Unit 3 Day 7

Exponential Growth & Decay

Warm Up - Zombies

SCENARIO 1

A pack of zombies is growing exponentially! After 1 hour, the original zombie infected 5 people, and those 5 zombies went on to infect 5 people, etc. After a zombie bite, it takes 1 hour to infect others. How many newly infected zombies will be created after 4 hours?
If possible, draw a diagram, create a table, a graph, and an equation.

SCENARIO 2

During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc. After a zombie bite, it takes an hour to infect others. Determine the number of newly infected zombies at 6 am this morning. If possible, draw a diagram, create a table, a graph, and an equation.

SCENARIO 3

At 9:00 am, the official count of the zombie infestation was 16384. Every hour the number of zombies quadruples. Around what time did the first zombie roll into town?

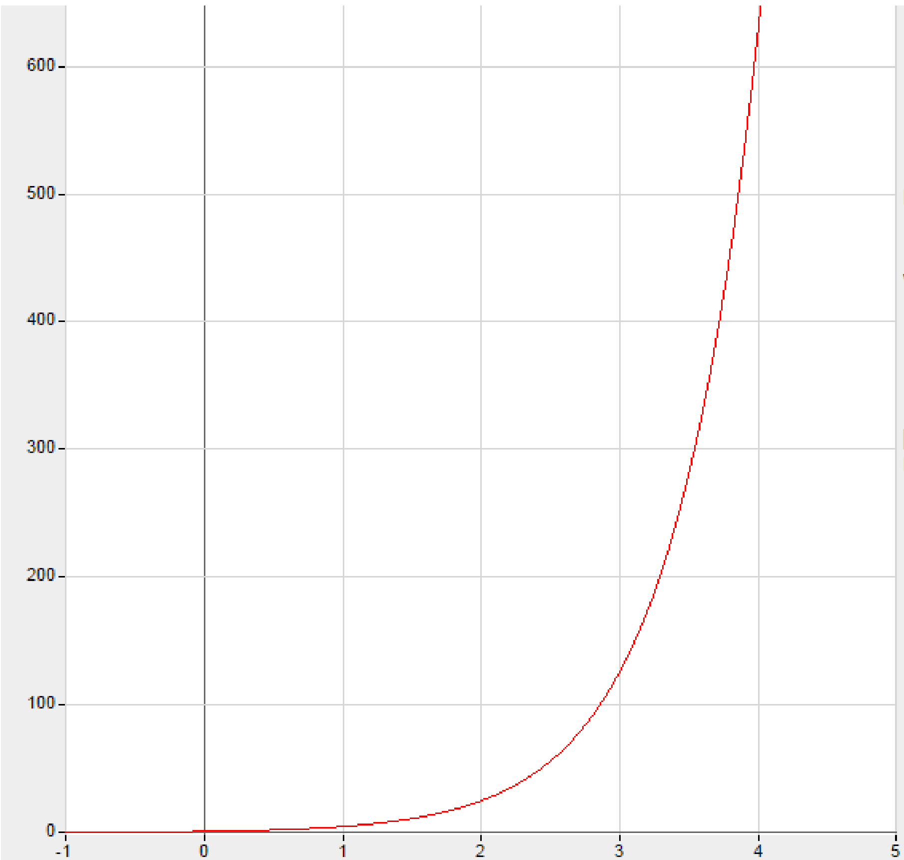
Warm-up: ZOMBIES!

SCENARIO 1

A pack of zombies is growing exponentially! After 1 hour, the original zombie infected 5 people, and those 5 zombies went on to infect 5 people, etc. After a zombie bite, it takes 1 hour to infect others. How many newly infected zombies will be created after 4 hours?

If possible, draw a diagram, create a table, a graph, and an equation.

New Zombies



Time (hours)

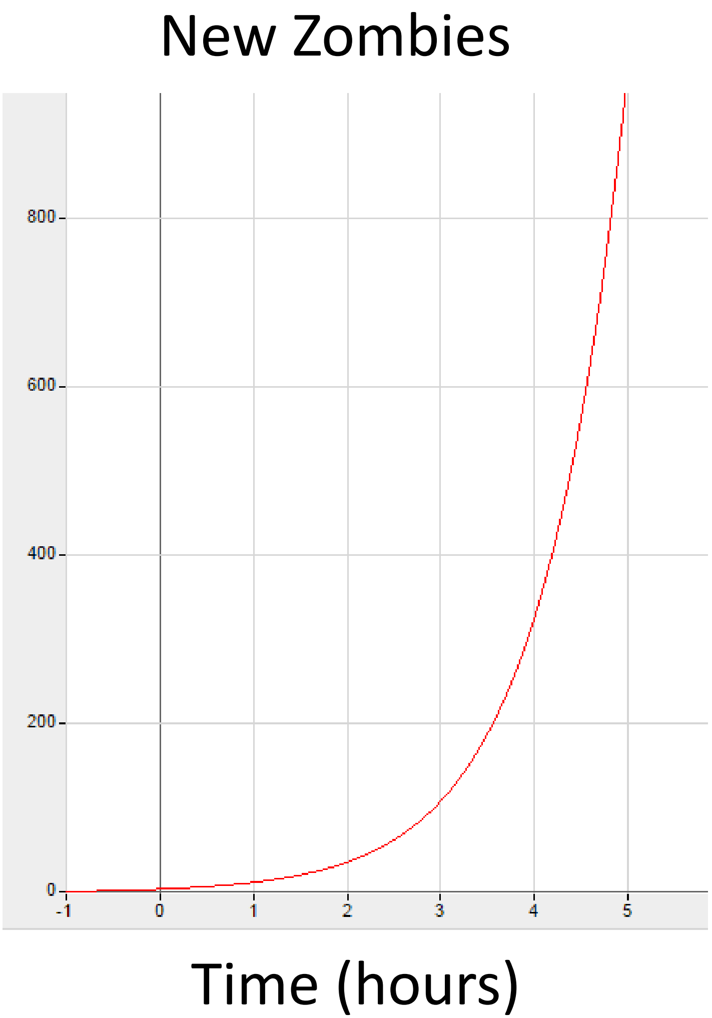
Time	New Zombies
0	1
1	5
2	25
3	125
4	625

$$y = 5^x$$

SCENARIO 2

During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc. After a zombie bite, it takes an hour to infect others. Determine the number of newly infected zombies at 6 am this morning.

If possible, draw a diagram, create a table, a graph, and an equation.



Time	New Zombies
0	4
1	12
2	36
3	108
4	324
5	972
6	2916

$$y = 4(3)^x$$

SCENARIO 3

At 9:00 am, the official count of the zombie infestation was 16384. Every hour the number of zombies quadruples. Around what time did the first zombie roll into town?

$$16384 = 4^x$$

$$x = 7$$

2 AM

Later in this unit, we'll learn how to solve these problems algebraically. For now, one way to solve is by using the calculator! ☺

Do y1 = 16384, y2 = $(4)^x$ then find intersection in calc.

Homework Answers – Packet p. 10-11 Odds & #4, 12

p. 10

1. $x = 9$

3. $x = 4$

4. No solution

5. $x = 4$

7. $x = 2$

9. $x = 4$

11. $x = 7/2$

p. 11

12. $x = 3$

13. $x = 8$

15.
    a) $h \approx 9.8$ ft.

    b) $h \approx 10.8$ ft.

    c) $h \approx 34722.2$ ft.

Homework:
Packet p. 12 & 13

Print Unit 3 Packet and Notes Days 7-12

Notes Day 7

Exponential Growth and Decay

What if???

- You find a bank that will pay you 3% interest annually, so you consider moving your account, but, your current bank wants to keep you. They offer to compound semiannually!

- You get 2.5% interest and 1.25% interest at the end of June and 1% interest at the end of December.

- Which bank is giving you a better deal?

We won't actually solve this problem…but wouldn't it be nice to know how to…in case you ran into a similar situation? Let's learn some about these types of problems today! ☺

Review of Vocabulary

Initial Value: The 1st term in a sequence or the dependent value associated with an independent value of 0.

Looking at Zombie Warm-up #2…
During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc.
Initial Value = 4

Common Ratio: The ratio of one term in a sequence and the previous term.

Looking at Zombie Warm-up #2… Common Ratio = 3

Common Difference: The difference between one term in a sequence and the previous term.

Looking at the sequence 4, 9, 14, 19, … Common Difference = 5

Review of Vocabulary

Recursive Function: a function that can be used to find any term in a sequence if you have the previous term

(i.e. NOW-NEXT)

Looking at Zombie Warm-up #2…

During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc.

Next = Now * 3   Start = 4

Explicit Function: a function that can be used to find any term in the sequence without having to know the previous term. (i.e. y = or f(x) =)

Looking at Zombie Warm-up #2…

$$y = 4(3)^x$$

Review of Vocabulary

Domain: the set of all input values for a function

Looking at Zombie Warm-up #2… $y = 4(3)^x$

the domain is all real numbers

Theoretical Domain: the set of all input values for a function without consideration for context

Looking at Zombie Warm-up #2… $y = 4(3)^x$

the theoretical domain is all real numbers

Practical Domain: the set of all input values for a function that are reasonable within context

Looking at Zombie Warm-up #2… $y = 4(3)^x$

the practical domain is $x \geq 0$ because x is time in hours after 6 AM and it doesn't make sense to think about negative time

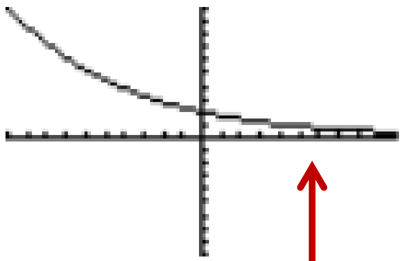
We'll discuss these terms more later today!!

Comparing Exponential Growth and Decay

Exponential Decay Model:

$y = C(1 - r)^t$

$0 < 1 - r < 1$

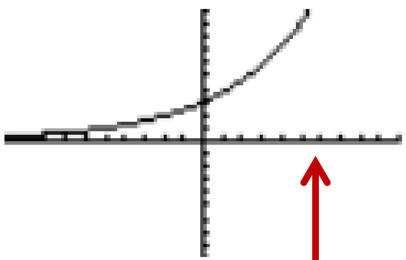


Note: Function is decreasing over time

Exponential Growth Model

$y = C(1 + r)^t$

$1 + r > 1$



Note: Function is increasing over time

Exponential Decay

$$y = a(b)^x$$

When $a > 0$ and b is between 0 and 1, the graph will be decreasing (decaying).

$$y = 1\left(\frac{1}{2}\right)^x$$

So for this example, each time x is increased by 1, y decreases to half of its previous value.

Exponential Decay

$$y = a(b)^x$$

b = $\underline{\quad 1 - r \quad}$

* r is the common ratio

This is because you begin with 100% (1 when written as a decimal) and subtract the same percentage each time.

Exponential Growth

$$y = a(b)^x$$

When **a > 0** and ***b* is greater than 1**, the graph will be increasing (growing).

$$y = 1(2)^x$$

So for this example, each time x is increased by 1, y increases by a factor of 2.

Exponential Growth

$$y = a(b)^x$$

b = <u>1 + r</u>

* r is the common ratio

This is because you begin with 100% (1 when written as a decimal) and add the same percentage each time.

Examples

1. A house that costs $200,000 in 2010 will appreciate in value by 3% each year.

a. Write a function that models the cost of the house over time. Use x for years after 2010, and y for the value of the house, in dollars. $y = 200,000(1+.03)^x$

b. Find the value of the house in 2020.

$\$268,783.28$

c. In what year will the house value have doubled?

Year 2033

More vocab you must know!

*Appreciate means increasing rate

*Depreciate means decreasing rate

Examples

2. The most recent virus that is making people ill is a fast multiplying one. On the first day of the illness, only 2 virus "bugs" are present. Each day after, the amount of "bugs" triples.

a. Write a function that models the "bugs'" growth over time. Let x represent the number of days since the onset of the illness and y represent the amount of "bugs".

$$y = 2(3)^x$$

b. Find the amount of "bugs" present by the 5th day.

$$y = 2(3)^5 = 486 \text{ bugs}$$

Examples

3. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.

a. Write a function that models the amount of menthol in your body over time. Let x represent minutes since you took the cough drops and y be the amount of menthol, in mg, remaining in your body.

$$y = 100(1 - \tfrac{1}{2})^x$$

b. It is safe to take a new cough drop when the amount of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop?

About 4.3 minutes

More vocab you must know!

*Half Life = Amount of time for a substance to be cut in half

For this problem, the half life is 1 minute

Example!

Let's try a tougher problem you may run in to!

4. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every 5 minutes, the amount of menthol in your body is cut in half.

Remember… Half Life = Amount of time for a substance to be cut in half

a. Write a function that models the amount of menthol in your body over time. Let x represent minutes since you took the cough drops and y be the amount of menthol, in mg, remaining in your body. **$y = 100(1/2)^{t/5}$ you must do t/5 exponent so that the exponent will determine the number of half lives that have occurred**

b. It is safe to take a new cough drop when the amount of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop? **$5 = 100(1/2)^{t/5}$**

Do $y1 = 5$, $y2 = 100(1/2)^{t/5}$ then find intersection in calc.

In the problems above, we wrote **explicit functions** for the exponential data.

$$y = a(b)^x$$

BUT we also need to know how to write **recursive functions** for the exponential data. Make sure you include the starting value of your data **a** – this will always be where the independent variable is **0** .

Next = Now * b , Starting at **a**

(You must tell the values of b and a)

Let's write the prior examples using Recursive Formulas!

Next = **Now * b** , Starting at **a**

1. A house that costs $200,000 in 2010 will appreciate in value by 3% each year.

   Next = Now * 1.03, Start = 200,000

2. The most recent virus that is making people ill is a fast multiplying one. On the first day of the illness, only 2 virus "bugs" are present. Each day after, the amount of "bugs" triples.

   Next = Now * 3, Start = 2

3. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.

   Next = Now * 0.5, Start = 100

You Try #4-6

4. Tobias ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

a. Write an explicit function that models the gnats' grown over time. Use x for nights after you first see the gnats and y for number of gnats.

$$y = 2(4)^x$$

b. Tobias' mom said that he would be grounded if the gnats number more than 120. On what night will Tobias be in trouble, if he doesn't step in and solve his gnat problem?

$$120 = 2(4)^x, \ so \ x \approx 3$$

c. Write a recursive function to model the gnats' growth.

Next = Now * 4, Start = 2

You Try #5

5. JaCorren is 60 inches and going through a growth spurt. For the next year, his growth will increase by 1% each month. Let x represent number of months and y represent JaCorren's height in inches.

a. Write an explicit function that models JaCorren's growth spurt over the next year. $y = 60(1+.01)^x$

$$y = 60(1.01)^x$$

b. Find JaCorren's height at the end of the year.

$$y = 60(1.01)^{12} = 67.61\ in.$$

c. Write a recursive function to model JaCorren's growth.

Next = Now * 1.01, Start = 60

You Try #6

6. Ian's new Mercedes cost him $75,000 in 2014. From the moment he drives it off the lot, it will depreciate by 20% each year for the first five years.

a. Write an explicit function that models the car's depreciation. Use x for years after 2014 and y for the car's value, in dollars.

$$y = 75000(1 - .20)^x$$

$$y = 75000(.80)^x$$

b. What will the car's value be in 2019?

$$y = 75000(.80)^5 = \$24,576$$

c. Write a recursive function to model the car's depreciation.

Next = Now * .80, Start = 75,000

More on Theoretical and Practical Domain

When you write an equation for a situation and use it to make predictions, you assume that other people who use it will understand the situation as well as you do. That's not always the case when you take away the context, so we sometimes need to provide some additional information to accompany the equation.

The **domain** of a function is the set of all the possible input values that can be used when evaluating it. If you remove your functions from the context of this situation and simply look at the table and/or graph of the function, what numbers are part of the **theoretical domain** of the function?

Will this be the case with all exponential functions? Why or why not?

When you consider the context, however, not all of the numbers in the theoretical domain really make sense. We call the numbers in the theoretical domain that make sense in our situation the practical domain. If you look at the tables for the problems, we can find what numbers would be a part of the **practical domain**.

More on Theoretical and Practical Domain

Let's find the theoretical domain and practical domain of some of the practice problems you just did.

Together…

4. Tobias ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

You try…

5. JaCorren is 60 inches and going through a growth spurt. For the next year, his growth will increase by 1% each month. Let x represent number of months and y represent JaCorren's height in inches.

Homework:
Packet p. 12 & 13

Print Unit 3 Packet and Notes Days 7-12

Practice

Exponential Word Problems

Scavenger Hunt

OR

Additional Practice