Sp14

Day 5: Modeling with Exponential Functions: Solving Equations with Rational Exponents and Radicals

Warm-up: Solve for the missing variable

Check it ✓

1. $-8+\sqrt{5a-5}=-3$; $+8$ $+8$; $\sqrt{5a-5}=5$; $(\sqrt{5a-5})^2=5^2$ → $5a-5=25$; $5a=30$; $\boxed{a=6}$; $-8+\sqrt{5\cdot 6-5}=-3$; $-8+\sqrt{25}=3$; $-8+5=3$ ✓

2. $10+\sqrt{10m-1}=13$; -10 -10 ; $\sqrt{10m-1}=3$; $(\sqrt{10m-1})^2=3^2$ → $10m-1=9$; $10m=10$; $\boxed{m=1}$; $10+\sqrt{10\cdot 1-1}=13$; check $10+\sqrt{9}=13$; $10+3=13$ ✓

3. $\frac{-12}{-6}=\frac{-6\sqrt{b+4}}{-6}$; $2=\sqrt{b+4}$; $(2)^2=(\sqrt{b+4})^2$ → $4=b+4$; -4 -4 ; $\boxed{b=0}$; $-12=-6\sqrt{0+4}$; $-12=-6\sqrt{4}$ → $-12=-6\cdot 2$ ✓

4. $-10\sqrt{v-10}=-60$; $\frac{}{-10}$ $\frac{}{-10}$; $\sqrt{v-10}=6$; $(\sqrt{v-10})^2=6^2$ → $v-10=36$; $\boxed{v=46}$; $-10\sqrt{46-10}=-60$; $-10\sqrt{36}=-60$; $-10\cdot 6=-60$ ✓

Part 2 - Solving equations with radicals:

The next few problems are...different. We're going to come across some equations that have no solution and some that have two solutions. Remember, you can always check your answers by substituting your solution into the equation to make sure it works. In fact, you really ***need*** to check your answers to these problems! When we solve an equation correctly, but the answer doesn't work when we check it, we call the solution extraneous.

$\sqrt{a+2}-2=a$

$+2$ $+2$

$\sqrt{a+2}=a+2$

$(\sqrt{a+2})^2=(a+2)^2$

$a+2=(a+2)(a+2)$ FOIL

$a+2=a^2+4a+4$

→ $a+2=a^2+4a+4$

$-a-2$ $-a-2$

$0=a^2+3a+2$ factor

$0=(a+2)(a+1)$

$\boxed{a=-2,-1}$

$\sqrt{-2+2}-2=-2$ check it ✓ ; $\sqrt{-1+2}-2=-1$ check it ✓

$\sqrt{3x-2}=-5$

$(\sqrt{3x-2})^2=(-5)^2$

$3x-2=25$

$+2$ $+2$

$3x=27$

$x=9$

$\boxed{\text{No Solution}}$

check it

$\sqrt{3\cdot 9-2}=-5$

$\sqrt{27-2}=-5$

$\sqrt{25}=-5$

$\neq$

9 doesn't work

9 is extraneous solution

$(2x+7)^{1/2}-x=2$

$(2x+7)^{1/2}=2+x$

$((2x+7)^{1/2})^2=(2+x)^2$

$2x+7=(2+x)(2+x)$ FOIL

$2x+7=4+4x+x^2$

→ $2x+7=x^2+4x+4$

$-2x-7$ $-2x-7$

$0=x^2+2x-3$ factor

$0=(x+3)(x-1)$

$x=-3,\ \boxed{1}$

$(2\cdot 1+7)^{1/2}-1=2$ check it — 1 does work!!

check it $(2\cdot -3+7)^{1/2}-3=2$ X ← doesn't work! ⇐ -3 is an extraneous solution

$x=1$ does work so that is the solution

$3x^{4/3}+5=53$

-5 -5

$\frac{3x^{4/3}}{3}=\frac{48}{3}$ → $x^{4/3}=16$

$(x^{4/3})^{3/4}=(16)^{3/4}$

$x=\sqrt[4]{16^3}$

$\boxed{x=8}$

check it $3(8)^{4/3}+5=53$

$53=53$ ✓

You're going to come across some tougher problems that involve multiple steps. Let's try a couple. ☺

$\sqrt{x-5}-\sqrt{x}=-2$

$+\sqrt{x}$ $+\sqrt{x}$

$\sqrt{x-5}=\sqrt{x}-2$

$(\sqrt{x-5})^2=(\sqrt{x}-2)^2$

$x-5=(\sqrt{x}-2)(\sqrt{x}-2)$

$x-5=x-2\sqrt{x}-2\sqrt{x}+4$

$x-5=x-4\sqrt{x}+4$

$-x$ -4 $-x$ -4

→ $-9=-4\sqrt{x}$

$\frac{}{-4}$ $\frac{}{-4}$

$9/4=\sqrt{x}$

$(9/4)^2=(\sqrt{x})^2$

$\boxed{\frac{81}{16}=x}$

check

$\sqrt{\frac{81}{16}-5}-\sqrt{\frac{81}{16}}=-2$ ✓

$\sqrt{3x+7}=x-1$

$(\sqrt{3x+7})^2=(x-1)^2$

$3x+7=(x-1)(x-1)$

$3x+7=x^2-x-x+1$

$3x+7=x^2-2x+1$

$0=x^2-5x-6$

$0=(x-6)(x+1)$

→ $\boxed{x=6}, -1$

check it $\sqrt{3\cdot 6+7}=6-1$

$\sqrt{25}=5$ ✓

$\sqrt{3\cdot -1+7}=-1-1$

$\sqrt{4}=-2$ X No!

-1 is extraneous solution

Applications of Equations with Rational Exponents or Radicals.

The distance between two points is $5\sqrt{2}$. If one of the points is located at $(4,2)$ and the other point has a x-value of -1, what are the possible y-values of the other point? Use $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ distance formula

$5\sqrt{2}=\sqrt{(4-\text{-}1)^2+(2-y)^2}$

$5\sqrt{2}=\sqrt{5^2+(2-y)(2-y)}$

→ $5\sqrt{2}=\sqrt{25+4-4y+y^2}$

$5\sqrt{2}=\sqrt{y^2-4y+29}$

$(5\sqrt{2})^2=(\sqrt{y^2-4y+29})^2$

→ $(5\sqrt{2})(5\sqrt{2})=y^2-4y+29$

$50=y^2-4y+29$

$0=y^2-4y-21$

→ $0=(y-7)(y+3)$

$y=7,-3$

2. The volume of a sphere is 2145. If the formula $V = (4/3)\pi r^3$ is used to calculate the volume of a sphere, what is the radius of the sphere?

$2145 = \frac{4}{3}\pi r^3$

$\frac{3}{4} \cdot 2145 = \frac{3}{4} \cdot \frac{4}{3}\pi r^3$

$\frac{1608.75}{\pi} = \frac{\cancel{\pi} r^3}{\cancel{\pi}}$

$\sqrt[3]{\frac{1608.75}{\pi}} = \sqrt[3]{r^3}$

$\boxed{8 = r}$

The equation $v = \sqrt{2.5r}$ allows you to calculate the maximum velocity, v, that a car can safely travel around a curve with a radius of r feet. This is used by the Department of Transportation to determine the best speed limit for a given stretch of road. If a road has a speed limit of 45 mph, what is the tightest turn on that road?

$45 = \sqrt{2.5r}$

$(45)^2 = (\sqrt{2.5r})^2$

$\frac{2025}{2.5} = \frac{2.5r}{2.5}$

$\boxed{810 = r \text{ ft}}$

Day 6: Quiz Day

Warm-up:

Simplify the following:

Space for Practice Problems:

1. $9\sqrt[3]{16} + \sqrt[3]{54}$

2. $6\sqrt{8x^3y^2} \cdot \sqrt{10xy^3}$

3. $\sqrt[4]{625x^5}$

Solve the following:

4. $\left(\frac{5^3}{3^x}\right)^{-2} = \frac{3^8}{5^y}$

5. $(x^{\frac{1}{2}})^3 = 27$

6. $\sqrt{a+4} - 4 = a$