation.

SCENARIO 2

During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc. After a zombie bite, it takes an hour to infect others. Determine the number of newly infected zombies at 6 am this morning.

If possible, draw a diagram, create a table, a graph, and an equation.

SCENARIO 3

At 9:00 am, the official count of the zombie infestation was 16384. Every hour the number of zombies quadruples. Around what time did the first zombie roll into town?

Notes Day 7: Exponential Growth and Decay

You find a bank that will pay you 3% interest annually, so you consider moving your account. Your current bank decides you're a good customer and offers you a special opportunity to compound your interest semiannually!!! (They make it sound like it's a really good deal, so you're curious). You don't play around with your money, so you ask what exactly that means. They explain that you'll still get 2.5% interest, but they'll give you 1.25% interest at the end of June and 1% interest at the end of December. You want to see if you make more money than you would if you just switched banks, so you do the calculations. Which bank is giving you a better deal? Explain your answer.

Review of Vocabulary

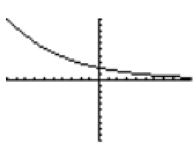
- **Initial Value:** The 1st term in a sequence or the dependent value associated with an independent value of 0.
- **Common Ratio:** The ratio of one term in a sequence and the previous term.
- **Common Difference:** The difference between one term in a sequence and the previous term.
- **Recursive Function:** function that can be used to find any term in a sequence if you have the previous term (i.e. NOW-NEXT)
- **Explicit Function:** a function that can be used to find any term in the sequence without having to know the previous term. (i.e. y = or f(x) =)
- **Domain:** the set of all input values for a function
- **Theoretical Domain:** the set of all input values for a function without consideration for context
- **Practical Domain:** the set of all input values for a function that are reasonable within context

Comparing Exponential Growth and Decay

Exponential Decay Model:

$y = C(1 - r)^t$

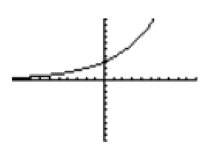
$0 < 1 - r < 1$



Exponential Growth Model

$y = C(1 + r)^t$

$1 + r > 1$



Exponential Decay

$y = a(b)^x$

When a > 0 and b is ________________________,

the graph will be ____________________________

(_________________)

$$y = 1\left(\frac{1}{2}\right)^x$$

So for this example, each time x is increased by 1,

y ____________________ of its previous value.

Exponential Growth

$y = a(b)^x$

When a > 0 and b is ________________________,

the graph will be ____________________________

(_________________)

$$y = 1(2)^x$$

So for this example, each time x is increased by 1,

y ____________________ by a factor of 2.

Exponential Decay	Exponential Growth
$y = a(b)^x$ b = __________ This is because you begin with 100% (1 when written as a decimal) and subtract the same percentage each time.	$y = a(b)^x$ b = __________ This is because you begin with 100% (1 when written as a decimal) and add the same percentage each time.

Examples

1. A house that costs $200,000 in 2010 will appreciate in value by 3% each year.
   a. Write a function that models the cost of the house over time. Use x for years after 2010, and y for the value of the house, in dollars.
   b. Find the value of the house in 2020.
   c. In what year will the house value have doubled?

2. The most recent virus that is making people ill is a fast multiplying one. On the first day of the illness, only 2 virus "bugs" are present. Each day after, the amount of "bugs" triples.
   a. Write a function that models the "bugs'" growth over time. Let x represent the number of days since the onset of the illness and y represent the amount of "bugs".
   b. Find the amount of "bugs" present by the 5^{th} day.

3. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.

   a. Write a function that models the amount of menthol in your body over time. Let x represent minutes since you took the cough drops and y be the amount of menthol, in mg, remaining in your body.

   b. It is safe to take a new cough drop when the amount of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop?

Let's try a tougher problem you may run in to!

4. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every **5** minutes, the amount of menthol in your body is cut in half.

   a. Write a function that models the amount of menthol in your body over time. Let x represent minutes since you took the cough drops and y be the amount of menthol, in mg, remaining in your body.

   b. It is safe to take a new cough drop when the amount of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop?

In the problems above, we wrote explicit functions for the exponential data.

$$y = a(b)^x$$

BUT we also need to know how to write recursive functions for the exponential data. Make sure you include the starting value of your data ______ – this will always be where the independent variable is ____.

Next = _______________ , Starting at _________

Let's write the prior examples using Recursive Formulas!

1.

2.

3.

You Try!

4. Tobias ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

   a. Write an explicit function that models the gnats' grown over time. Use x for nights after you first see the gnats and y for number of gnats.

   b. Tobias' mom said that he would be grounded if the gnats number more than 120. On what night will Tobias be in trouble, if he doesn't step in and solve his gnat problem?

   c. Write a recursive function to model the gnats' growth.

5. JaCorren is 60 inches and going through a growth spurt. For the next year, his growth will increase by 1% each month. Write a function that models JaCorren's growth spurt over the next year. Let x represent number of months and y represent JaCorren's height in inches.

a. Write an explicit function that models JaCorren's growth spurt over the next year.

b. Find JaCorren's height at the end of the year.

c. Write a recursive function to model JaCorren's growth.

6. Ian's new Mercedes cost him $75,000 in 2014. From the moment he drives it off the lot, it will depreciate by 20% each year for the first five years.

a. Write an explicit function that models the car's depreciation. Use x for years after 2014 and y for the car's value, in dollars.

b. What will the car's value be in 2019?

c. Write a recursive function to model the car's depreciation.

When you write an equation for a situation and use it to make predictions, you assume that other people who use it will understand the situation as well as you do. That's not always the case when you take away the context, so we sometimes need to provide some additional information to accompany the equation.

The **domain** of a function is the set of all the possible input values that can be used when evaluating it. If you remove your functions from the context of this situation and simply look at the table and/or graph of the function, what numbers are part of the **theoretical domain** of the function?

Will this be the case with all exponential functions? Why or why not?

When you consider the context, however, not all of the numbers in the theoretical domain really make sense. We call the numbers in the theoretical domain that make sense in our situation the **practical domain**. If you look at the tables for the problems, we can find what numbers would be a part of the practical domain.

Classwork: Exponential Word Problems Scavenger Hunt or additional practice