Day 7: Exponential Growth and Decay

Warm-up: ZOMBIES!

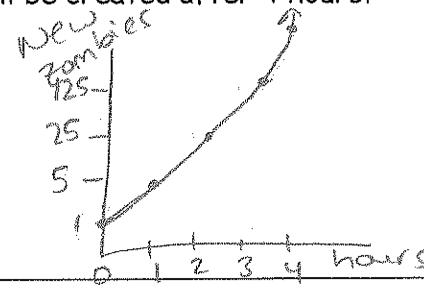
SCENARIO 1

A pack of zombies is growing exponentially! After 1 hour, the original zombie infected 5 people, and those 5 zombies went on to infect 5 people, etc. After a zombie bite, it takes 1 hour to infect others. How many newly infected zombies will be created after 4 hours?

If possible, draw a diagram, create a table, a graph, and an equation.

Time	New Zombies
0	1
1	5
2	25
3	125
4	625

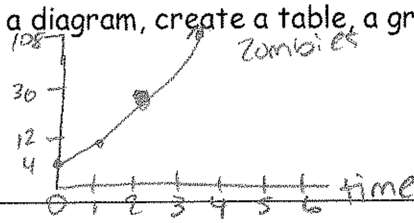
$\boxed{y = 5^x}$



SCENARIO 2

During this attack, a pack of 4 zombies walked into town last night around midnight. Each zombie infected 3 people total, and those 3 zombies went on to infect 3 people, etc. After a zombie bite, it takes an hour to infect others. Determine the number of newly infected zombies at 6 am this morning.

If possible, draw a diagram, create a table, a graph, and an equation.



$\boxed{y = 4(3)^x}$

Time	New Zombies
midnight → 0	4
1	12
2	36
3	108
4	324
5	972
6	2916

SCENARIO 3

At 9:00 am, the official count of the zombie infestation was 16384. Every hour the number of zombies quadruples. Around what time did the first zombie roll into town?

$16384 = 4^x \qquad x = 7$

7 hours after 9:00 am is $\boxed{2\text{ Am}}$

solve using table in calc! put 4^x in y_1 and find $= 16384$.

Notes Day 7: Exponential Growth and Decay

You find a bank that will pay you 3% interest annually, so you consider moving your account. Your current bank decides you're a good customer and offers you a special opportunity to compound your interest semiannually!!! (They make it sound like it's a really good deal, so you're curious). You don't play around with your money, so you ask what exactly that means. They explain that you'll still get 2.5% interest, but they'll give you 1.25% interest at the end of June and 1% interest at the end of December. You want to see if you make more money than you would if you just switched banks, so you do the calculations. Which bank is giving you a better deal? Explain your answer.

Review of Vocabulary

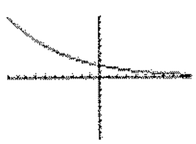
- Initial Value: The 1st term in a sequence or the dependent value associated with an independent value of 0.
  zombie Warm-up #2 → initial value = 4
- Common Ratio: The ratio of one term in a sequence and the previous term.
  zombie Warmup #2 → Common ratio = 3
- Common Difference: The difference between one term in a sequence and the previous term.
  Ex: 4, 9, 14, 19 → common difference = 5
- Recursive function: a function that can be used to find any term in a sequence if you have the previous term (i.e. NOW-NEXT) zombie #2
  $\boxed{\text{Next} = \text{Now} \cdot 3}$ start = 4
- Explicit function: a function that can be used to find any term in the sequence without having to know the previous term. (i.e. y = or f(x) =) zombie #2 $\boxed{y = 4(3)^x}$
- Domain: the set of all input values for a function zombie #2 → domain (x) is all real #'s
- Theoretical Domain: the set of all input values for a function without consideration for context same for this example
- Practical Domain: the set of all input values for a function that are reasonable within context
  Zombie #2 → domain $x \geq 0$ because x is hours after 6AM

Comparing Exponential Growth and Decay

Exponential Decay Model:

$y = C(1 - r)^t$

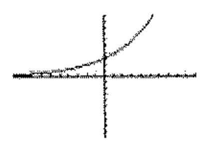
$0 < 1 - r < 1$



Exponential Growth Model

$y = C(1 + r)^t$

$1 + r > 1$



Exponential Decay

$y = a(b)^x$

When a > 0 and b is between 0 and 1, the graph will be decreasing (decaying)

$$y = 1\left(\frac{1}{2}\right)^x$$

So for this example, each time x is increased by 1, y decreases to half of its previous value.

Exponential Growth

$y = a(b)^x$

When a > 0 and b is greater than 1, the graph will be increasing (growing)

$$y = 1(2)^x$$

So for this example, each time x is increased by 1, y increases by a factor of 2.

Exponential Decay	Exponential Growth
$y = a(b)^x$ b = $1-r$ r = rate as a decimal This is because you begin with 100% (1 when written as a decimal) and subtract the same percentage each time.	$y = a(b)^x$ b = $1+r$ r = rate as a decimal This is because you begin with 100% (1 when written as a decimal) and add the same percentage each time.

Examples

1. A house that costs $200,000 in 2010 will appreciate in value by 3% each year.

   a. Write a function that models the cost of the house over time. Use x for years after 2010, and y for the value of the house, in dollars.

   $\boxed{y = 200{,}000(1+.03)^x}$

   b. Find the value of the house in 2020. $x = 10$ years since 2010

   $y = 200{,}000(1.03)^{10}$

   $\boxed{y = \$268{,}783.28}$

   c. In what year will the house value have doubled?

   $400{,}000 = 200{,}000(1.03)^x$ → plug into y_1 and look at table $x = 23$

   $\boxed{\text{year } 2033}$

2. The most recent virus that is making people ill is a fast multiplying one. On the first day of the illness, only 2 virus "bugs" are present. Each day after, the amount of "bugs" triples.

   a. Write a function that models the "bugs'" growth over time. Let x represent the number of days since the onset of the illness and y represent the amount of "bugs".

   $\boxed{y = 2(3)^x}$

   b. Find the amount of "bugs" present by the 5th day.

   $y = 2(3)^5 = \boxed{486 \text{ bugs}}$

3. You have a bad cough and have to attend your little sister's choir concert. You take cough drops that contain 100 mg of menthol in each drop. Every minute, the amount of menthol in your body is cut in half.

   a. Write a function that models the amount of menthol in your body over time. Let x represent minutes since you took the cough drops and y be the amount of menthol, in mg, remaining in your body.

   $y = 100(1-\tfrac{1}{2})^x$

   b. It is safe to take a new cough drop when the amount of menthol in your body is less than 5 mg. How long will it be before you can take another cough drop?

   $5 = 100(\tfrac{1}{2})^x$ → put in y_1 and find when $y_1 = 5$

   $y_2 = 100(\tfrac{1}{2})^x$

   after about 5 hours

x	y
1	50
2	25
3	12.5
4	6.25
5	3.125

In the problems above, we wrote explicit functions for the exponential data.

$y = a(b)^x$

BUT we also need to know how to write recursive functions for the exponential data. Make sure you include the starting value of your data __a__ - this will always be where the independent variable is __0__.

Next = __Now * b__ , Starting at __a__

Let's write the prior examples using Recursive Formulas!

1. Next = Now * 0.03 Start = 200,000
2. Next = Now * 3 Start = 2
3. Next = Now * $\frac{1}{2}$ Start = 100

You Try!

4. Tobias ate half a banana in his room and forgot to throw the rest away. That night, two gnats came to visit the banana. Each night after, there were four times as many gnats hanging around the banana.

a. Write an explicit function that models the gnats' grown over time. Use x for nights and y for number of gnats.

$y = 2(4)^x$

b. Tobias' mom said that he would be grounded if the gnats number more than 120. On what night will Tobias be in trouble, if he doesn't step in and solve his gnat problem?

$120 = 2(4)^x$ $x = 3$

x	y
1	8
2	32
3	128
4	

c. Write a recursive function to model the gnats' growth.

Next = Now * 4 Start = 2

5. JaCorren is 60 inches and going through a growth spurt. For the next year, his growth will increase by 1% each month. Write a function that models JaCorren's growth spurt over the next year. Let x represent number of months and y represent JaCorren's height in inches.

a. Write an explicit function that models JaCorren's growth spurt over the next year.

$y = 60(1 + .01)^x$

b. Find JaCorren's height at the end of the year.

$y = 60(1.01)^{12} = \boxed{67.61 \text{ inches}}$

c. Write a recursive function to model JaCorren's growth.

Next = Now * 1.01 Start = 60

6. Ian's new Mercedes cost him $75,000 in 2014. From the moment he drives it off the lot, it will depreciate by 20% each year for the first five years.

a. Write an explicit function that models the car's depreciation. Use x for years after 2014 and y for the car's value, in dollars.

$y = 75{,}000(1 - .20)^x$ $y = \boxed{75{,}000(.80)^x}$

b. What will the car's value be in 2019?

$y = 75{,}000(.80)^5$ ← 5 years since 2014

$= \boxed{\$24{,}576}$

c. Write a recursive function to model the car's depreciation.

Next = Now * .80 Start = 75,000