Day 8: Point Ratio Form

Warm Up: Exponential Regression

When handed to you at the drive-thru window, a cup of coffee was 200°F. Some data has been collected about how the coffee cools down on your drive to school.

Time (minutes)	Temperature (°F)
0	200
1	190
2	180.5
3	171.48
4	162.9
5	154.76
6	147.02

1. Would you say that the data has a positive, negative, or no correlation? Explain.

negative correlation (decreasing over time)

2. Without using your calculator, would you say that the data appears to be more linear or exponential? Explain.

$r = \frac{190}{200} = \frac{180.5}{190} = \frac{171.48}{180.5} = .95$

Exponential because values have a common ratio (not common difference)

3. Explain the steps you take to enter the data in the calculator and find a regression equation. Consult a neighbor if you need some help.

① Stat Edit Enter x's in L1 and y's in L2 ② Stat Calc ExpReg

4. Calculate the linear regression and use it to predict how cool the coffee will be in 15 minutes.

$\boxed{y = -8.82x + 198.85}$ Do Y1(15) or $y = -8.82(15) + 198.85$ $\boxed{66.51°F}$

5. Calculate the exponential regression and use it to predict how cool the coffee will be in 15 minutes.

$\boxed{y = 200.00(.95)^x}$ Do Y1(15) or $y = 200.00(.95)^{15}$ $\boxed{92.66°F}$

6. Which regression do you think provides a more accurate prediction? Explain your reasoning.

Exponential because coffee should still be pretty warm after just 15 minutes (not 66.51°, below room temp). Also

Point Ratio Form

When you ring a bell or clang a pair of cymbals, the sound seems loud at first, but the sound becomes quiet very quickly, indicating exponential decay. This can be measured in many different ways. The most famous, the decibel, was named after Alexander Graham Bell, inventor of the telephone. We can also use pressure to measure sound.

While conducting an experiment with a clock tower bell it was discovered that the sound from that bell had an intensity of 40 lb/in^2 four seconds after it rang and 4.7 lb/in^2 seven seconds after it rang.

1. Using this information, what would you consider to be the independent and dependent values in the experiment?

independent: time (the one you can manipulate)
dependent: sound intensity (the one you measure)

2. What two points would be a part of the exponential curve of this data?

(4, 40) (7, 4.7)

3. What was the initial intensity of the sound?

695.04 lb/in^2

4. Find an equation that would fit this exponential curve.

$\boxed{y = 695.04(.4898)^x}$

Find with Exponential Regression OR Point Ratio Form

You can find this equation using exponential regression on your graphing calculator, but what if you don't have a graphing calculator?

You may recall learning how to write equations for linear information using *point-slope form*. This is a little bit different, obviously, because we're working with an exponential curve, but when we use *point-ratio form*, you should see some similarities. Point-Slope $y - y_1 = m(x - x_1)$ for linear equations

Point-Ratio Form

$$y = y_1 \cdot b^{x-x_1}$$ for exponential equations

Where (x, y) and (x_1, y_1) are ordered pairs that can be used to find the value of b, the common ratio.

OR

Where (x_1, y_1) is an ordered pair and the common ratio, b, can be used to write an explicit equation for the scenario or identify other possible values for the scenario.

Let's look back at the Bell problem to see how to solve for an exponential formula by hand using the following steps!

To find the value of b

Step 1: Identify two ordered pairs.

$(\underset{x_1}{4}, \underset{y_1}{40})$ $(\underset{x}{7}, \underset{y}{4.7})$

Step 2: Substitute the values into Point-Ratio Form

$y = y_1 \cdot b^{x-x_1}$

$4.7 = 40 \cdot b^{7-4}$

Step 3: Simplify the exponents.

$4.7 = 40 \cdot b^3$

Step 4: Divide to get the power alone.

$\frac{4.7}{40} = b^3$

Step 5: Use the properties of exponents to isolate b.

$\left(\frac{4.7}{40}\right)^{1/3} = (b^3)^{1/3}$ $.4898 = b$

To find the initial value, a

Step 1: Identify one ordered pair and the b value.

$y = a(b)^x$ $(4, 40)$ $b = .4898$

Step 2: Substitute the values into exponential form.

$\frac{40}{((.4898)^4)} = \frac{a(.4898)^4}{((.4898)^4)}$

Step 3: Simplify to solve for a.

$a = \frac{40}{((.4898)^4)}$ $a = 695.04$

$\boxed{y = 695.04(.4898)^x}$

The above steps can be used to find any value of x or y if you have one ordered pair and the b value.

Guided Practice: Point – Ratio Form

1) The intensity of light also decays exponentially with each additional colored gel that is added over a spotlight. With three gels over the light, the intensity of the light was 900 watts per square centimeter. After two more gels were added, the intensity dropped to 600 watts per square centimeter.

a. Write an equation for the situation. Show your work.

x = # colored gels

y = light intensity

$(\underset{x_1}{3}, \underset{y_1}{900})$ $(\underset{x}{5}, \underset{y}{600})$

3+2 "two more"

$\boxed{y = 1653.41(.8165)^x}$

① $y = y_1 \cdot b^{x-x_1}$

$600 = 900 \cdot b^{5-3}$

$\frac{600}{900} = b^2$

$\sqrt{600/900} = b = .8165$

② $y = a(b)^x$

$y = a(.8165)^x$

$\frac{600 = a(.8165)^5}{((.8165)^5)}$

$1653.41 = a$

b. Use your equation to determine the intensity of the light with 7 colored gels over it.

x = 7

$y = 1653.41(.8165)^7$

$y = 1653.41(\underset{STO}{X})^7$

$\boxed{400 \text{ w/cm}^2}$

* use $\boxed{STO}$ and $\boxed{X}$ keys on calculator to store the base "b" in the calc.

2) Using the points (2, 12.6) and (5, 42.525).

a. Find an exponential function that contains the points. Show your work.

① $y = y_1 \cdot b^{x-x_1}$

$12.6 = 42.525 \cdot b^{2-5}$

$\frac{12.6}{42.525} = \frac{42.525 \cdot b^{-3}}{42.525}$

$\left(\frac{12.6}{42.525}\right)^{-1/3} = \left(b^{-3}\right)^{-1/3}$

$1.5 = b$

$\boxed{y = 5.6(1.5)^x}$

② $y = a(b)^x$

$y = a(1.5)^x$

$\frac{12.6}{((1.5)^2)} = \frac{a(1.5)^2}{((1.5)^2)}$

$5.6 = a$

b. Rewrite the function so that the exponent of the function is simply x.

$\boxed{y = 5.6(1.5)^x}$

3) When the brown tree snake was introduced to Guam during World War II by the US military it devastated the local ecosystem when its population grew exponentially. If 1 snake was accidentally brought to Guam in 1945 and it was estimated that there were 625 snakes in 1949, write an equation for the situation and use it to predict how many snakes there were in 1955. In your equation, let x = number of years after 1945. Show your work.

(0, 1) 1945 1 snake (4, 625) 1949 snakes

① $y = y_1 \cdot b^{x-x_1}$

$1 = 625 \cdot b^{0-4}$

$\left(\frac{1}{625}\right)^{-1/4} = \left(b^{-4}\right)^{-1/4}$

$\left(\frac{1}{625}\right)^{-1/4} = b$

$5 = b$

② $y = a(5)^x$

$y = 1(5)^x$ 1 snake in 1945

$\boxed{y = 5^x}$

③ 1955 − 1945 = 10 yrs

x = 10

$y = 5^{10}$

$\boxed{9765625}$ snakes in 1955

4) The water hyacinth that was introduced to North Carolina from Brazil ended up clogging our waterways and altering the chemistry of the water. We're not certain exactly when the water hyacinth was introduced, but there were 76.9 square miles of water hyacinths in 1984. Ten years later, there were 80793.6 square miles of watch hyacinth. Using this information, calculate when there was less than 0.1 square miles of this invasive plant in our waterways.

Let x = yrs since 1984

(0, 76.9) (10, 80793.6)

① $y = y_1 \cdot b^{x-x_1}$

$\frac{80793.6}{76.9} = \frac{76.9(b)^{10-0}}{76.9}$

$\left(\frac{80793.6}{76.9}\right)^{1/10} = \left(b^{10}\right)^{1/10}$

$2.0051 = b$

$\boxed{\text{STO}} \rightarrow \boxed{X}$

② $y = a(2.0051)^x$

$\boxed{y = 76.9(2.0051)^x}$ because 76.9 when first measured

③ $\underbrace{0.1}_{y_1} = \underbrace{76.9(2.0051)^x}_{y_2}$ + intersect

x = −9.55

1984 − 9.55 = $\boxed{1974}$.45

5) You recently discovered that your great aunt Sue set up a bank account for you several years ago. You've never received bank statements for it until recently. The first month you received a statement for the account, you had $1234.56 in the account. One month later, the account had $1296.29. What is the approximate percentage of monthly interest that this account is earning each month?

a. What is the approximate percentage of monthly interest that this account is earning each month?

(0, 1234.56) (1, 1296.29)

$y = y_1 \cdot b^{x-x_1}$

$\frac{1234.56}{1296.29} = \frac{1296.29 \cdot b^{0-1}}{1296.29}$

$\left(\frac{1234.56}{1296.29}\right)^{-1} = \left(b^{-1}\right)^{-1}$

$1.0500 = b$

$1.0500 = 1 + r$ because % growth

$-1 \quad -1$

$.0500 = r$ $\boxed{5\%}$ growth per month

b. In what month will your account have doubled?

$y = 1234.56(1.05)^x$

$(1234.56)(2) = 1234.56(1.05)^x$

$\underbrace{2469.12}_{y_1} = \underbrace{1234.56(1.05)^x}_{y_2}$ + intersect → x = 14.21

in the $\boxed{\text{14th month}}$

6) When the wolf population of the Midwest was first counted in 1980, there were 100 wolves. In 1993, that population had grown to 3100 wolves! Assuming the growth of this population is exponential, complete each of the following.

a. Write an explicit equation to model the data. Show your work!

Let x = yrs after 1980 (0, 100) (13, 3100)

x y x_1 y_1

$a = 100$ starting amount in 1980

① $y = y_1 \cdot b^{x - x_1}$

$100 = 3100(b)^{0-13}$

$\frac{100}{3100} = b^{-13}$

$\left(\frac{100}{3100}\right)^{-1/13} = \left(b^{-13}\right)^{-1/13}$

$1.3023 = b$

$\boxed{y = 100(1.3023)^x}$

b. Write a recursive (NOW-NEXT) equation for the data.

next = now · 1.3023

start = 100

c. Use one of your equations to complete the table below.

Time Since 1980 (in years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Estimated Wolf Population	100	130.2	169.6	220.9	287.7	374.6	487.9	635.4	827.5	1077.7	1403.5	1827.8	2380.4	3,100

d. Identify the theoretical and practical domain of this situation. Explain how you decided upon your answers.

theoretical domain: all real numbers $(-\infty, \infty)$

→ according to the equation, all x-values can be used

practical domain: $y > 0$ or $(0, \infty)$

because time cannot really be negative

7) A 2008 Ford Focus cost $15452 when purchased. In 2013 it is worth $8983 if it's in excellent condition.

a. Assuming the value of the Focus depreciates by the same percentage each year, what is that percentage?

(0, 15452) (5, 8983) Let x be years after 2008

2008 value 2013 value

$y = y_1(b)^{x - x_1}$

$15452 = 8983(b)^{0-5}$

$\frac{15452}{8983} = \frac{8983}{8983}(b)^{0-5}$

$\left(\frac{15452}{8983}\right)^{-1/5} = \left(b^{-5}\right)^{-\frac{1}{5}}$

$(1.7201)^{-1/5} = b = .8972$

$b = 1 - r$ because depreciate is decay

$.8972 = 1 - r$

$-1 \quad -1$

$-.1028 = -r$

$\boxed{10.28\%}$

b. Write an explicit function for the value of the Ford Focus.

$\boxed{y = 15452(.8972)^x}$

↑

starting value in 2008

c. If the owner of the car plans on selling it in 2018, how much should she expect to get for it?

-2008

x = 10 years

$y = 15452(.8972)^{10} = \boxed{\$5222.49}$

An application of point-ratio form: finding the monthly rate of depreciation.

Together: The value, V, of a car can be modeled by the function $V(t) = 13{,}000(0.82)^t$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?

A. 1.5% B. 1.6% C. 9.2% D. 18.0%

* we can not just do $b = .82 = 1 - r$ because that gives yearly rate

we must find the monthly rate

(0, 13000) (1, 10660) from "a" value in yr $= 13000(.82)^1$ ⇒ (0, 13000) (12, 10660)

$y = y_1(b)^{x-x_1}$ $13000 = 10660(b)^{0-12}$ $\left(\frac{13000}{10660}\right)^{-1/12} = (b^{-12})^{-1/12}$ $b = .9836$ $1 - r = .9836$ $r = .0164$ $\boxed{1.64\%}$

You Try!

The value of a truck can be modeled by the function $V(t) = 16{,}000(0.76)^t$, where t is the number of years since the car was purchased. To the nearest tenth of a percent, what is the monthly rate of depreciation?

(0, 16000) from "a" value on equation

(1, 12160) from t = 1 year $y = 16000(.76)^1$

⇒ for monthly rate, let t = 12 months for 1 year (12, 12160) (0, 16000)

$y = y_1 \cdot b^{x-x_1}$ $12160 = 16000 \cdot b^{12-0}$ $\frac{12160}{16000} = \frac{16000 \cdot b^{12}}{16000}$ $(.76)^{1/12} = (b^{12})^{1/12}$ $.9774 = b$

for depreciation (decay) $b = 1 - r$ $.9774 = 1 - r$ $r = .0226$

monthly rate is $\boxed{r = 2.3\%}$

Unit 3 Practice

1) Simplify $5\sqrt[3]{72x^5y^7} \bullet 2\sqrt[3]{343x^4y^3}$

$10\sqrt[3]{72 \cdot 343x^9y^{10}}$

$10\sqrt[3]{8 \cdot 9 \cdot 343\, x^3x^3x^3\, y^3y^3y^3y}$

$10x^3y^3\sqrt[3]{8 \cdot 9 \cdot 343y}$

$10 \cdot 2 \cdot 7x^3y^3\sqrt[3]{9y} = \boxed{140x^3y^3\sqrt[3]{9y}}$

Isolate before Inverse

2) Solve $\sqrt{2x-4} + x = 6$

$(\sqrt{2x-4})^2 = (6-x)^2$

$2x - 4 = (6-x)(6-x)$

$2x - 4 = x^2 - 12x + 36$

$0 = x^2 - 14x + 40$

$0 = (x-10)(x-4)$

$x = \cancel{10}, 4$ — 10 doesn't work

$\sqrt{2 \cdot 10 - 4} + 10 \neq 6$ $\sqrt{2 \cdot 4 - 4} + 4 = 6$ $\boxed{x = 4}$

* Isolate before Inverse

3) Solve $5(2x+4)^{4/3} = 80$

$\frac{5(2x+4)^{4/3}}{5} = \frac{80}{5}$

$((2x+4)^{4/3})^{3/4} = (16)^{3/4}$

$2x + 4 = 8$

$2x = 4$ $\boxed{x = 2}$

$5(2 \cdot 2 + 4)^{4/3} = 80$

4) Sr-85 is used in bone scans. It has a half-life of 64.9 hours. Write the exponential function for a 12 mg sample. Find the amount remaining after 72 hours. Show work algebraically.

$y = a(1/2)^{t/\text{half life time}}$

$y = 12(1/2)^{t/64.9}$ → $t = 72$ $y = 12(1/2)^{72/64.9}$ $y = \boxed{5.56 \text{ mg}}$ remaining after 72 hours

5) Write exponential function given (2, 18.25) and (5, 60.75). Show your work algebraically.

$y = y_1 \cdot b^{x-x_1}$

$\frac{60.75}{18.25} = \frac{18.25 \cdot b^{2-5}}{18.25}$ $(3.329)^{-1/3} = (b^{-3})^{-1/3}$ $.6697 = b$

$y = a(b)^x$ $\frac{18.25}{(.6697)^2} = \frac{a(.6697)^2}{(.6697)^2}$ $a = \frac{18.25}{(.6697)^2} = 40.69$ $\boxed{y = 40.69(.6697)^x}$

6) A plant 4 feet tall grows 3 percent each year. How tall will the tree be at the end of 12 years? Round your answer to the hundredths place.

$a = 4$ = initial amount $b = 1 + r = 1 + .03$ $b = 1.03$ $x = 12$

$y = a(b)^x$ $y = 4(1.03)^x$ $y = 4(1.03)^{12}$ $\boxed{5.70 \text{ ft}}$

7) A BMX bike purchased in 1990 and valued at \$6000 depreciates 7% each year.

$b = 1 - r = 1 - .07 = .93 = b$

a) Write a next-now formula for the situation

$\boxed{\text{next} = \text{now} \cdot (.93) \text{ and start} = 6000}$

b) Write an explicit formula for the situation

$\boxed{y = 6000(.93)^x}$

c) How much will the BMX bike be worth in 2006?

find y $-1990 \rightarrow x = 16$ $y = 6000(.93)^{16}$ $\boxed{\$1878.79}$

d) In what year will the BMX bike be worth \$3000?

find x $y = 3000$ $3000 = 6000(.93)^x$

Do y_1 + y_2 in calculator + 2nd Trace Intersect Enter Enter Enter

$x = 9.55$ + 1990 1999.55 ⇐ $\boxed{1999}$