Unit 3 Day 9

Transformations Of Exponentials
&
Inverses Of Functions

Day 9 Warm Up

How are the following graphs changed from their parent graph, $y = x^2$? (If you don't remember some of the transformations, graph the equation and the parent in the calculator ☺)

1) $y = (x - 4)^2$

2) $y = x^2 - 4$

3) $y = x^2 + 1$

4) $y = (x + 3)^2$

5) $y = 4x^2$

6) $y = \frac{1}{2} x^2$

7) $y = 3(x - 4)^2$

8) $y = 1/4\ x^2 + 3$

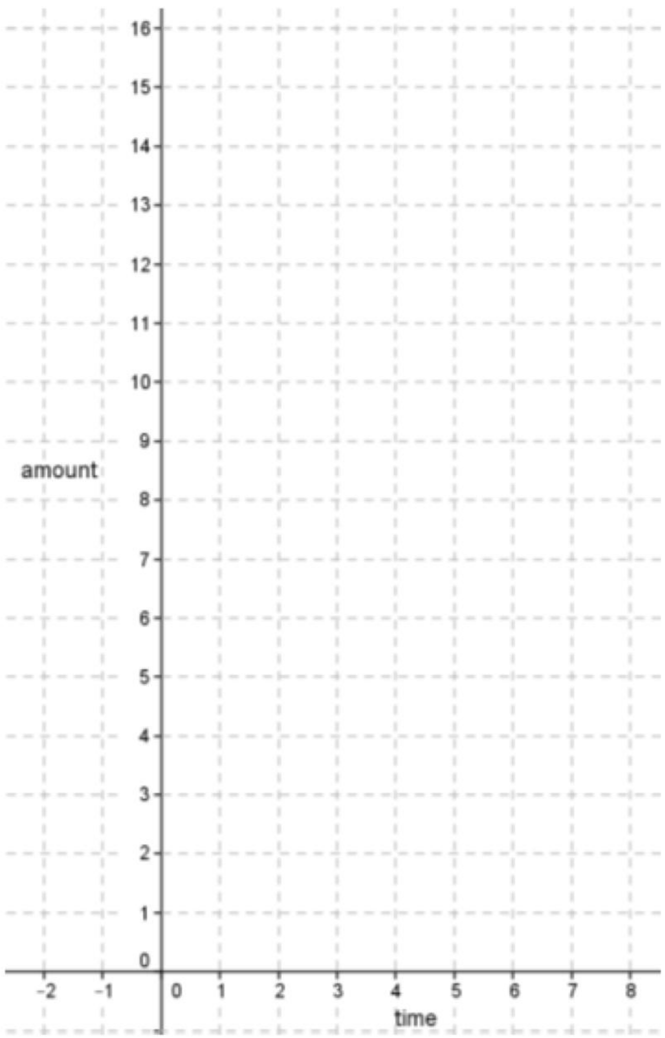
You may need to do your work on a piece of paper!
Good practice for the next quiz!! ☺

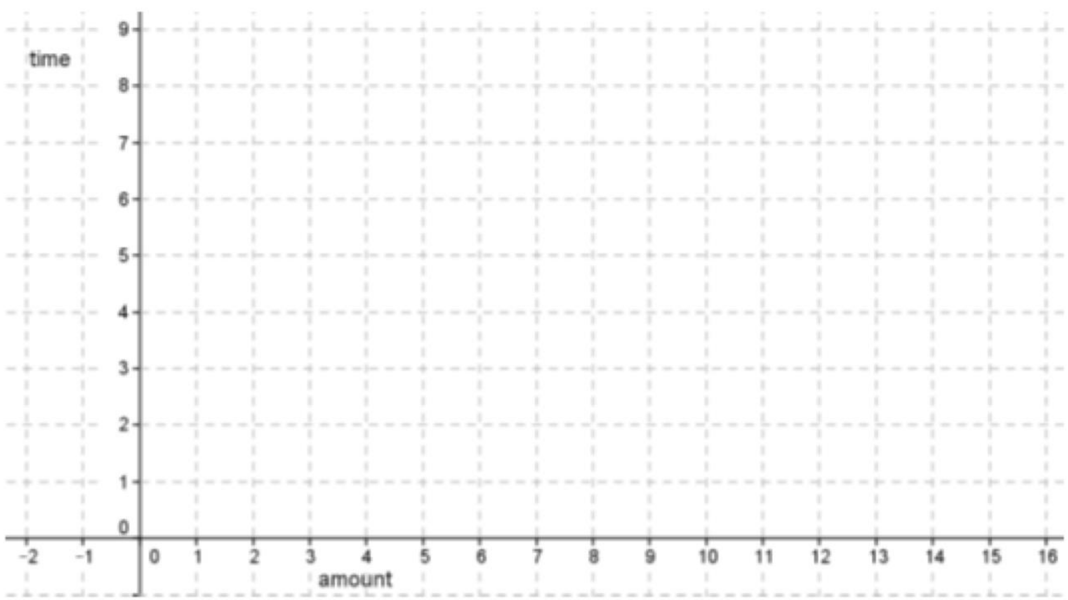
Warm Up continued ->

Warm-Up Continued…

the following data to graph on each of the following coordinate planes.
Be sure to pay attention to the labels on each axis.

Time	Amount of radioactive material
0	½
1	1
2	2
3	4
4	8
5	16





Day 9 Warm Up

You may need to do your work on a piece of paper!

Good practice for the next quiz!! ☺

1) $2(4x-5)^{\frac{2}{3}}=18$

2. $(3x-11)^{\frac{1}{2}}-x=-3$

3. How long will it take \$500 to triple if it is invested in a stock that increases 6% a year?

4. Write an exponential function using the points (4, 10) and (12, 15.6). Show work algebraically.

5. A car driving 102 mph decreases speed 15% per second. Write a recursive AND explicit equation to represent the car speed over time in seconds.

Day 9 Warm Up

How are the following graphs changed from their parent graph, $y = x^2$? (If you don't remember some of the transformations, graph the equation and the parent in the calculator ☺)

1) $y = (x - 4)^2$

Translated Right 4

2) $y = x^2 - 4$

Translated Down 4

3) $y = x^2 + 1$

Translated Up 1

4) $y = (x + 3)^2$

Translated Left 3

5) $y = 4x^2$

Vertical stretch by factor of 4

6) $y = \frac{1}{2} x^2$

Vertical compression by factor of 1/2

7) $y = 3(x - 4)^2$

Vertical stretch by factor of 3 and translated right 4

8) $y = 1/4\ x^2 + 3$

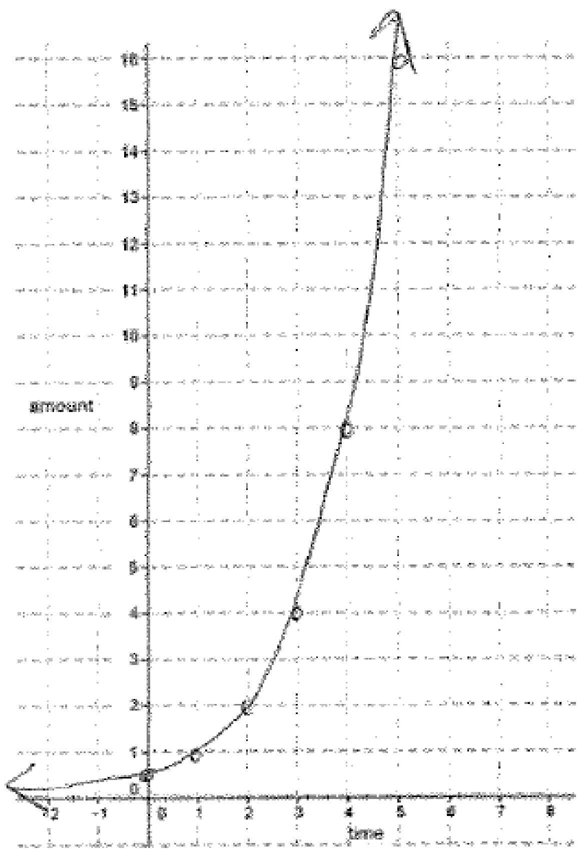
Vertical compression by factor of ¼ and translated up 3

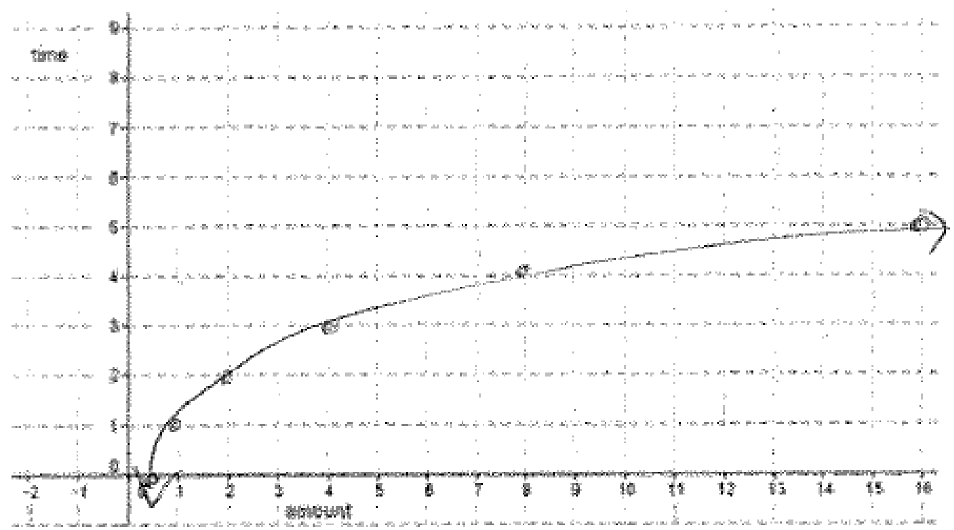
Warm Up continued ->

Day 9 Warm Up Answers

9) Use the following data to graph on each of the following coordinate planes. Be sure to pay attention to the labels on each axis.

Time	Amount of radio-active material
0	$\frac{1}{2}$
1	1
2	2
3	4
4	8
5	16





Explain how the graphs are alike.-Both are curved, level off in one place.

Are different. -They are reflections over y=x. 1st flattens at y = 0 and 2nd flattens at x = 0.

11) $2(4x-5)^{\frac{2}{3}}=18$

$x=8$

12) $(3x-11)^{\frac{1}{2}}-x=-3$

$x=4,\ 5$

13. How long will it take $500 to triple if it is invested in a stock that increases 6% a year?

About 19 years

14. Write an exponential function using the points (4, 10) and (12, 15.6). Show work algebraically.

$y=8.01(1.0572)^x$

15. A car driving 102 mph decreases speed 15% per second. Write a recursive AND explicit equation to represent the car speed over time in seconds.

Next = Now*.85, Start = 102

y = $102(.85)^x$

Check all solutions ☺

Homework Answers
Packet p. 14

1) a. 50%

   b. 50 grams

   c. $Y = 50(1/2)^{x/5730}$

   d. 44.30 g

2) a. $Y = 88.7(1.0077)^x$

   b. 23.6 clicks

3) 24.66 minutes

4) a. $Y = 5(2.5)^x$

   b. Next = now • 2.5; start = 5

   c. 150% increase

Homework Answers
Notes Packet p. 24 (Questions 6 and 7)

6) When the wolf population of the Midwest was first counted in 1980, there were 100 wolves. In 1993, that population had grown to 3100 wolves! Assuming the growth of this population is exponential, complete each of the following.

$$y = 100(1.3023)^x$$

c)

Time Since 1980 (in years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Estimated Wolf Population	100	130.2	169.6	220.9	287.6	374.6	487.8	635.3	827.4	1007.5	1403.2	1827.3	2379.8	3,100

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d) Identify the theoretical and practical domain of this situation. Explain how you decided upon your answers.

Theoretical domain: All real #s

Practical Domain: $x \geq 0$

7) A 2008 Ford Focus cost $15452 when purchased. In 2013 it is worth $8983 if it's in excellent condition.

a) Assuming the value of the Focus depreciates by the same percentage each year, what is that percentage?

x = years after 2008

$$(0,15452) \quad (5,\ 8983)$$

$$8983 = 15452b^{(5-0)}$$

$$0.5813 = b^5$$

$$b = (.5813)^{\frac{1}{5}} \qquad b \approx 0.8972$$

$$0.8972 = b$$

$$1 + r = b$$

$$0.8972 = 1 + r$$

$$r = 10.28\%\ decrease$$

b) Write an explicit function for the value of the Ford Focus.

$$y = 15452(0.8972)^x$$

c) If the owner of the car plans on selling it in 2018, how much should she expect to get for it?

$$y = 15452(0.8972)^{10} = \$5222.25$$

Unit 3 Practice (Notes p. 25)

1) Simplify $5\sqrt[3]{72x^5y^7} \bullet 2\sqrt[3]{343x^4y^3}$ $140x^3y^3\sqrt[3]{9y}$

2) Solve $\sqrt{2x-4}+x=6$ $x=4$

3) Solve $5(2x+4)^{4/3}=80$ $x=2$

Unit 3 Practice (continued)

4) Sr-85 is used in bone scans. It has a half-life of 34 hours. Write the exponential function for a 15 mg sample. Find the amount remaining after 86 hours. Show work algebraically.

$2.598\ mg$

5) Write exponential function given (2, 18.25) and (5, 60.75). Show your work algebraically.

$$y = 8.19(1.4931)^x$$

6) A plant 4 feet tall grows 3 percent each year. How tall will the tree be at the end of 12 years? Round your answer to the hundredths place.

$5.70\ ft$

7) A BMX bike purchased in 1990 and valued at $6000 depreciates 7% each year. Next = Now * .93, Start = 6000

a) write a next-now formula for the situation

b) write an explicit formula for the situation $y = 6000(.93)^x$

c) how much will the BMX bike be worth in 2006? $\$1,878.79$

d) in what year will the motorcycle be worth $3000?

1999

Tonight's Homework:

Packet Pages 15-16

Translations of Exponential Graphs

We'll do the first graph y = 3^x together!!

The Lesson

Graph the parent function $y = (3)^x$ using a table of x and y values. Also, CLEARLY INDICATE the horizontal asymptote.

x	y
-1	**1/3**
0	**1**
1	**3**
2	**9**
3	**27**

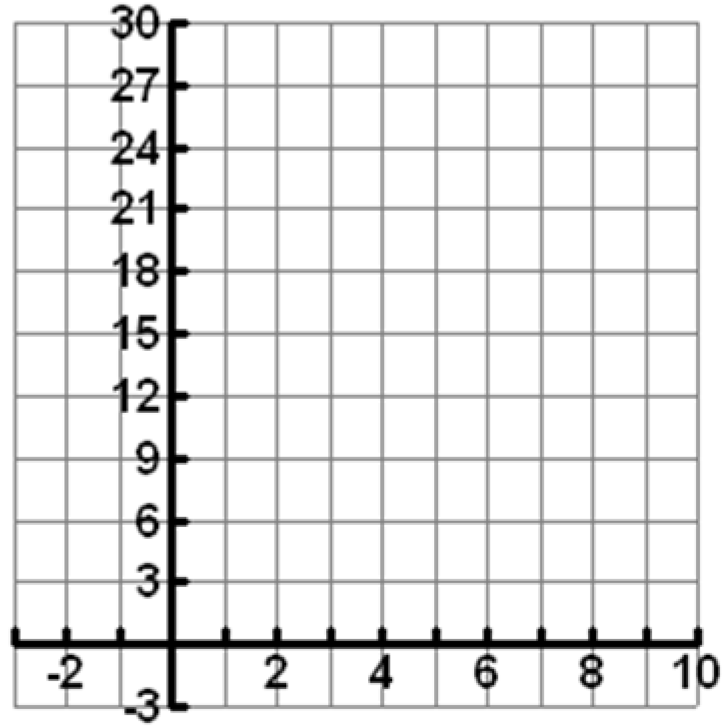
HA: **y = 0**

Domain:

$(-\infty, \infty)$

Range:

$(0, \infty)$



***Asymptote: a line that a curve approaches as it approaches infinity**

Translations of Exponential Graphs

On the same grid, graph y = $(3)^{x+1}$

using a different color or mark.

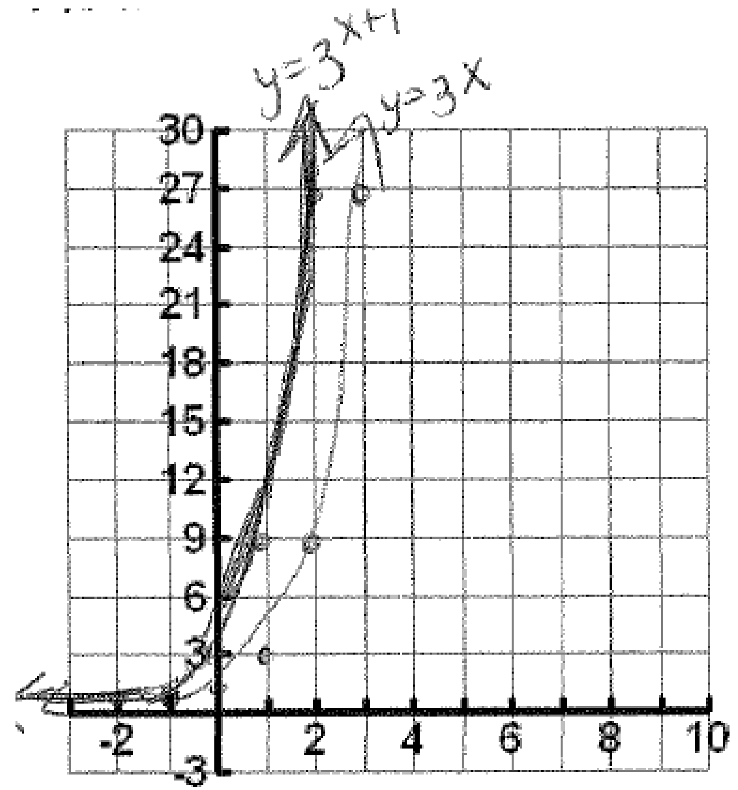
CLEARLY INDICATE the horizontal asymptote.

x	y
-2	1/3
-1	1
0	3
1	9
2	27

HA: **y = 0**

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$



Explain how the graph shifted from the parent graph: **Left 1**

Try it out !

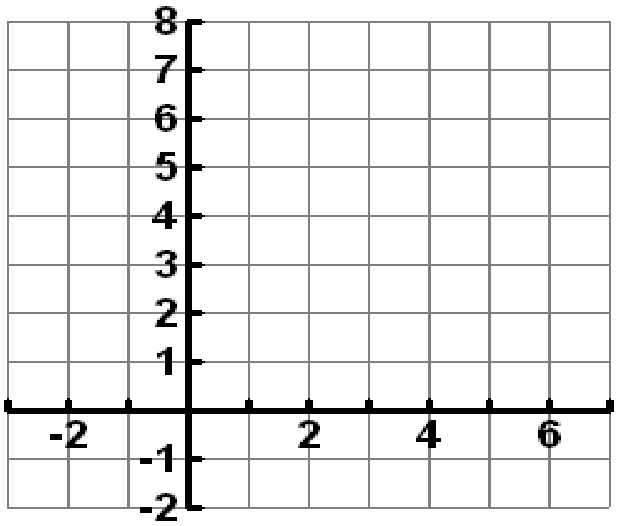
Graph the parent function: $F(x) = (2)^x$

x	F(x)
-1	

HA:___________

Domain: ____________

Range: ____________



On the same grid, graph $F(x) = (2)^{x-4}$ using a different color or mark.

x	F(x)
3	

HA:___________ Domain: ____________ Range: ____________

Explain how the graph shifted from the parent graph:

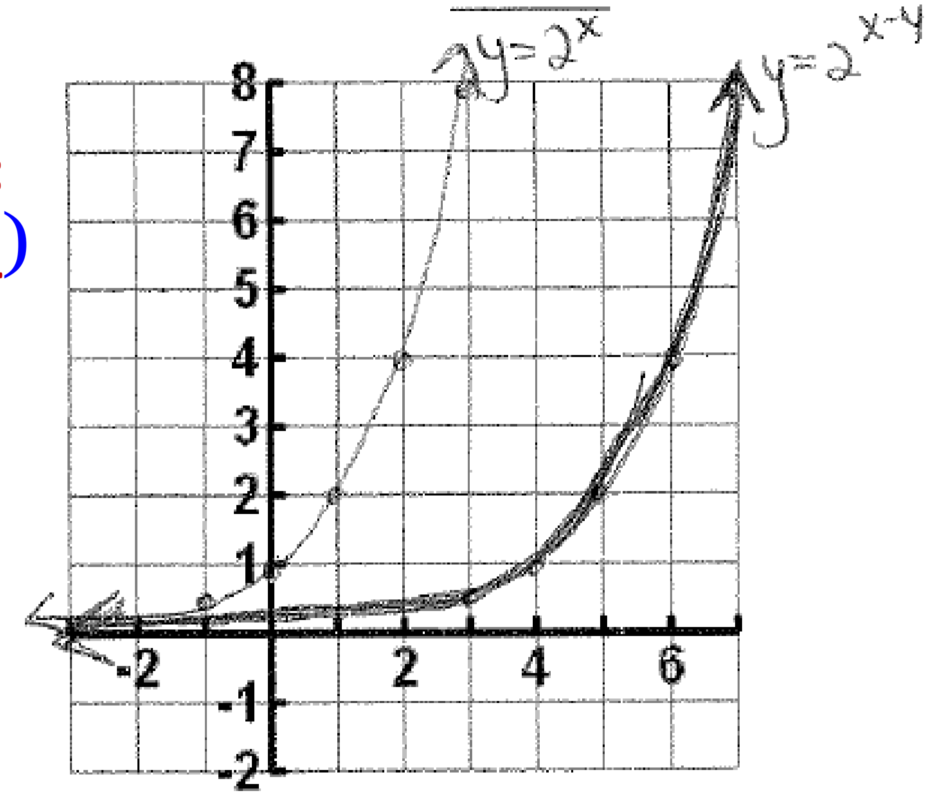
Graph the parent function: $y = (2)^x$

x	y
-1	1/2
0	1
1	2
2	4
3	8

HA: y = 0

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$



On the same grid, graph $y = (2)^{x-4}$ using a different color or mark.

x	y
3	1/2
4	1
5	2
6	4
7	8

HA: y = 0

Explain how the graph shifted from the parent graph:

right 4

SUMMARY of Horizontal Translations

Adding "c" in the exponent shifts the graph to the _left_ c units.

Subtracting "c" in the exponent shifts the graph to the _right_ c units.

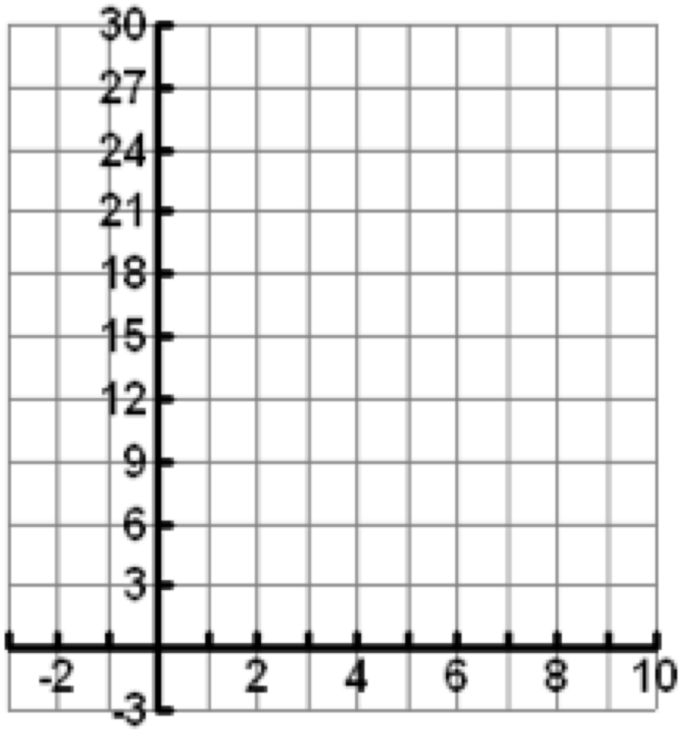
Try It Continued... **Top of p. 28**

Graph the parent function again: $F(x) = (3)^x$

HA:___________

Domain: ____________

Range: ____________



On the same grid, graph $F(x) = (3)^x + 1$ using a different color or mark.

x	F(x)
-1	

HA:___________

Domain: ____________

Range: ____________

On the same grid, graph $F(x) = (3)^x - 2$ using a different color or mark.

x	F(x)
-1	

HA:___________

Domain: ____________

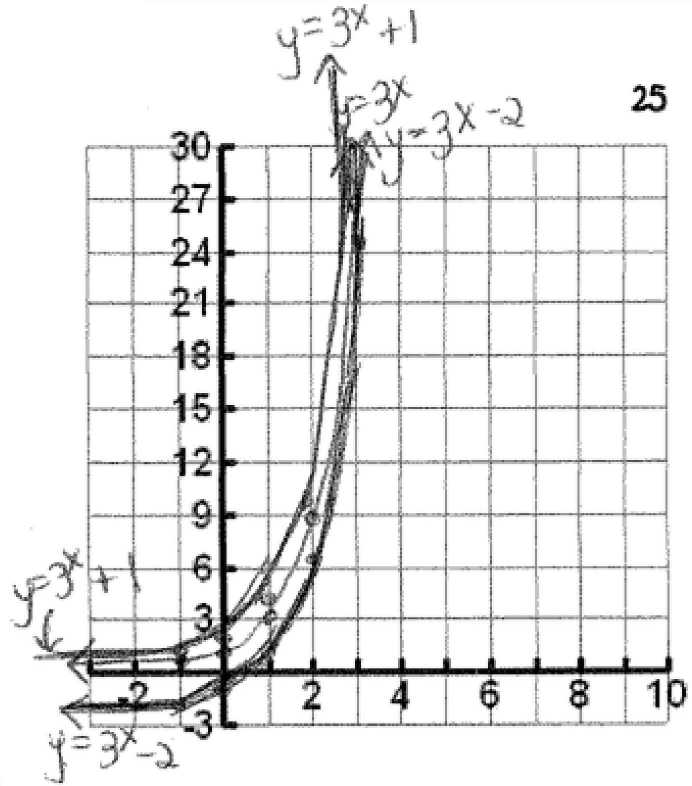
Range: ____________

Top of p. 28

Graph the parent function again: $y = (3)^x$

x	y
-1	1/3
0	1
1	3
2	9
3	27

HA: $y = 0$



On the same grid, graph $y = (3)^x + 1$ using a different color or mark.

x	y
-1	1 1/3
0	2
1	4
2	10
3	28

HA: $y = 1$

Explain how the graph shifted from the parent graph: up 1

Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

On the same grid, graph $y = (3)^x - 2$ using a different color or mark.

x	y
-1	-1 2/3
0	-1
1	1
2	7
3	25

HA: $y = -2$

Explain how the graph shifted from the parent graph: down 2

Domain: $(-\infty, \infty)$

Range: $(-2, \infty)$

SUMMARY of Vertical Translations

Adding "c" to the whole equation shifts the graph __up__ c units.

Subtracting "c" from the whole equation shifts the graph _down_ c units.

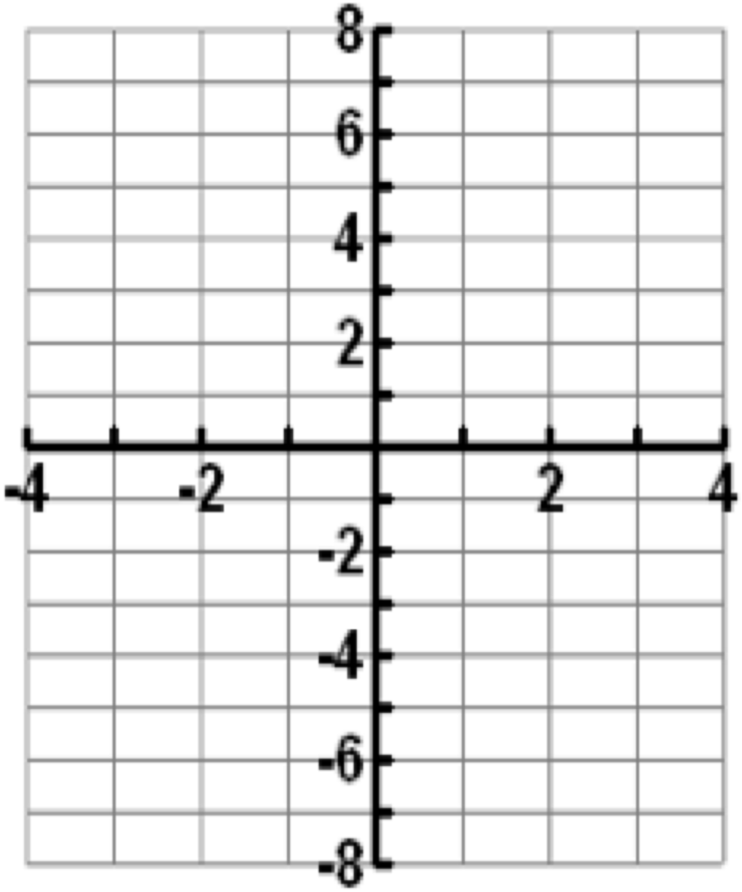
NOTE: The range is not y > 0 for these graphs!

Try It Continued… Chart on p. 28

Use a different color or mark to graph each of the following on the same grid. Be sure to LABEL your curves.

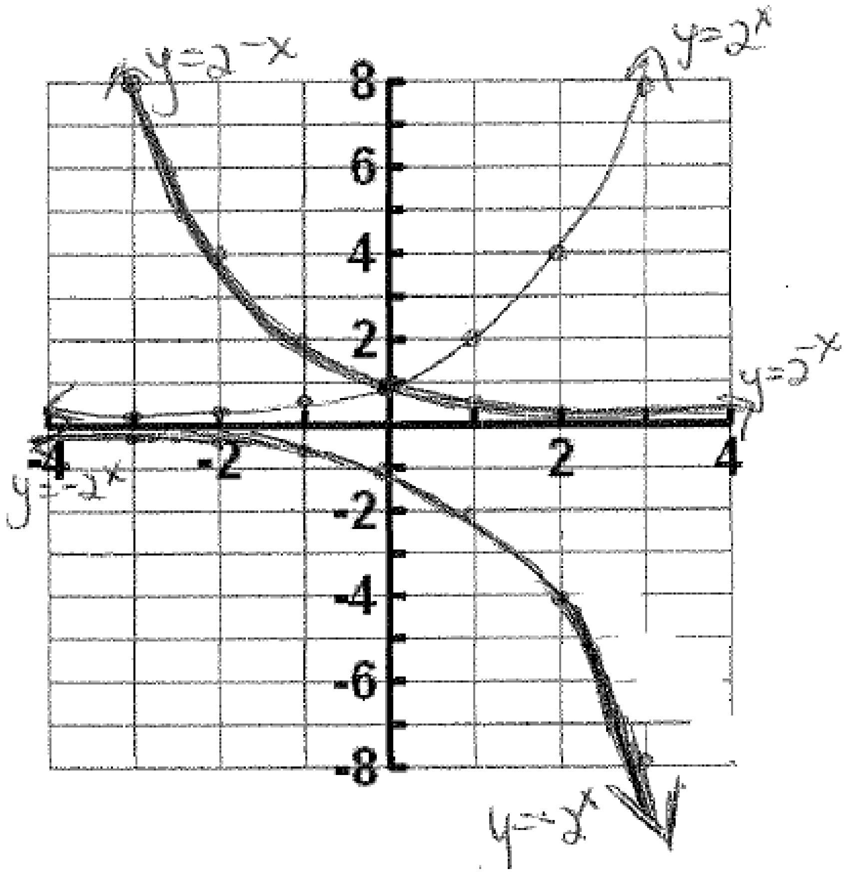
x	$F(x) = (2)^x$	$F(x) = (2)^{-x}$	$F(x) = -(2)^x$
-3			
-2			
-1			
0			
1			
2			
3			

HA			
D			
R			



Check Chart on p. 28

x	$y = (2)^x$	$y = (2)^{-x}$	$y = -(2)^x$
-3	1/8	8	-1/8
-2	¼	4	-¼
-1	½	2	-½
0	1	1	-1
1	2	½	-2
2	4	¼	-4
3	8	1/8	-8



HA	**y = 0**	**y = 0**	**y = 0**
R:	$(0, \infty)$	$(0, \infty)$	$(-\infty, 0)$

For all 3 graphs,
Domain: All real #'s

<u>SUMMARY of Reflections</u>: p. 29

<u>**Negative**</u> on x causes the graph to reflect in the

<u>**opposite**</u> direction over the <u>**Y**</u> axis.

<u>**Negative**</u> on the front of the equation causes the graph to

reflect in the <u>**opposite**</u> direction over the <u>**X**</u> axis.

Sum It All Up!!

SUMMARY of Horizontal Translations

Adding "c" in the exponent shifts the graph to the **LEFT** c units.

Subtracting "c" in the exponent shifts the graph to the **RIGHT** c units.

SUMMARY of Vertical Translations

Adding "c" to the whole equation shifts the graph **UP** c units.

Subtracting "c" to the whole equation shifts the graph **DOWN** c units.

SUMMARY of Reflections:

Negative on ' x ' causes the graph to reflect in the **X-value** direction over the **Y-** axis.

Negative on the front of the equation causes the graph to reflect in the **Y-value** direction over the **X -axis**.

Inverse Notes

In mathematics, when the independent and dependent variables are reversed, we have what is called the **inverse of the function**.

The inverse of a function helps to get back to an original value of x.

Investigation: Inverses

Consider the following functions:

f(x) = 6 + 3x

2) f(x) = $\frac{1}{3}(x - 6)$

Part 1: Graphs of Inverse Functions

For each of the functions above, follow steps 1 – 4.

1. Make a table of 5 values and graph the function on a separate sheet of graph paper.
2. Make another table by switching the x and y values and graph the inverse on the same coordinate plane.
3. What do you notice about the two graphs?
4. What line are the inverses reflected over?

Inverse Notes

Part 2: Equations of Inverse Functions

We saw in part 1 of the investigation that functions 1 and 2 are inverse functions. We also know that we can find inverses of tables by switching the x and y values in a table. So the question we want to explore now is how to find the equation of an inverse function.

For each of the functions above, follow steps 5 – 6.
5. Take the function and switch the x and y values.
6. Then solve for y.

The equation for the function 1's inverse should be function 2 and the inverse for function 2 should be the equation for function 1.

This process will work for any function which has an inverse. So, let's try some different types in the problems below.

Let's try a few algebraically together.

Find the inverse of the functions below.

1. $y = 2x + 5$

2. $y = \frac{1}{3}x - 4$

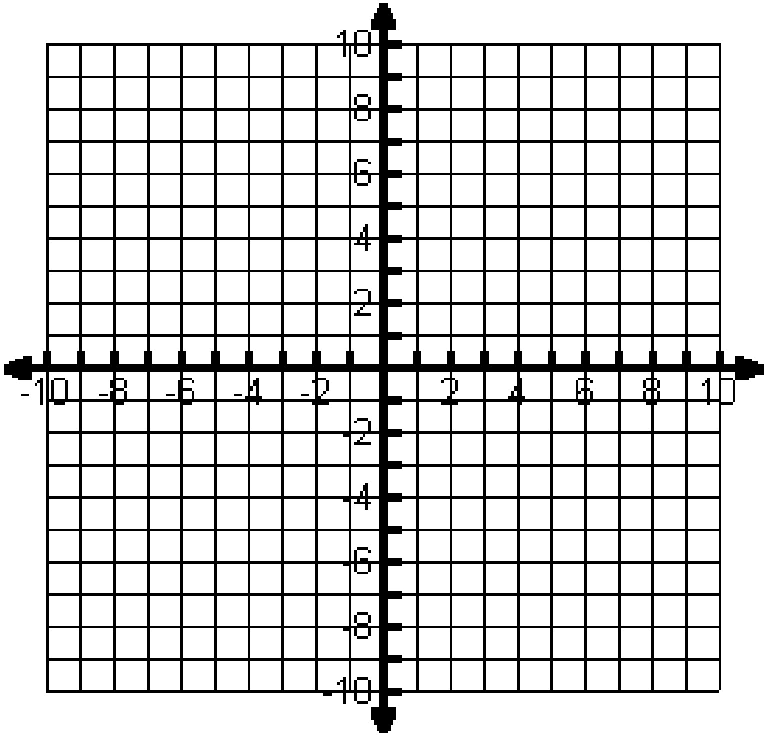
$$y = \frac{1}{2}x - \frac{5}{2}$$

$$y = 3x - 12$$

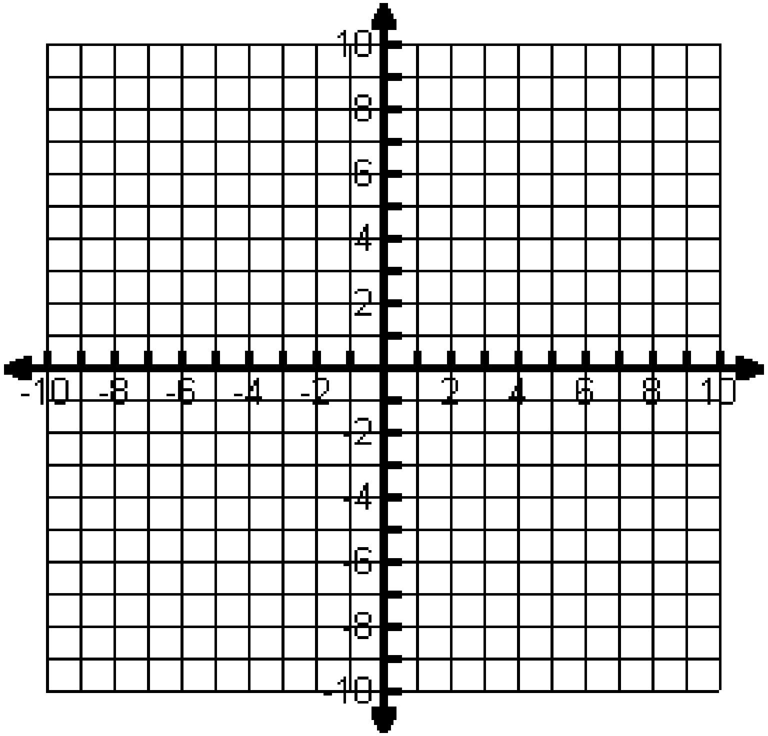
Now Try p. 32

Find the inverses of the functions below. Graph the function and its inverse below.

1) $f(x) = x^3$



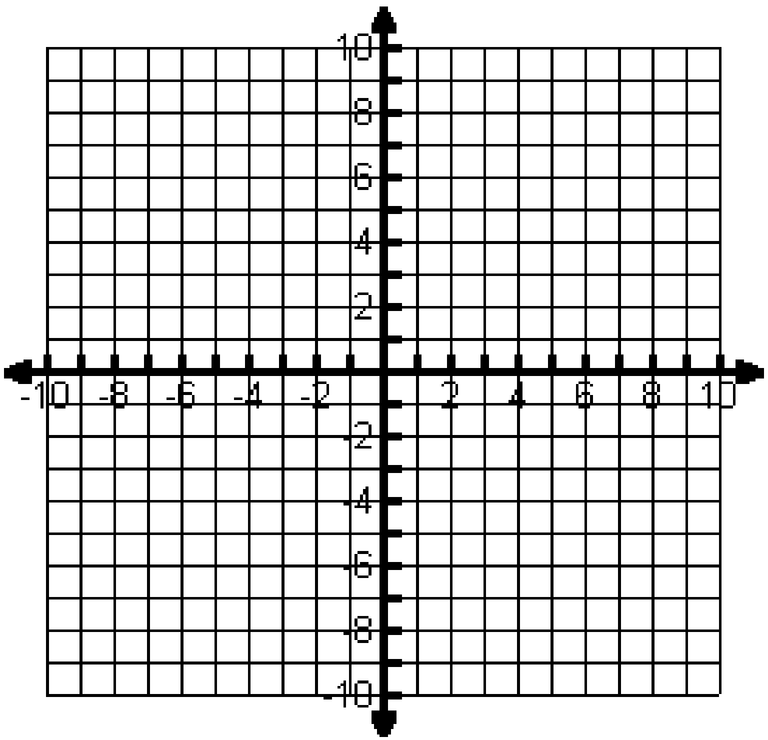
2) $y = -3x + 4$



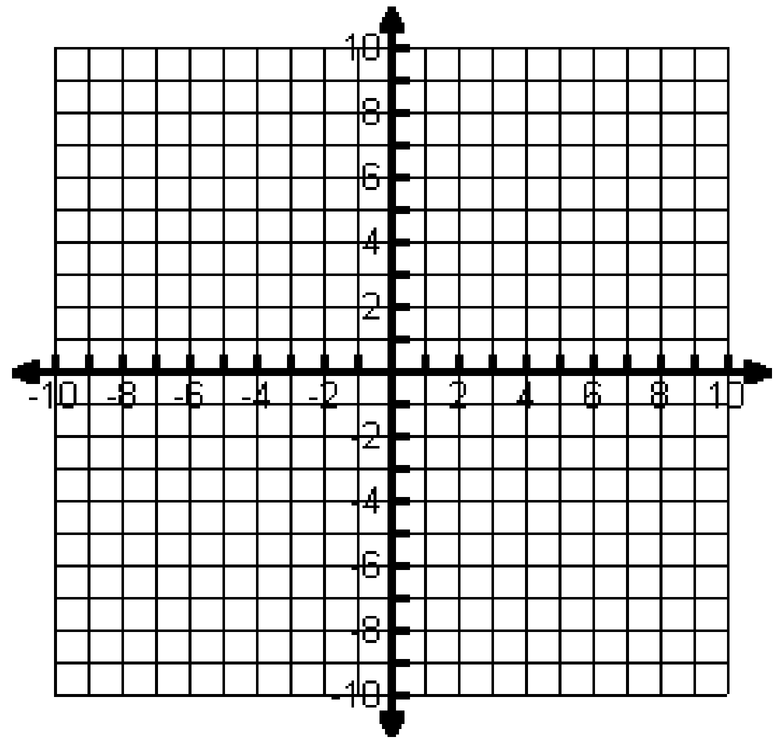
Now Try p. 32

Find the inverses of the functions below. Graph the function and its inverse below.

3) $f(x) = \sqrt{x-5}$

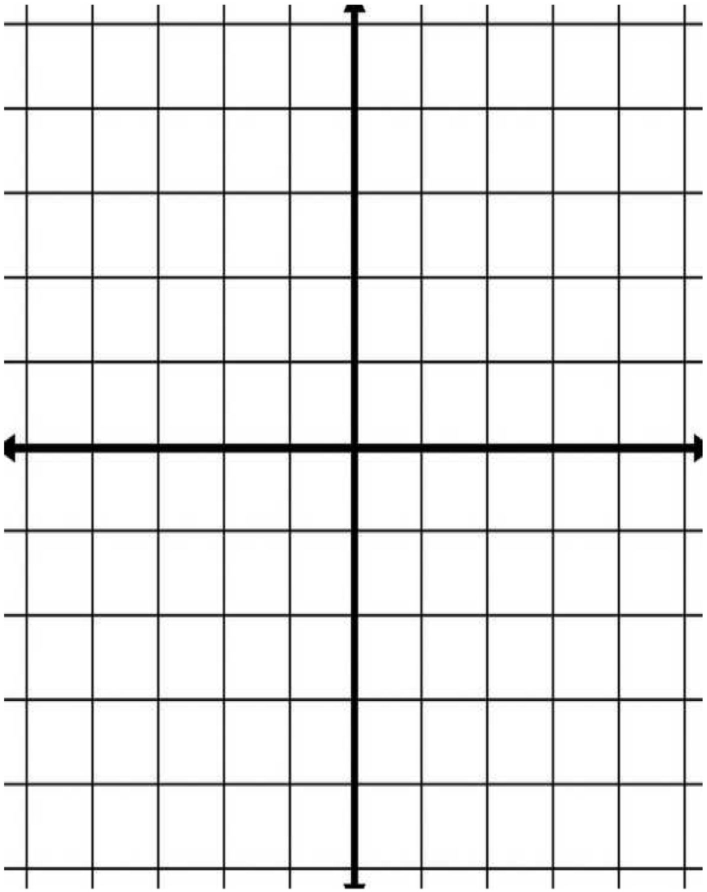


4) $y = 2^x$



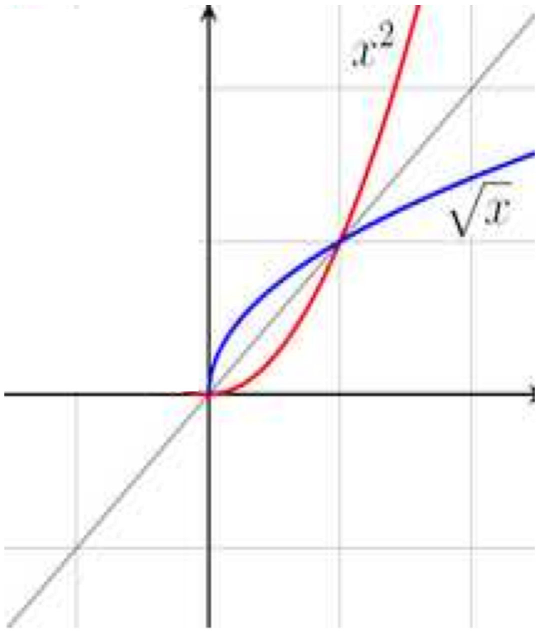
Now You Try!

Graph $y = x^2 + 3$ and find the inverse by interchanging the x and y values of several ordered pairs. Is the inverse a function? Check by graphing both $y = x^2 + 3$ and the inverse on the graph on the right.



Inverse Notes

Let's look at a simpler function $y = x^2$. If you only consider the positive values of x in the original function, the graph is half of a parabola. When you reflect the "half" over the $y = x$ line, it will pass the vertical line test as shown on the right.



Algebraically, let's switch the x and y values and solve for y.

Function: $y = x^2$, where $x \geq 0$

Switch the x and y values:

$$x = y^2$$

Solve for y:

$$\sqrt{x} = y$$

As shown, both graphs pass the vertical line tests and are inverse functions of each other. The above example means that if you restrict the domains of some functions, then you will be able to find inverse functions of them.

Additional Practice!

Write an exponential function for a graph that includes the points (4, 36) and (6, 15). Show your work algebraically using the point ratio formula. Round your "b" value to 4 decimal places.

Point-Ratio Form

Where (x, y) and (x_1, y_1) are ordered pairs that can be used to find the value of b, the common ratio.

$$\boxed{y = y_1 \bullet b^{x-x_1}}$$

Write an exponential function for a graph that includes the points (4, 36) and (6, 15).

<u>To find the value of b</u>

Step 1: Identify two ordered pairs. $(4, 36), \ (6, 15)$

Labeling the points can help -> x y x_1 y_1

Step 2: Substitute the values into point-ratio form. $36 = (15)b^{(4-6)}$

Step 3: Simplify the exponents $36 = (15)b^{(-2)}$

Step 4: Divide to get the power alone. $\frac{36}{15} = b^{-2}$

Step 5: Use the properties of exponents to isolate b.

$$b = \left(\frac{36}{15}\right)^{\left(-\frac{1}{2}\right)} \approx .6455$$

Round b to 4 places for the formula!!

Here's what we have so far! $\boxed{y = a \bullet (.6455)^x}$

Now, we need to find the initial value, a, to finish the formula!!

Next slide ->

Point-Ratio Form

Where (x, y) and (x_1, y_1) are ordered pairs that can be used to find the value of b, the common ratio.

$$y = y_1 \bullet b^{x-x_1}$$

Write an exponential function for a graph that includes the points (4, 36) and (6, 15).

Here's what we have so far!

$$\boxed{y = a \bullet (.6455)^x}$$

To find the initial value, a

Step 1: Identify one ordered pair and the b value.

$$(4, 36),\ \ b \approx .6455$$

Step 2: Substitute the values into exponential form.

$$36 = a(.6455)^4$$

Step 3: Simplify to solve for a.

$$\frac{36}{(.6455)^4} = \frac{a\cancel{(.6455)^4}}{\cancel{(.6455)^4}}$$

Remember, use the ANS button on the calculator!!

$$207.36 = a$$

Final Point Ratio Formula! ☺

$$\boxed{y = 207.36 \bullet (.6455)^x}$$

An application of point-ratio form: finding the monthly rate of depreciation.

Read the following question…then we'll discuss it!

The value, V, of a car can be modeled by the function $V(t) = 13{,}000(0.82)^t$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?

A 1.5%

B 1.6%

C 9.2%

D 18.0%

Steps

1) Use the info in the problem to get 2 coordinate pairs.
   (0, 13000) use coefficient = initial value
   (1, 10660) use equation $13000(0.82)^1$ to find the value after y year

2) Since they want a monthly rate, convert the ordered pairs to months. (0, 13000) (12, 10660) 12 months = 1 year

3) Use point-ratio form to find the b value! ☺

$$(0,13000) \quad (12, 10660) \qquad 10660 = 13000b^{(12-0)} \qquad 0.82 = b^{12}$$

$$b = (.82)^{\frac{1}{12}} \qquad b \approx 0.9836 \qquad r = 1 - 0.9836 = 0.0164$$

You Try!

The value of a truck can be modeled by the function $V(t) = 16{,}000(0.76)^t$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?

2.3% per month

Tonight's Homework:

Packet Pages 15-16