Day 4 : Modeling with Exponential Functions: Solving Equations with Rational Exponents and Radicals

Warmup: You know a lot about inverses in mathematics – we use them every time we solve equations. Write down the inverse operation for each of the following (there could be more than one correct answer) and then give a definition for "inverse" in your own words.

If you get stuck, it may be helpful for you to write the expression out and think what you would do to solve an equation that had that expression on one side of the equation.

The phrase…	Is the expression…	And its inverse is…
adding 5 to a number	$x+5$	Subtracting 5 from a number
subtracting 7 from a number	$x-7$	adding 7 to a number
multiplying a number by $\frac{1}{2}$	$\frac{1}{2}x$	multiplying a # by 2
Multiplying a number by $\frac{2}{5}$	$\frac{2}{5}x$	multiplying a number by 5/2
dividing a number by 3	$x/3$	multiplying a number by 3
squaring a number	x^2	square rooting a number
Taking the square root of a number	$\sqrt{x}$	squaring a number
Raising a number to the 5^{th} power	x^5	taking the 5th root of the number
Taking the 5^{th} root of a number	$\sqrt[5]{x}$	raising a number to the 5th power
Raising a number to the $\frac{2}{5}$ power	$x^{2/5}$	raising a number to the 5/2 power

An "inverse" is…

Make sure you have all of the answers correct to the table above before going on. Now that you're finished, we're going to combine our skills from this unit so far to solve some more challenging equations.

Work with a partner – Decide who will be Partner A and who will be Partner B.

Skill	Partner A	Partner B
1	$(\sqrt[4]{a})^4=(14)^4$; $a=38416$	$(\sqrt[5]{b})^5=(50)^5$; $b=312{,}500{,}000$
2	$(\sqrt[5]{a^9})^5=(26)^5$ → $a^9=26^5$; $a^9=11881376$ → $\sqrt[9]{a^9}=\sqrt[9]{11881376}$; $a=6.11$	$(\sqrt[4]{b^3})^4=(27)^4$ → $b^3=27^4$; $b^3=531441$ → $\sqrt[3]{b^3}=\sqrt[3]{531441}$; $b=81$
3	$\sqrt[3]{(\sqrt[7]{a})^3}=\sqrt[3]{21}$; $\sqrt[7]{a}=\sqrt[3]{21}$ → $(\sqrt[7]{a})^7=(\sqrt[3]{21})^7=\sqrt[3]{21^7}$; $a=441\sqrt[3]{21}$	$\sqrt[5]{(\sqrt{b})^5}=\sqrt[5]{12}$; $\sqrt{b}=\sqrt[5]{12}$; $(\sqrt{b})^2=(\sqrt[5]{12})^2=\sqrt[5]{12^2}$; $b=\sqrt[5]{144}$
4	$(a^{\frac{1}{5}})^5=(50)^5$; $a=50^5$; $a=312{,}500{,}000$	$(b^{\frac{1}{4}})^4=(14)^4$; $b=14^4$; $b=38416$
5	$a^{\frac{3}{4}}=27$; $(a^{3/4})^{4/3}=(27)^{4/3}$ → $a=\sqrt[3]{27^4}$; $27\sqrt[3]{27}$; $a=81$	$b^{\frac{9}{5}}=26$; $(b^{9/5})^{5/9}=(26)^{5/9}$; $b=\sqrt[9]{26^5}\approx 6.11$
6	$(a^{\frac{1}{2}})^5=12$; $a^{5/2}=12$; $(a^{5/2})^{2/5}=(12)^{2/5}$; $a=\sqrt[5]{12^2}$; $a=\sqrt[5]{144}$	$(b^{\frac{1}{7}})^3=21$; $b^{3/7}=21$; $(b^{3/7})^{7/3}=(21)^{7/3}$; $b=\sqrt[3]{21^7}$; $b=21^2\sqrt[3]{21}$; $b=441\sqrt[3]{21}$

Compare the answers you got when you practiced Skills 1-3 with the answers you got when you practiced Skills 4-6. Work with your partner to explain your findings.

Same answers in other column for Skills 1-3 as Skills 4-6

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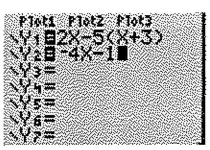
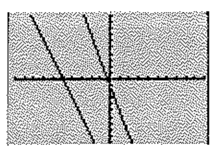
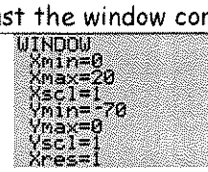
Warm-up: Solving Equations using the Calculator
Solving equations is usually easier to do by hand, but every now and then, it's nice to have a back-up plan…just in case. If you have a graphing calculator, you can use it to help you check your work.

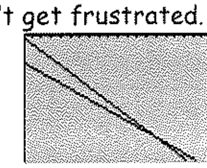
Here are the equations we're going to solve…from easiest to most difficult. Use the steps on the next page to solve each equation. The first one is used as the example. Some of them you may be able to solve without even picking up a pencil or calculator, but the idea is to practice your calculator skills. If you don't practice them, you won't be able to use them if you need to.

Level 1:	$2x - 5(x+3) = -4x - 1$	X=14
Level 2:	$2x^2 - 5x = 7x + 1$	X= -0.082, 6.082
Level 3:	$-\frac{1}{3}x^3 + 5x = 2x^2 - 3$	X= 0.51, 2.29
Level 4:	$\sqrt{x-5} - 12 = -8$	X=21
Level 5:	$2^x = 3$	X=1.58
Level 6:	$3x^{\frac{2}{3}} = 9$	X= 5.20, -5.20

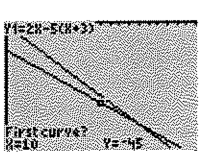
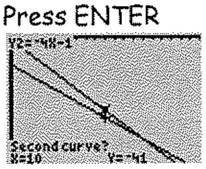
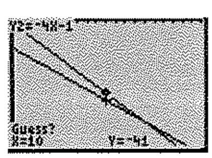
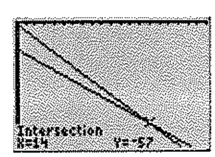
Here's how this works…

<table>
<tr>
<td>STEP 1: Go to Y= and Type the entire LEFT side of the equation into Y1 and the entire RIGHT side of the equation into Y2.

</td>
<td>STEP 2: Press GRAPH
</td>
<td colspan="2">STEP 3: Adjust the window (Press WINDOW or ZOOM) so that you can see where the two graphs intersect – there could be more than one point of intersection!

Sometimes this procedure will work without this step, but not always. It takes some practice and patience to adjust the window correctly. Don't get frustrated. ☺


</td>
</tr>
<tr>
<td>STEP 4: Press 2nd TRACE and choose 5: intersect.

Your calculator will ask you the question "First curve?" and the cursor will flash on one of the graphs.

Press ENTER

</td>
<td>STEP 5: Your calculator will ask you the question "Second curve?" and the cursor should begin flashing on the other graph. If it doesn't, use your left & right arrow keys until the other graph is highlighted.
Press ENTER

</td>
<td>STEP 6: Next your calculator will ask you if you want to "Guess?" At this point, if there is more than one point of intersection, move your cursor close to the one you're looking for. Then press ENTER.

</td>
<td>STEP 7: Your calculator will show you an ordered pair. In this case (14, -57). The x-value is the solution to your equation, and the y-value is what you'll get when you substitute the x-value into either side of the equation.

</td>
</tr>
</table>

The lesson: Yesterday's problems only had one step. You cannot do this step until the radical or rational exponent is isolated on one side of the equation. You can isolate the radical using the inverses discussed at the beginning of the lesson. There are also some problems below in which the rational exponent or radical is applied to the entire side of the equation. Only in these situations will you undo the rational exponents or radicals first. Before solving the entire problem, make sure you know what the first step will be.

Step 1

$x^{1/4} - 2 = 3$

$+2 \quad +2$

$x^{1/4} = 5$

$(x^{1/4})^4 = (5)^4$

$x = 625$

Step 1

$4x^7 - 6 = -2$

$+6 \quad +6$

$\frac{4x^7}{4} = \frac{4}{4}$

$x^7 = 1$

$\sqrt[7]{x^7} = \sqrt[7]{1}$

$x = 1$

Step 1

$\frac{3(x^{2/3} + 5)}{3} = \frac{207}{3}$

$x^{2/3} + 5 = 69$

$x^{2/3} = 64$

$(x^{2/3})^{3/2} = (64)^{3/2}$

$= (\sqrt{64})^3 = 8^3 = 512$

Step 1

$\frac{1450}{800} = \frac{800\left(1 + \frac{x}{12}\right)^{7.8}}{800}$

$1.8125 = \left(1 + \frac{x}{12}\right)^{7.8}$

$(1.8125)^{1/7.8} = \left(\left(1 + \frac{x}{12}\right)^{7.8}\right)^{\frac{1}{7.8}}$

$1.0792 = 1 + \frac{x}{12}$

$0.0792 = \frac{x}{12}$

$x = .95$

Step 1

$\frac{14.2}{222.1} = \frac{222.1 \cdot x^{3.5}}{222.1}$

$.0639 = x^{3.5}$

$(.0639)^{2/7} = (x^{7/2})^{2/7}$

$0.46 = x$

Step 1

$3x^{3/4} + 5 = 53$

$-5 \quad -5$

$\frac{3x^{3/4}}{3} = \frac{48}{3}$

$x^{3/4} = 16$

$(x^{3/4})^{4/3} = (16)^{4/3}$

$x = \sqrt[3]{16^4}$

$x = \sqrt[3]{16^3 \cdot 16}$

$x = \sqrt[3]{16^3 \cdot 2^3 \cdot 2}$

$x = 32\sqrt[3]{2}$

Step 1

$x^{1/2} - 5 = 0$

$+5 \quad +5$

$x^{1/2} = 5$

$(x^{1/2})^2 = 5^2$

$x = 25$

Step 1

$(2x + 7)^{1/2} = 3$

$((2x+7)^{1/2})^2 = (3)^2$

$2x + 7 = 9$

$-7 \quad -7$

$2x = 2$

$x = 1$

Step 1

$\sqrt[3]{x - 2} = 4$

$(\sqrt[3]{x-2})^3 = 4^3$

$x - 2 = 64$

$+2 \quad +2$

$x = 66$

Step 1

$\sqrt{a + 2} - 2 = 12$

$+2 \quad +2$

$\sqrt{a+2} = 14$

$(\sqrt{a+2})^2 = 14^2$

$a + 2 = 196$

$-2 \quad -2$

$a = 194$

Step 1

$\sqrt{2x - 5} = 9$

$(\sqrt{2x-5})^2 = (9)^2$

$2x - 5 = 81$

$+5 \quad +5$

$\frac{2x}{2} = \frac{86}{2}$

$x = 43$

Step 1

$\sqrt[4]{3x + 1} - 5 = 0$

$+5 \quad +5$

$\sqrt[4]{3x+1} = 5$

$(\sqrt[4]{3x+1})^4 = 5^4$

$3x + 1 = 625$

$-1 \quad -1$

$3x = 624$

$x = 208$