Day 3: Rational Exponents & Radicals

Warm-up (Properties of Exponents…with FRACTIONS!) → practices fraction skills AND Exponent Rules

Even though they seem more complicated, fractions are numbers too. You can use all the same properties with fraction (rational) exponents as you can with integer exponents. Write down those properties first.

$a^m \cdot a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$ $\frac{1}{a^n} = a^{-n}$ $(a^m)^n = a^{mn}$ $(a \cdot b)^n = a^n b^n$

warm-up continues on the next page

Work with a partner to write each expression in simplified form (one coefficient, each base used only once with one exponent each, no negative exponents). Each of you should complete one column, taking turns. Check each other's work.

COLUMN A	COLUMN B
$\left(x^{\frac{2}{3}}\right)^{-3} = x^{2/3 \cdot -3} = x^{-6/3} = x^{-2} = \boxed{\frac{1}{x^2}}$	$\left(x^{-\frac{4}{7}}\right)^{7} = x^{\frac{-4}{7} \cdot 7} = x^{-4} = \boxed{\frac{1}{x^4}}$
$\left(3x^{\frac{2}{3}}\right)^{-1} = 3^{-1} x^{-2/3} = \boxed{\frac{1}{3x^{2/3}}}$	$5\left(x^{\frac{2}{3}}\right)^{-1} = 5x^{-2/3} = \boxed{\frac{5}{x^{2/3}}}$
$(-27x^{-9})^{\frac{1}{3}} = (-27)^{1/3}(x^{-9 \cdot 1/3}) = -3x^{-9/3} = -3x^{-3} = \boxed{\frac{-3}{x^3}}$	$(-32x^{15})^{\frac{1}{5}} = (-32)^{1/5} \cdot x^{15 \cdot 1/5} = \boxed{-2x^3}$
$\left(\frac{x^3}{x^{-1}}\right)^{-\frac{1}{4}} = \frac{x^{-3/4}}{x^{1/4}} = \frac{1}{x^{1/4} \cdot x^{3/4}} = \boxed{\frac{1}{x}}$ *	$\left(\frac{x^2}{x^{-10}}\right)^{\frac{1}{3}} = \frac{x^{2 \cdot 1/3}}{x^{-10 \cdot 1/3}} = \frac{x^{2/3}}{x^{-10/3}} = x^{2/3} \cdot x^{10/3} = x^{12/3} = \boxed{x^4}$
$\left(x^{\frac{1}{2}}y^{-\frac{2}{3}}\right)^{-6} = x^{1/2 \cdot -6} y^{-2/3 \cdot -6} = x^{-3}y^{12/3} = \boxed{\frac{y^4}{x^3}}$	$\left(x^{\frac{2}{3}}y^{-\frac{1}{6}}\right)^{-12} = x^{2/3 \cdot -12} y^{-1/6 \cdot -12} = x^{-24/3} y^{12/6} = x^{-8}y^{2} = \boxed{\frac{y^2}{x^8}}$
$\left(\frac{x^{\frac{1}{4}}}{y^{-\frac{3}{4}}}\right)^{12} = \frac{x^{1/4 \cdot 12}}{y^{-3/4 \cdot 12}} = \frac{x^{12/4}}{y^{-36/4}} = \frac{x^3}{y^{-9}} = \boxed{x^3y^9}$ *	$\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15} = \frac{x^{-2/3 \cdot 15/1}}{y^{-1/3 \cdot 15/1}} = \frac{x^{-30/3}}{y^{-15/3}} = \frac{x^{-10}}{y^{-5}} = \boxed{\frac{y^5}{x^{10}}}$

Raising a number to the power of $\frac{1}{2}$ is the same as performing a familiar operation. Let's take a look at the graph of $y = x^{1/2}$ to discover that operation.

Step 1: Type $x^{1/2}$ into the y= screen on your graphing calculator. $y_1 = x \wedge (1/2)$ ← use parentheses around exponent
Step 2: Look at the table of values generated by this function. Verify that you have the same values as the rest of your class. (It is very easy to make a mistake when you type in the exponents here!)
Step 3: Discuss with your classmates what you believe to be the relationship between the x and y values in the table. Where have you seen this relationship before? Summarize your findings in a sentence.
Step 4: Type $x^{1/3}$ into the y= screen on your graphing calculator. → $y = \sqrt{x}$ gives same #s as $x^{1/2}$. $x^{1/2} = \sqrt{x}$
Step 5: Look at the table of values generated by this function. Verify that you have the same values as the rest of your class. (It is very easy to make a mistake when you type in the exponents here!)
Step 6: Discuss with your classmates what you believe to be the relationship between the x and y values in the table. Have you seen this relationship before? Summarize your findings in a sentence.

$y = \sqrt[3]{x}$ gives same values as $x^{1/3}$. $x^{1/3} = \sqrt[3]{x}$

Step 7: Type 25^x into the y= screen on your graphing calculator.
Step 8: Adjust your table so that the values go up by $\frac{1}{2}$ and begin at 0. Verify that your table contains the same values as the rest of your class. (Δtbl = 1/2 ; tblstart = 0)
Step 9: Discuss with your classmates the pattern you see. Use the table below to help you see the pattern. (One row has been completed for you). Summarize your findings in the space beside the table.

X (exponent)	X (exponent) as a fraction with a denominator of 2	$Y_1=(25^x)$	Rewrite Y_1 as a power of 25 with fraction exponents	Rewrite Y_1 as a power of $\sqrt{25}$
0	0/2	1	$25^{0/2}$	$(\sqrt{25})^0$
.5	1/2	5	$25^{1/2}$	$(\sqrt{25})^1$
1	2/2	25	$25^{2/2}$	$(\sqrt{25})^2$
1.5	$\frac{3}{2}$	125	$25^{3/2}$	$(\sqrt{25})^3$
2	4/2	625	$25^{4/2}$	$(\sqrt{25})^4$
2.5	5/2	3125	$25^{5/2}$	$(\sqrt{25})^5$
3	6/2	15625	$25^{6/2}$	$(\sqrt{25})^6$
3.5	7/2	78125	$25^{7/2}$	$(\sqrt{25})^7$

Summary
- A fraction exponent is the same thing as a radical.
- A fraction exponent with denominator of 2 in fraction is really a square root.
- The denominator of a fraction exponent is the index.
- The numerator of fraction exponent gives exponent of radical.

How could you use this pattern to find the value of $36^{3/2}$? Check your answer in the calculator.
$(\sqrt{36})^3 = 6^3 = 6^2 \cdot 6 = 36 \cdot 6 = \boxed{216}$

How could you use this pattern to find the value of $27^{2/3}$? Check your answer in the calculator.
$(\sqrt[3]{27})^2 = 3^2 = \boxed{9}$

How could you use this pattern to find the value of $81^{5/4}$? Check your answer in the calculator.
$(\sqrt[4]{81})^5 = (\sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3})^5 = 3^5 = \boxed{243}$

*Know your vocabulary ⟹ $x^{\frac{a}{b}} = \sqrt[b]{x^a}$ (index, exponent, radicand, radical symbol)

Step 10: Generally speaking, how can you find the value of an expression containing a rational exponent. Use the expression $a^{m/n}$ to help you in your explanation.

$a^{m/n}$ is equivalent to $\sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$

The numerator of exponent is exponent of radical. The denominator of fraction exponent is index of radical.

You try: Rewrite each of the following expressions in radical form.

$x^{\frac{3}{2}}$ $(\sqrt{x})^3 = \sqrt{x^3}$ $\boxed{x\sqrt{x}}$	$(-27)^{\frac{2}{3}}$ $\sqrt[3]{(-27)^2} = \boxed{9}$ $(\sqrt[3]{-27})^2 = (-3)^2$	$(16x)^{\frac{5}{4}}$ $= (\sqrt[4]{16})^5 \sqrt[4]{x^5}$ $\sqrt[4]{16^5x^5}$ $32x\sqrt[4]{x} = \boxed{32\sqrt[4]{x^5}}$ $\boxed{32x\sqrt[4]{x}}$	$y^{-9/8}$ $\frac{1}{y^{9/8}} = \boxed{\frac{1}{\sqrt[8]{y^9}}} = \boxed{\frac{1}{y\sqrt[8]{y}}}$
$2a^{\frac{1}{4}} = 2\sqrt[4]{a^1}$ $= \boxed{2\sqrt[4]{a}}$	$4^{\frac{-7}{2}} = \sqrt{4^{-7}} = \frac{1}{\sqrt{4^7}}$ $\frac{1}{4^{7/2}}$ $= \frac{1}{\sqrt{4^6}\sqrt{4}} = \frac{1}{4^3 \cdot 2} = \boxed{\frac{1}{128}}$	$(3^{\frac{2}{5}})^5 = 3^{\frac{2}{5}\cdot 5}$ OR $(\sqrt[5]{3^2})^5 = 3^2$ $= 3^2 = \boxed{9}$	$x^{1.2} = x^{\frac{12}{10}} = x^{6/5}$ $= \boxed{\sqrt[5]{x^6}} = \boxed{x\sqrt[5]{x}}$

Now, reverse the rule you developed to change radical expressions into rational expressions.

$\sqrt[5]{2} = \sqrt[5]{2^1} = \boxed{2^{1/5}}$	$(\sqrt[3]{6})^5 = (\sqrt[3]{6^1})^5 = (6^{1/3})^5$ $= 6^{1/3\cdot 5} = \boxed{6^{5/3}}$	$(\sqrt{5})^7 = (\sqrt[2]{5^1})^7 = (5^{1/2})^7$ $= 5^{1/2\cdot 7} = \boxed{5^{7/2}}$
$\sqrt{7} = \sqrt[2]{7^1} = \boxed{7^{1/2}}$	$(\sqrt[4]{9^3}) = \boxed{9^{3/4}}$	$(\sqrt[7]{3x})^2 = (\sqrt[7]{3^1x^1})^2 = (3^{1/7}x^{1/7})^2$ $= 3^{1/7\cdot 2}x^{1/7\cdot 2} = \boxed{3^{2/7}x^{2/7}}$

Earlier in this unit, you learned that when written in radical form, it's only possible to write two multiplied radicals as one if the index is the same. However, if you convert the radical expressions into expressions with rational exponents, you CAN multiply or divide them (as you saw in your warm-up)! Give it a try ☺ Write your final answer as a simplified radical.

if no fraction exponents, no radicals!

$\frac{12\sqrt[3]{y}}{4\sqrt{y}} = \frac{12y^{1/3}}{4y^{1/2}}$ $= 3y^{1/3-1/2} = 3y^{-1/6} = \frac{3}{y^{1/6}} = \boxed{\frac{3}{\sqrt[6]{y}}}$	$\left(\frac{\sqrt[3]{a^2}}{\sqrt{b}}\right)^{-6} = \left(\frac{a^{2/3}}{b^{1/2}}\right)^{-6}$ $= \frac{a^{2/3\cdot -6}}{b^{1/2\cdot -6}} = \frac{a^{-4}}{b^{-3}} = \boxed{\frac{b^3}{a^4}}$	$(2\sqrt[4]{a})^3 \cdot \sqrt{a^3} = (2^3)(\sqrt[4]{a})^3 \cdot \sqrt{a^3}$ $= 8(\sqrt[4]{a^1})^3 \cdot (\sqrt{a})^3 = 8(a^{1/4})^3 \cdot (a^{1/2})^3$ $= 8a^{3/4}a^{3/2} = 8^{9/4} = \sqrt[4]{8^9} = 8^2\sqrt[4]{8}$ $\boxed{64\sqrt[4]{8}}$
$\sqrt[4]{x^{12}} \cdot \sqrt{y^{-2}}$ $x^{12/4} \cdot y^{-2/2} = x^3 \cdot y^{-1}$ $= \boxed{\frac{x^3}{y}}$ simplified with no fractions → no radical needed	$\frac{\sqrt{64x^3}}{\sqrt[3]{512x^9}} = \frac{64^{1/2}x^{3/2}}{512^{1/3}x^{9/3}}$ $= \frac{8x^{3/2}}{8x^3} = 1x^{3/2-3} = x^{-3/2}$ $= \frac{1}{x^{3/2}} = \frac{1}{\sqrt{x^3}} = \boxed{\frac{1}{x\sqrt{x}}}$	$\sqrt[4]{625x^8} = (625)^{1/4}(x)^{8/4}$ $= \boxed{5x^2}$
$\sqrt[7]{x^2} \cdot \sqrt[14]{x^3}$ $= x^{2/7} \cdot x^{3/14} = x^{\frac{4}{14}} \cdot x^{\frac{3}{14}}$ $= x^{\frac{7}{14}} = x^{1/2}$ $\boxed{\sqrt{x}}$	* $\frac{1}{\sqrt[3]{-27x^9}}$ $= \frac{1}{(-27)^{1/3}x^{9/3}} = \boxed{\frac{1}{-3x^3}}$	$(\sqrt{x} \cdot \sqrt[3]{y^2})^{-6}$ $= (x^{1/2}y^{2/3})^{-6} = x^{1/2\cdot -6}y^{2/3\cdot 6}$ $= x^{-6/2}y^{-12/3} = x^{-3}y^{-4}$ $= \boxed{\frac{1}{x^3y^4}}$

*remember to watch for odd index and negatives under radical ∴ then you need a group of negatives

$\sqrt[3]{-27}$

$\sqrt[3]{-3\cdot -3 \cdot -3}$

" -3

How does the idea of simplifying radicals relate to the idea of rational exponents? There are several ways to approach this. Develop your own method for calculating simplest radical form of an expression without converting to radical form until the very last step!

$a^{\frac{3}{2}}$ $= a^{1+\frac{1}{2}} = a^1 \cdot a^{\frac{1}{2}} = \boxed{a\sqrt{a}}$

$\sqrt{a^3} = \sqrt{a\cdot a\cdot a} = \sqrt{a^2}\sqrt{a} = a^{2/2}\cdot a^{1/2} = a^{1+1/2} = a^{3/2}$

$a\sqrt{a}$

$b^{\frac{6}{4}}$ $= b^{1+\frac{2}{4}} = b^1 \cdot b^{\frac{1}{2}} = \boxed{b\sqrt{b}}$

$c^{\frac{10}{5}}$ $= \boxed{c^2}$

$d^{\frac{25}{3}}$ $= d^{8+\frac{1}{3}} = d^8 \cdot d^{\frac{1}{3}} = \boxed{d^8\sqrt[3]{d}}$

$\sqrt[3]{d^{25}}$

$\sqrt[3]{(ddd)(ddd)(ddd)(ddd)(ddd)(ddd)(ddd)(ddd)\,d}$

Describe your method for simplifying radicals from rational exponents. Share your method with the class.

- change exponent into a whole # plus a fraction
- convert that expression into two exponentials multiplied together
- change the (fraction one) rational exponent into a radical

I have, who has activity