Day 2: Simplifying Radicals and Basic Operations

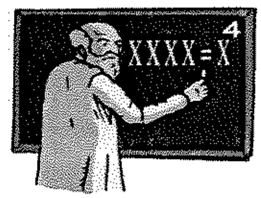
Warm-up: Match the radical on the left with a radical on the right with the equivalent value. Use your calculator if necessary.

1. $\sqrt{98} = \sqrt{49}\sqrt{2} = 7\sqrt{2}$ A
2. $\sqrt{108} = \sqrt{36}\sqrt{3} = 6\sqrt{3}$ G
3. $3\sqrt{2} \cdot 4\sqrt{3} = 12\sqrt{6}$ F
4. $-\sqrt{8} \cdot 3\sqrt{2} = -3\sqrt{16} = -3 \cdot 4 = -12$ D
5. $4\sqrt{6} + 3\sqrt{6} = 7\sqrt{6}$ I
6. $\sqrt{8} + \sqrt{18} = \sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2} = 2\sqrt{2} + 3\sqrt{2}$ $5\sqrt{2}$ C
7. $3\sqrt{2} - 2\sqrt{2} = 1\sqrt{2} = \sqrt{2}$ E
8. $\sqrt{48} - \sqrt{27} = \sqrt{16}\sqrt{3} - \sqrt{9}\sqrt{3}$ $\sqrt{3}$ H
   $4\sqrt{3} - 3\sqrt{3}$ $1\sqrt{3}$

A	$7\sqrt{2}$
B	$4\sqrt{2}$
C	$5\sqrt{2}$
D	-12
E	$\sqrt{2}$
F	$12\sqrt{6}$
G	$6\sqrt{3}$
H	$\sqrt{3}$
I	$7\sqrt{6}$

The lesson:

We are familiar with taking square roots ($\sqrt{\ }$) or with taking cubed roots ($\sqrt[3]{\ }$), but you may not be as familiar with the elements of a radical.



index → $\sqrt[n]{x} = r$ ← root

radicand

An index in a radical tells you how many times you have to multiply the root times itself to get the radicand. For example, in the equation $\sqrt{81} = 9$, 81 is the radicand, 9 is the root, and the index is 2 because you have to multiply the root by itself twice to get the radicand ($9 \cdot 9 = 9^2 = 81$). When a radical is written without an index, there is an understood index of 2.

$\sqrt[3]{64} = ?$
Radicand = 64
Index = 3
Root is 4
because $4 \cdot 4 \cdot 4 = 4^3 = 64$

$\sqrt[3]{64} = \boxed{4}$

$\sqrt[5]{32x^5} = ?$
Radicand = $32x^5$
Index = 5
Root is $2x$
because $2x \cdot 2x \cdot 2x \cdot 2x \cdot 2x = (2x)^5 = 32x^5$

$\sqrt[5]{32x^5} = \boxed{2x}$

You can use your calculator to do this, but for some of the more simple problems, you should be able to figure them out in your head.

Reminder: To use your calculator:
Step 1: Type in the index.
Step 2: Press MATH
Step 3: Choose 5: $\sqrt[x]{\ }$
Step 4: Type in the radicand.

Index tells how many #s you need to have a full group

You Try:

$\sqrt[5]{243y^5}$

$\sqrt[5]{243} \cdot \sqrt[5]{y^5}$

$\sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ $\boxed{3y}$ $y \cdot y \cdot y \cdot y \cdot y$

1 full group 1 full group

$\sqrt[4]{1296m^4n^8}$

$\sqrt[4]{1296} \cdot \sqrt[4]{m^4} \cdot \sqrt[4]{n^8}$

$\boxed{6mn^2}$ mmmm → 1 group

nnnn nnnn → 2 groups

$\sqrt[4]{6 \cdot 6 \cdot 6 \cdot 6}$ 1 group $6m \cdot n \cdot n$

$\sqrt{144v^8}$

$\boxed{12v^4}$ $\sqrt{144}$ $\sqrt{vv\,vv\,vv\,vv}$

$\sqrt{12 \cdot 12}$ 1 group 4 groups

ended here Day 1

BUT not every problem will work out that nicely!

Try using your calculator to find an ***exact*** answer for $\sqrt[3]{24}$ = ≈ 2.8845

The calculator will give us an estimation, but we can't write down an irrational number like this exactly – it can't be written as a fraction and the decimal never repeats or terminates. The best we can do for an exact answer is use simplest radical form.

Here are some examples of how to write these in simplest radical form. See if you can come up with a method for doing this. Compare your method with your neighbor's and be prepared to share it with the class. (Hint: do you remember how to make a factor tree?)

$\sqrt{12} = 2\sqrt{3}$	$\sqrt[3]{24} = 2\sqrt[3]{3}$	$\sqrt[4]{48} = 2\sqrt[4]{3}$
$\sqrt{4}\sqrt{3}$ $2\sqrt{3}$ $\sqrt{2\cdot 2}\sqrt{3}$ 1 group leftover = no group	$\sqrt[3]{8}\cdot\sqrt[3]{3}$ $\sqrt[3]{2\cdot 2\cdot 2}$ $2\sqrt[3]{3}$ 1 group leftover = no group!!	$\sqrt[4]{16}\cdot\sqrt[4]{3} = \sqrt[4]{2\cdot 2\cdot 2\cdot 2}\cdot\sqrt[4]{3}$ $2\sqrt[4]{3}$ 1 group leftover = no group

Simplifying Radicals:
1) break down radicand using perfect roots or factor tree
2) you need a perfect root or group # matching index to pull #s outside
3) always check that radicand is fully simplified (break down more when you can!)

Examples:

$\sqrt{16x^2} = \sqrt{16}\sqrt{x\cdot x}$ $\sqrt{4\cdot 4}\sqrt{x\cdot x}$ need group of 2 $\boxed{4x}$	$\sqrt{8x} = \sqrt{8}\sqrt{x}$ $= \sqrt{2\cdot 2\cdot 2}\sqrt{x} = \boxed{2\sqrt{2x}}$ 1 group leftover	$\sqrt{15x^3} = \sqrt{15}\cdot\sqrt{x\cdot x}\,x = \boxed{x\sqrt{15x}}$ $\sqrt{5\cdot 3}$ 1 group no pairs
$\sqrt[3]{-8} = \sqrt[3]{(-2)(-2)(-2)}$ 1 group need groups of 3 $\boxed{-2}$	$\sqrt[3]{80n^5} = \sqrt[3]{8\cdot 10}\sqrt[3]{n\cdot n\cdot n}\,n\cdot n$ $\sqrt[3]{2\cdot 2\cdot 2}\sqrt[3]{10}$ 1 group 1 group $\boxed{2n\sqrt[3]{10n^2}}$	$\sqrt[4]{96} = \sqrt[4]{16}\cdot\sqrt[4]{6}$ $= \sqrt[4]{2\cdot 2\cdot 2\cdot 2}\cdot\sqrt[4]{6}$ 1 group $\boxed{2\sqrt[4]{6}}$
$\sqrt[4]{81} = \sqrt[4]{3\cdot 3\cdot 3\cdot 3}$ 1 group need groups of 4 $\boxed{3}$	$\sqrt[5]{486} = \sqrt[5]{2\cdot 243}$ $= \sqrt[5]{2}\cdot\sqrt[5]{243}$ $\sqrt[5]{2}\cdot\sqrt[5]{3\cdot 3\cdot 3\cdot 3\cdot 3}$ 1 group $\boxed{3\sqrt[5]{2}}$	$\sqrt[3]{-40} = \sqrt[3]{-8}\cdot\sqrt[3]{5}$ $\sqrt[3]{(-2)(-2)(-2)}\cdot\sqrt[3]{5}$ $\boxed{-2\sqrt[3]{5}}$
$\sqrt[3]{18x^4}$ $\sqrt[3]{9\cdot 2}\cdot\sqrt[3]{x\cdot x\cdot x\cdot x}$ $\sqrt[3]{3\cdot 3\cdot 2}\cdot\sqrt[3]{x\cdot x\cdot x}\,x$ need groups of 3 $\boxed{x\sqrt[3]{18x}}$	$\sqrt[4]{64x^3} = \sqrt[4]{64}\cdot\sqrt[4]{x^3}$ $= \sqrt[4]{16\cdot 4}\cdot\sqrt[4]{x^3}$ $\sqrt[4]{2\cdot 2\cdot 2\cdot 2\cdot 4}\cdot\sqrt[4]{x^3}$ $\boxed{2\sqrt[4]{4x^3}}$	$\sqrt[5]{-32x^3y^6} = \sqrt[5]{-32}\cdot\sqrt[5]{x^3}\cdot\sqrt[5]{y^6}$ $= \sqrt[5]{(-2)^5}\cdot\sqrt[5]{x^3}\cdot\sqrt[5]{y\cdot y\cdot y\cdot y\cdot y}\,y$ -2 $\boxed{-2y\sqrt[5]{x^3}}$
$\sqrt[3]{81x^3y^2z^4}$ $\sqrt[3]{27\cdot 3}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{y^2}\cdot\sqrt[3]{z^4}$ $\sqrt[3]{3\cdot 3\cdot 3}\cdot 3\cdot\sqrt[3]{x\cdot x\cdot x}\sqrt[3]{y^2}\cdot\sqrt[3]{z\cdot z\cdot z}\,z$ $\boxed{3xz\sqrt[3]{3y^2z}}$	$\sqrt[3]{192x^5y^7z^2}$ $\sqrt[3]{8\cdot 8\cdot 3}\sqrt[3]{x^5}\sqrt[3]{y^7}\sqrt[3]{z^2}$ $\sqrt[3]{8}\cdot\sqrt[3]{8}\cdot\sqrt[3]{3}\sqrt[3]{x^3}\sqrt[3]{x^2}\sqrt[3]{y^6}\sqrt[3]{y}\sqrt[3]{z^2}$ $2\cdot 2\cdot\sqrt[3]{3}\cdot x\cdot\sqrt[3]{x^2}\cdot y^2\sqrt[3]{y}\sqrt[3]{z^2}$ $\boxed{4xy^2\sqrt[3]{3x^2yz^2}}$	$\sqrt[4]{1875x^4z^2}$ $\sqrt[4]{5^4\cdot 3}\sqrt[4]{x^4}\cdot\sqrt[4]{z^2}$ $\sqrt[4]{5^4}\cdot\sqrt[4]{3}\cdot x\sqrt[4]{z^2}$ $5\sqrt[4]{3}\cdot x\sqrt[4]{z^2}$ $\boxed{5x\sqrt[4]{3z^2}}$

Multiplying Radicals: When written in radical form, it's only possible to write two multiplied radicals as one if the index is the same. As long as this requirement is met,

1) multiply the outsides

2) multiply the insides

3) Simplify!

outside · outside $\sqrt[n]{\text{inside} \cdot \text{inside}}$

$2\sqrt{3}\cdot 5\sqrt{2}$

$2\cdot 5\sqrt{3\cdot 2}$

$\boxed{10\sqrt{6}}$

$-3\sqrt{8}\cdot\sqrt{2}$

$-3\cdot 1\sqrt{8\cdot 2}$

$-3\sqrt{16}$

$-3\cdot 4$

$\boxed{-12}$

$4\sqrt{5}\cdot 3\sqrt{10} = 4\cdot 3\sqrt{5\cdot 10}$

$12\sqrt{50}$

$12\sqrt{25}\sqrt{2}$

$12\cdot 5\sqrt{2}$

$\boxed{60\sqrt{2}}$

$\sqrt{3x^2y}\cdot\sqrt{5xy}$

$\sqrt{3\cdot 5x^2\cdot xy^2}$

$\sqrt{15 x\cdot x\, x\, y\cdot y}$

$\boxed{xy\sqrt{15x}}$

$6\sqrt{8x^3y^2}\cdot\sqrt{10xy^3}$

$6\sqrt{80x^4y^5}$

$6\sqrt{16\cdot 5\, xxxx\, yyyyy}$

$4\cdot 4 \cdot 5$

$6\cdot 4x^2y^2\sqrt{5y} = \boxed{24x^2y^2\sqrt{5y}}$

$-\sqrt{5x^4y^3}\cdot\sqrt{15x^2y^5}$

$-\sqrt{5\cdot 15\cdot x^6y^8}$

$-\sqrt{5\cdot 5\, 3\, x\cdot x\, x\cdot x\, x\cdot x\, yy\, yy\, yy\, yy}$

$\boxed{-5x^3y^4\sqrt{3}}$

$\sqrt[3]{4x^2}\cdot 5\sqrt[3]{8xy}$

$\boxed{10x\sqrt[3]{4y}}$

$5\sqrt[3]{32x^3y}$

$5\sqrt[3]{8\cdot 4x^3y}$

$5\sqrt[3]{2\cdot 2\cdot 2}\sqrt[3]{4}\sqrt[3]{x^3}\sqrt[3]{y}$

$5\cdot 2\cdot\sqrt[3]{4}\cdot x\cdot\sqrt[3]{y}$

$\sqrt[4]{2x^5}\cdot\sqrt[4]{40x^3y^3}$

$\sqrt[4]{80x^8y^3}$

$\sqrt[4]{16\cdot 5x^8y^3}$

$\sqrt[4]{16}\sqrt[4]{5}\sqrt[4]{x^8}\sqrt[4]{y^3}$

$\boxed{2x^2\sqrt[4]{5y^3}}$

$4\sqrt[5]{27x^3}\cdot\sqrt[5]{9x^3y^5}$

$4\sqrt[5]{3\cdot 3\cdot 3\cdot x\cdot x\cdot x}\cdot\sqrt[5]{3\cdot 3\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y}$

$4\sqrt[5]{3\cdot 3\cdot 3\cdot 3\cdot 3\, x\cdot x\cdot x\cdot x\cdot x\, x\, y\cdot y\cdot y\cdot y\cdot y}$

$4\cdot 3xy\sqrt[5]{x}$

$\boxed{12xy\sqrt[5]{x}}$

$3\sqrt[3]{5x^3}\cdot 2\sqrt[3]{50y}$

$6\sqrt[3]{250\, x\cdot x\cdot x\, y}$

$6x\sqrt[3]{5\cdot 5\cdot 5\cdot 2y}$

$6x\cdot 5\sqrt[3]{2y}$

$\boxed{30x\sqrt[3]{2y}}$

$\sqrt[3]{9}\cdot\sqrt[3]{-24}$

$\sqrt[3]{3\cdot 3\cdot -8\cdot 3}$

$\sqrt[3]{3\cdot 3\cdot 3\, -2\cdot -2\cdot -2}$

$3\cdot -2$

$\boxed{-6}$

$\sqrt[4]{8}\cdot\sqrt[4]{32}$

$\sqrt[4]{2\cdot 2\cdot 2}\cdot\sqrt[4]{8\cdot 4}$

$\sqrt[4]{2\cdot 2\cdot 2}\cdot\sqrt[4]{2\cdot 2\cdot 2\cdot 4}$

$\sqrt[4]{2\cdot 2\cdot 2\cdot 2\, 2\cdot 2\cdot 2\cdot 2}$

$2\cdot 2$

$\boxed{4}$

Adding and Subtracting Radicals: You've been combining like terms in algebraic expressions for a long time! Show your skills by simplifying the following expressions.

$2x - x + 4x =$ $5x$

$1x + 4x$

$3y - 2x + y - 6y =$ $-2x - 2y$

$4y - 2x - 6y$

Usually we say that like terms are those that contain the same variable expression, but they can also contain the same radical expression. *When you add or subtract radicals, you can only do so if they contain the same index and radicand.* Just like we don't change the variable expression when we add or subtract, we're not going to change the radical expression either. All we are going to do is add or subtract the coefficients.

Always simplify the radical before you decide that you can't add or subtract.

$3\sqrt{3} + 4\sqrt{3}$

$(3+4)\sqrt{3}$

$\boxed{7\sqrt{3}}$

$\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$

$1\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$

$(1+2+3)\sqrt{5}$

$\boxed{6\sqrt{5}}$

$4\sqrt{12} - \sqrt{75}$

$4\cdot\sqrt{4}\sqrt{3} - \sqrt{25}\sqrt{3}$

$4\cdot 2\sqrt{3} - 5\sqrt{3}$

$8\sqrt{3} - 5\sqrt{3}$

$(8-5)\sqrt{3}$

$\boxed{3\sqrt{3}}$

$\sqrt{45x^3} - \sqrt{20x^3}$

$\sqrt{9}\sqrt{5}\sqrt{x^2}\sqrt{x} - \sqrt{4}\sqrt{5}\sqrt{x^2}\sqrt{x}$

$3x\sqrt{5x} - 2x\sqrt{5x}$

$(3x-2x)\sqrt{5x}$

$1x\sqrt{5x}$

$\boxed{x\sqrt{5x}}$

$5\sqrt[3]{32} - 2\sqrt[3]{108}$

$5\sqrt[3]{8}\sqrt[3]{4} - 2\sqrt[3]{27}\sqrt[3]{4}$

$5\cdot 2\sqrt[3]{4} - 2\cdot 3\sqrt[3]{4}$

$10\sqrt[3]{4} - 6\sqrt[3]{4}$

$\boxed{4\sqrt[3]{4}}$

$3\sqrt[3]{16} + \sqrt[3]{54}$

$3\sqrt[3]{8}\sqrt[3]{2} + \sqrt[3]{27}\sqrt[3]{2}$

$3\cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$

$6\sqrt[3]{2} + 3\sqrt[3]{2}$

$\boxed{9\sqrt[3]{2}}$

$2\sqrt[3]{125a^4} - 5\sqrt[3]{8a}$

$2\sqrt[3]{125}\sqrt[3]{a^3}\sqrt[3]{a} - 5\sqrt[3]{8}\sqrt[3]{a}$

$2\cdot 5\cdot a\sqrt[3]{a} - 5\cdot 2\sqrt[3]{a}$

$\boxed{10a\sqrt[3]{a} - 10\sqrt[3]{a}}$

$9\sqrt[3]{40a} - 7\sqrt[3]{135a}$

$9\sqrt[3]{8}\sqrt[3]{5}\sqrt[3]{a} - 7\sqrt[3]{27}\sqrt[3]{5}\sqrt[3]{a}$

$9\cdot 2\sqrt[3]{5a} - 7\cdot 3\sqrt[3]{5a}$

$18\sqrt[3]{5a} - 21\sqrt[3]{5a}$

$\boxed{-3\sqrt[3]{5a}}$

$5\sqrt[3]{16y^4} + 7\sqrt[3]{2y}$

$5\sqrt[3]{8}\sqrt[3]{2}\sqrt[3]{y^3}\sqrt[3]{y} + 7\sqrt[3]{2y}$

$5\cdot 2\cdot y\sqrt[3]{2y} + 7\sqrt[3]{2y}$

$\boxed{10y\sqrt[3]{2y} + 7\sqrt[3]{2y}}$

$6\sqrt{18} + 3\sqrt{50}$

$6\sqrt{9}\sqrt{2} + 3\sqrt{25}\sqrt{2}$

$6\cdot 3\sqrt{2} + 3\cdot 5\sqrt{2}$

$18\sqrt{2} + 15\sqrt{2}$

$\boxed{33\sqrt{2}}$

$\sqrt[3]{54} + \sqrt[3]{16}$

$\sqrt[3]{27}\sqrt[3]{2} + \sqrt[3]{8}\sqrt[3]{2}$

$3\sqrt[3]{2} + 2\sqrt[3]{2}$

$\boxed{5\sqrt[3]{2}}$

$\sqrt[4]{32} + \sqrt[4]{48}$

$\sqrt[4]{16}\sqrt[4]{2} + \sqrt[4]{16}\sqrt[4]{3}$

$\boxed{2\sqrt[4]{2} + 2\sqrt[4]{3}}$