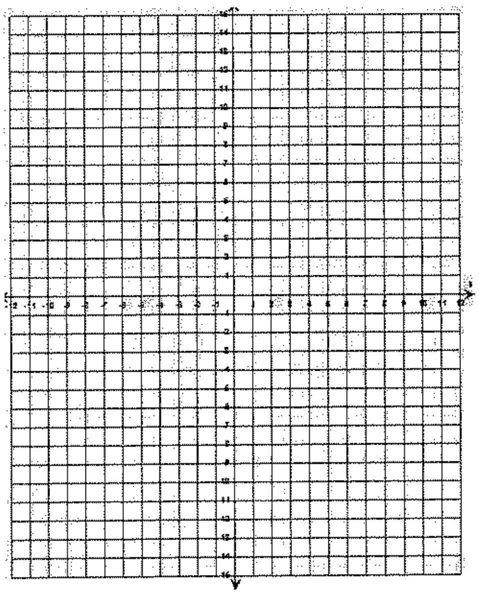
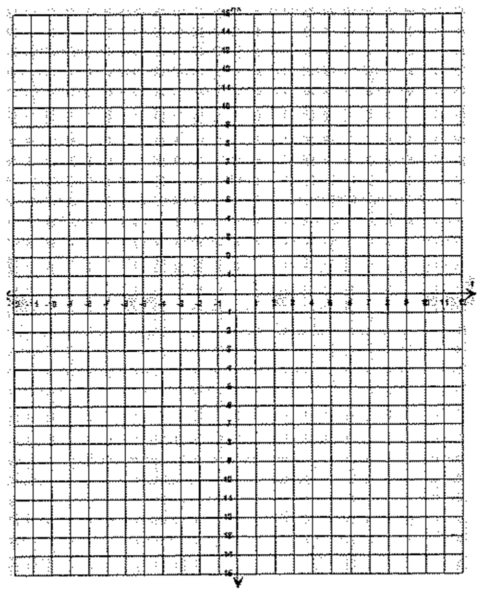
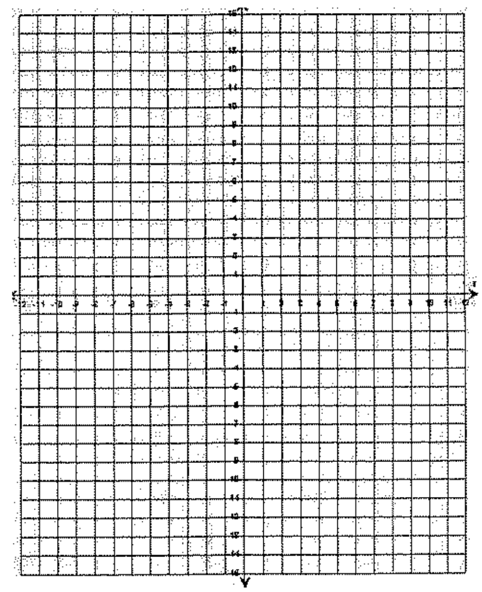
Graph the following transformations of the function $y = \log_{10} x$ on the coordinate planes. Determine the domain, range, and asymptotes of each transformation. Describe the transformations.

5) $y = \log_{10} x - 6$	6) $y = -\log_{10}(x + 2)$	7) $y = \log_{10} 2x$
Domain:	Domain:	Domain:
Range:	Range:	Range:
Asymptotes:	Asymptotes:	Asymptotes:
Description:	Description:	Description:

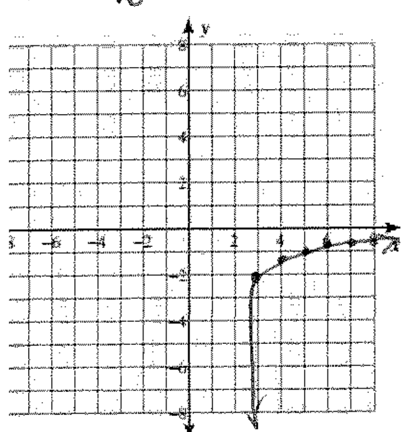




Day ~~9~~ 13: Common Logs: Introduction to Solving Equations and Word Problems

Warm-up: Key Features of Logarithmic Functions

Graph the following function. Identify the domain, range, asymptotes, and transformations of the parent function $y = \log_{10}(x)$.

$y = \log_{10}(4x - 11) - 2$



Domain: $4x - 11 > 0$ $4x > 11$ $\boxed{x > \frac{11}{4}}$ or $x > 2.75$

Range: $\boxed{\text{all real numbers}}$ or $\mathbb{R}$

Asymptote: $\boxed{x = \frac{11}{4}}$ or $x = 2.75$

Description of transformations:

Translated right 2.75 and down 2

3	-2
4	-1.3
5	-1.05
7	-.77
9	-.68

Express each of the numbers below as accurately as possible as a power of 10. You can find exact values for some of the required exponents by thinking about the meanings of positive and negative exponents. Others might require some calculator exploration of ordered pairs that satisfy the exponential equation $y = 10^x$.

a. 100 10^x=100, x=2 10^2	b. 10,000 10^4	c. 1,000,000 10^6
d. 0.01 10^{-2}	e. -0.001 -10^{-3}	f. 3.45 $10^{0.538}$
g. -34.5 $-10^{1.538}$	h. 345 $10^{2.538}$	i. 0.0023 $10^{-2.63}$

takes a lot of time if not an exact power of 10... there's a better way!

Day 13 Notes Part 1 Common Logs: Solving Equations and Word Problems

Common Logarithms

We have already discussed logarithms in the previous lesson, but in this lesson we will solve exponent problems which focus on a base of 10. When we solve logarithmic problems in base 10, we call them **common logarithms**.

The definition of common logarithms is usually expressed as:

use log loop $\log_{10} a = b$ if and only if $10^b = a$

steps: start at base, loop to other side, put = on paper, loop over to other side

$\text{Log}_{10}\, a$ is pronounced "log base 10 of a". Because base 10 logarithms are so commonly used, $\log_{10} a$ is often written as log a. Most calculators have a built-in log function that automatically finds the required exponent value.

1) Use your calculator to find the following logarithms. Then compare the results with your work on Problem 1.

a. log 100 = 2 because $10^2 = 100$	b. log 10,000 = 4 because $10^4 = 10{,}000$	c. log 1,000,000 = 6 because $10^6 = 1{,}000{,}000$
d. log 0.01 = -2 because $10^{-2} = 0.01$	e. log -0.001 → non real answer! remember domain $x > 0$	f. log 3.45 = 0.538 because $10^{0.538} = 3.45$
g. log -34.5 → non real answer! remember domain $x > 0$ must be true!!	h. log 345 = 2.538 because $10^{2.538} = 345$	i. log 0.0023 = -2.638 because $10^{-2.638} = 0.0023$

2) What do your results from Problem 1 (especially Parts e and g) suggest about the kinds of numbers that have logarithms? See if you can explain your answer by using the connection between logarithms and the exponential function $y = 10^x$.

Domain of logs must be positive because a base that's positive raised to a power can never be negative number

Summarize the Mathematics

A) How would you explain to someone who did not know about logarithms what the expression *log y = x* tells us about the numbers x and y? $10^x = y$ 10 is base, x is exponent, y is 10 raised to a power x

B) What can be said about the value of *log y* in each case below? Give brief justifications of your answers.

i. $0 < y < 1$ log y will be a negative value because $10^x = y$ with $0 < y < 1$ only if x is negative values (like #2d, 2i)

ii. $1 < y < 10$ log y will be >0 and <1 ($0 < \log y < 1$) because $10^0 = 1$ and $10^1 = 10$ (like #2f)

iii. $10 < y < 100$ $1 < \log y < 2$ (log y will be >1 and <2) because $10^1 = 10$ and $10^2 = 100$ (like #2a)

iv. $100 < y < 1{,}000$ $2 < \log y < 3$ (log y will be >2 and <3) because $10^2 = 100$ and $10^3 = 1000$ (like #2a, 2h)

Check Your Understanding

Use your understanding of the relationship between logarithms and exponents to help complete these tasks.

a. Find these common (base 10) logarithms without using a calculator.

i. $\log_{10} 1{,}000 = x$
$10^x = 1000$
$10^x = 10^3$ $\boxed{x=3}$

ii. $\log_{10} 0.001 = x$
$10^x = 0.001$
$10^x = 10^{-3}$ $x = \boxed{-3}$

iii. $\log_{10} 10^{3.2} = x$
$10^x = 10^{3.2}$
$x = 3.2$ $\boxed{3.2}$

b. Use the function $y = 10^x$, but not the logarithm key of your calculator to estimate each of these logarithms to the nearest tenth. Explain how you arrived at your answers.

i. $\log_{10} 75 = x$
$10^x = 75$
$10^x = 10^{1.875}$ $\boxed{1.875}$

ii. $\log_{10} 750 = x$
$10^x = 750$
$10^x = 10^{2.875}$ $\boxed{2.875}$

iii. $\log_{10} 7.5 = x$
$10^x = 7.5$
$10^x = 10^{0.875}$ $\boxed{0.875}$

Day 13 Notes Part 2 Common Logs: Solving for Expenents and Word Problems

Investigation 2: Solving for Exponents

Logarithms can be used to find exponents that solve equations like $10^x = 9.5$. For this reason, they are an invaluable tool in answering questions about exponential growth and decay. For example, the world population is currently about 6.2 billion and growing exponentially at a rate of about 1.14% per year. To find the time when this population is likely to double, you need to solve the equation

$$6.2(1.0114)^t = 12.4, \text{ or } (1.0114)^t = 2$$

As you work on the problems of this investigation, look for ways to answer this question:

How can common logarithms help in finding solutions of exponential equations?

STEPS
1. Isolate where variable is
2. take log of both sides
3. use log power property $\log_a b^x = x\log_a b$
4. Solve

1) Use number sense and what you already know about logarithms to solve these equations.

a. $10^x = 1{,}000$ isolate ✓ $\log 10^x = \log 1000$, $x\log 10 = \log 1000$, $\frac{x\log 10}{\log 10} = \frac{\log 1000}{\log 10}$ $\boxed{x=3}$

b. $10^{x+2} = 1{,}000$ isolated ✓ $\log 10^{x+2} = \log 1000$, $(x+2)\log 10 = \log 1000$, $x+2 = \frac{\log 1000}{\log 10}$, $x = \frac{\log 1000}{\log 10} - 2$ $\boxed{x=1}$

c. $10^{3x+2} = 1{,}000$ isolated ✓ $\log 10^{3x+2} = \log 1000$, $(3x+2)\log 10 = \log 1000$, $3x+2 = \frac{\log 1000}{\log 10}$, $3x+2=3$, $3x=1$ $\boxed{x=\frac{1}{3}}$

d. $2(10)^x = 200$, $10^x = 100$, $\log 10^x = \log 100$, $x\log 10 = \log 100$, $x = \frac{\log 100}{\log 10}$ $\boxed{x=2}$

e. $3(10)^{x+4} = 3{,}000$ isolate, $10^{x+4} = 1000$, $\log 10^{x+4} = \log 1000$, $(x+4)\log 10 = \log 1000$, $x+4=3$ $\boxed{x=-1}$

f. $10^{2x} = 50$ $\log 10^{2x} = \log 50$, $\frac{2x\log 10}{2\log 10} = \frac{\log 50}{(2\log 10)}$ $\boxed{x=0.85}$ be careful with parentheses, 2 AND log 10 are divided!

g. $10^{3x+2} = 43$ $\log 10^{3x+2} = \log 43$, $(3x+2)\log 10 = \log 43$, $3x+2 = \frac{\log 43}{\log 10}$, $3x = \frac{\log 43}{\log 10} - 2$ $\boxed{x=-.12}$

h. $12(10)^{3x+2} = 120$, $10^{3x+2} = 10$, $\log 10^{3x+2} = \log 10$, $(3x+2)\log 10 = \log 10$, $3x+2=1$ $\boxed{x=-\frac{1}{3}}$

i. $3(10)^{x+4} + 7 = 28$, $3(10)^{x+4} = 21$, $10^{x+4} = 7$, $\log 10^{x+4} = \log 7$, $(x+4)\log 10 = \log 7$, $x = \frac{\log 7}{\log 10} - 4$ $\boxed{x=-3.15}$

Unfortunately, many of the functions that you have used to model exponential growth and decay have not used 10 as the base. On the other hand, it is not too hard to transform any exponential expression in the form b^x into an equivalent expression with base 10. You will learn how to do this after future work with logarithms. The next three problems ask you to use what you already know about solving exponential equations with base 10 to solve several exponential growth problems.

2) If a scientist counts 50 bacteria in an experimental culture and observes that one hour later the count is up to 100 bacteria, the function $P(t) = 50(10^{0.3t})$ provides an exponential growth model that matches these data points.

a. Explain how you can sure that P(0) = 50.

$P(0) = 50(10^{0.3\cdot 0})$
$P(0) = 50(10^0) = 50(1) = \boxed{50}$ because $x^0 = 1$

b. Show that $P(1) \approx 100$.

$P(1) = 50(10^{0.3\cdot 1})$
$= 50(10)^{0.3} \approx 99.76$ in calculator

c. Use the given function to estimate the time when the bacteria population would be expected to reach 1,000,000. Explain how to find this time in two ways – one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning.

logs: $\frac{1000000}{50} = \frac{50(10^{0.3t})}{50}$, $20{,}000 = 10^{0.3t}$, $\log 20000 = \log 10^{0.3t}$, $\frac{\log 20000}{0.3\log 10} = \frac{0.3t\log 10}{0.3\log 10}$

$\boxed{t = 14.3368 \text{ hours}}$

calc: can check in calc $y_1 = 1000000$, $y_2 = 50(10^{0.3x})$ + intersect

3) The world population in 2005 was 6.2 billion and growing exponentially at a rate of 1.14% per year. The function $P(t) = 6.2(10^{0.005t})$ provides a good model for the population growth pattern.

a. Explain how you can be sure that P(0) = 6.2

$P(0) = 6.2(10^{0.005\cdot 0})$
$= 6.2(10^0) = 6.2(1) = \boxed{6.2}$

b. Show that P(1) = 6.2 + 1.14%(6.2)

$P(1) = 6.2 + 1.14\%(6.2)$
$6.2(10^{.005+1}) = 6.2(10^{1.005}) = 6.2(10.1158)$

c. Find the time when world population would be expected to reach 10 billion if growth continues at the same exponential rate. Explain how to find this time in two ways – one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning.

logs: $\frac{10}{6.2} = \frac{6.2(10^{.005t})}{6.2}$, $1.6129 = 10^{.005t}$, $\log 1.6129 = \log 10^{.005t}$, $\frac{\log 1.6129}{(.005\log 10)} = \frac{.005t\log 10}{.005\log 10}$

$\boxed{41.5217 = t}$

CHECK YOUR UNDERSTANDING: Practice

Use logarithms and other algebraic methods as needed to complete the following tasks.

a. Solve these equations.

I. $\frac{5(10)^x}{5} = \frac{450}{5}$ $10^x = 90$ $\log 10^x = \log 90$ $\frac{x \log 10}{\log 10} = \frac{\log 90}{\log 10}$ $\boxed{x = 1.9542}$

II. $\frac{4(10)^{2x}}{4} = \frac{40}{4}$ $10^{2x} = 10$ $10^{2x} = 10^1$ $2x = 1$ $\boxed{x = 1/2}$ OR $\frac{4(10)^{2x}}{4} = \frac{40}{4}$ $(10)^{2x} = 10$ $\log(10)^{2x} = \log(10)$ $\frac{2x \log 10}{\log 10} = \frac{\log 10}{\log 10}$ $2x = 1$ $x = 1/2$

III. $\frac{5(10)^{4x-2}}{5} = \frac{500}{5}$ $10^{4x-2} = 100$ $10^{4x-2} = 10^2$ $4x-2 = 2$ $4x = 4$ $\boxed{x = 1}$ OR $\frac{5(10)^{4x-2}}{5} = \frac{500}{5}$ $10^{4x-2} = 100$ $\log 10^{4x-2} = \log 100$ $\frac{(4x-2)\log 10}{\log 10} = \frac{\log 100}{\log 10}$ $4x - 2 = \frac{\log 100}{\log 10}$ $\frac{4x}{4} = \frac{\log 100}{4\log 10} + \frac{2}{4}$ $\boxed{x = \frac{\log 100}{4\log 10} + \frac{1}{2}}$ $\boxed{x = 1}$

IV. $8x^2 + 3 = 35$ $8x^2 = 32$ $x^2 = 4$ $\sqrt{x^2} = \sqrt{4}$ $\boxed{x = \pm 2}$

b. The population of the United States in 2006 was about 300 million and growing exponentially at a rate of about 0.7% per year. If that growth rate continues, the population of the country in year 2006 + t will be given by the function $P(t) = 300(10^{0.003t})$. According to that population model, when is the U.S. population predicted to reach 400 million? Check the reasonableness of your answer with a table or a graph of P(t).

$\frac{400}{300} = \frac{300(10^{0.003t})}{300}$ $4/3 = 10^{0.003t}$ $\log(4/3) = \log 10^{0.003t}$ $\frac{\log(4/3)}{0.003 \log 10} = \frac{0.003t \log 10}{0.003 \log 10}$ $41.65 = t$

$\boxed{2006 + 41.65 = 2047.65}$ 2047

Day 14: Wrap-Up Review

Day 14 Warm-up:

1. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% each year. Write a function describing the value of the land as a function of time (let time be years after 1960).

a.) Write an explicit equation to model the relationship:

b.) Write a recursive (Now-Next) equation to model the relationship:

c.) What was the value of the land in 2011?

d.) In what year will the land be worth $50,000?