Unit 3 Day 12

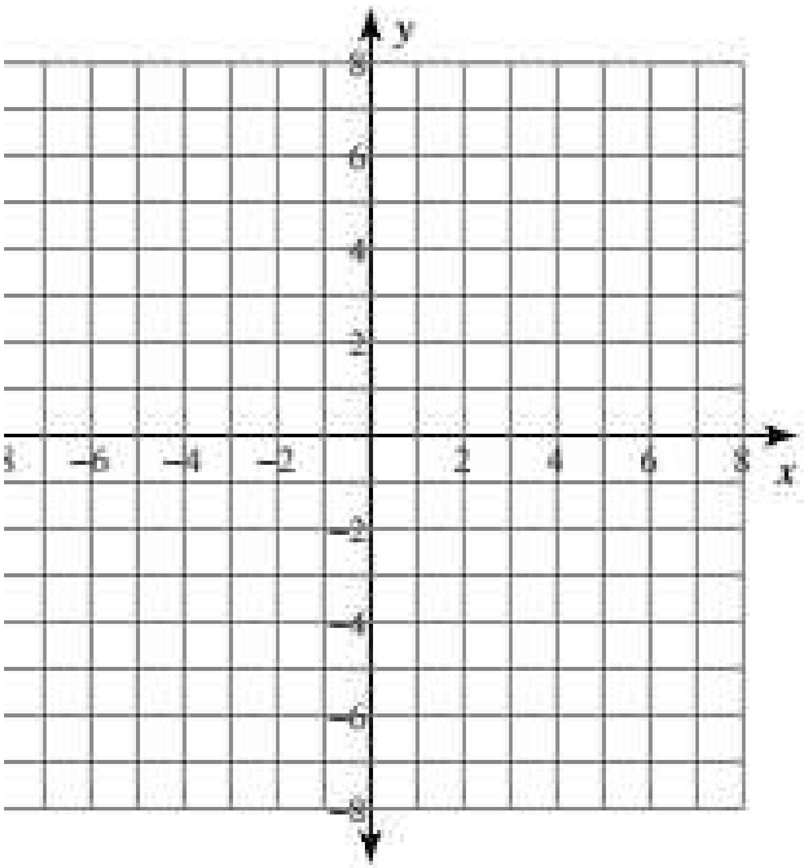
Common Logs and Applications

Warm Up Day 12

1. Key Features of Logarithmic Functions

Graph the following function. Identify the domain, range, asymptotes, and transformations of the parent function y = log (x).

$y = \log_{10}(4x - 11) - 2$



Domain: **x > 2.75** **(or 11/4)**
(2.75, ∞)

Range: **All Real #s** (−∞, ∞)

Asymptote: **x = 2.75**

Description of transformations:

Right 2.75 and down 2

2. Convert to exponential form then evaluate.

a) $\log_2 16$ **4**

b) log 10000 **4**

3. Solve.

a) $10^x = 1000$ **x = 3**

b) $10^{x-2} = 10000$ **x = 6**

Tonight's Homework

Packet p. 22 - 23 and
Finish Notes p. 38 - 39

Homework Answers from Day 11 material
Packet p. 17-18 All

1. a) Translated 2 units right.
   b) Translated 3 units up.
   c) Translated 2 units right and 3 units up.

2. Domain : $(2, \infty)$ Range : $(-\infty, \infty)$
   x-intercept (2.001, 0) Vertical Asymptote: x = 2

3. a) Reflected over the x-axis.
   b) Translated 2 units right.
   c) Reflected over the x-axis and translated 2 units right.

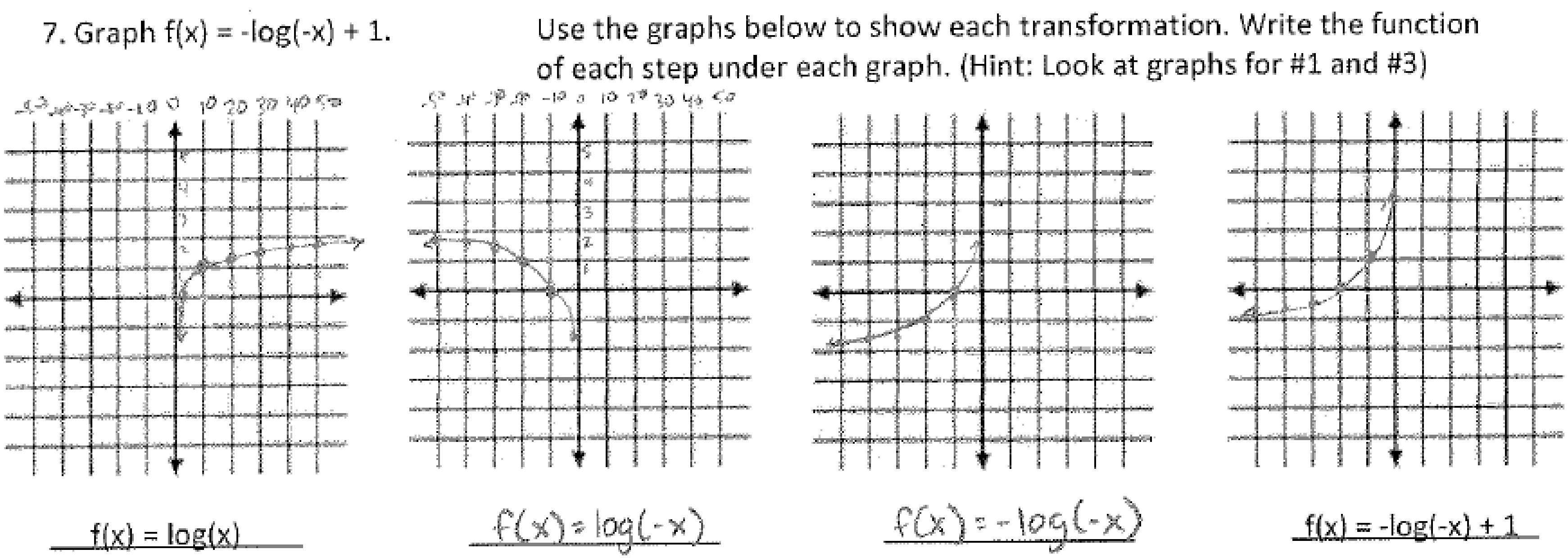
4. Domain : $(2, \infty)$ Range : $(-\infty, \infty)$
   x-intercept (2, 0) Vertical Asymptote x = 2

Homework Answers from Day 11 material

5. a) Reflected over the y-axis.
   b) Translated 2 units up.
   c) Reflected over the y-axis and translated 2 units up.

6. Domain : $(-\infty, 0)$ Range : $(-\infty, \infty)$
   x-intercept: (-0.01, 0) Vert. Asymptote: x = 0

7.

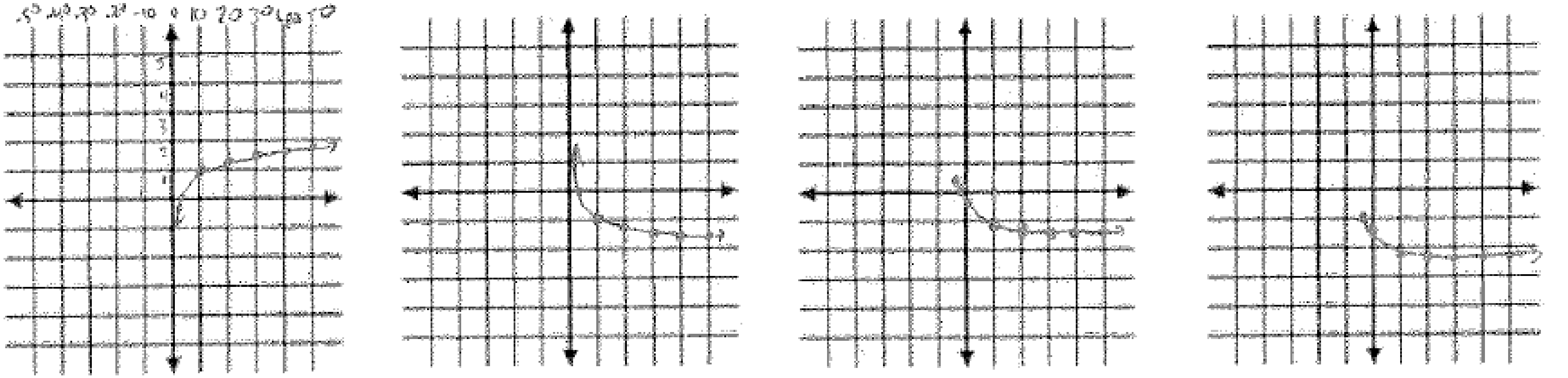


#7 Graph Scale: x-values increase by 10, y values increase by 1.

Homework Answers

8.

8. Graph $f(x) = -\log(x + 2) - 1$. Write the function of each step under each graph.



$f(x) = \log(x)$ | $f(x) = -\log(x)$ | $f(x) = -\log(x+2)$ | $f(x) = -\log(x+2) - 1$

#8 Graph Scale: x values increase by 10, y values increase by 1.

Homework Answers for HW p. 19 – 20 Odds

p. 19

1. $x = 15/2$
2. $x = \{-5, 5\}$
3. $x = -6$
4. $x = 169$
5. $x = \{-5, 6\}$
6. $x = 9$
7. $x = 3$
8. $x = 7/3$

p. 20

1. $b = 10$
2. $x = 8$
3. $a = -4$
4. $x = -4$
5. $r = 28$
6. $m = 8$
7. $k = \{0, 8\}$
8. $b = -1$
9. $x = -2$
10. $k = 4$

Homework Answers for HW p. 21 Odds

p. 21

11. r = 4	12. b = 1
13. r = 7	14. p = {2, 5}
15. n = 16	16. v = 4
17. n = 7	18. x = {-1, -3}
9. x = 9	10. b = 6

Investigation

Notes p. 37 - 38

Investigation #2

Logarithms can be used to find exponents that solve equations like $10^x = 9.5$. For this reason, they are an invaluable tool in answering questions about exponential growth and decay. For example, the world population is currently about 6.2 billion and growing exponentially at a rate of about 1.14% per year. To find the time when this population is likely to double, you need to solve the equation

$$6.2(1.0114)^t = 12.4, \text{ or } (1.0114)^t = 2$$

As you work on the problems of this investigation, look for ways to answer this question:

- How can common logarithms help in finding solutions of exponential equations?

Use number sense and what you already know about logarithms to solve these equations.

a. $10^x = 1{,}000$

x = 3

b. $10^{x+2} = 1{,}000$

x = 1

<u>Steps</u>

1) **Isolate the exponential part (where your variable is).**
2) **Take the log of both sides.**
3) **Use log power property**

   $\log_a b^x = x \log_a b$
4) **Solve.**

Use number sense and what you already know about logarithms to solve these equations.

c. $10^{3x + 2} = 1{,}000$

d. $2(10)^{x} = 200$

$x = 1/3$

$x = 2$

★ = You Try!

★ e. $3(10)^{x+4} = 3{,}000$

x = -1

f. $10^{2x} = 50$

x = 0.85

★ g. $10^{3x+2} = 43$

x = -.12

★ h. $12(10)^{3x+2} = 120$

x = -1/3

★ i. $3(10)^{x+4} + 7 = 28$

x = -3.15

Complete Applications & Practice

p. 38-39

Application #2

2) If a scientist counts 50 bacteria in an experimental culture and observes that one hour later the count is up to 100 bacteria, the function $P(t) = 50(10^{0.3t})$ provides an exponential growth model that matches these data points.

a. Explain how you can be sure that $P(0) = 50$.

50 is our initial value

AND $P(0) = 50(10^{0.3*0}) = 50(10^{0}) = 50(1) = 50$

b. Show that $P(1) \approx 100$.

$P(1) = 50(10^{0.3*1}) \approx 99.76 \approx 100$

c. Use the given function to estimate the time when the bacteria population would be expected to reach 1,000,000. Explain how to find this time in two ways – one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning.

$1{,}000{,}000 = 50(10^{0.3*t})$ -> solve using logs

t = 14.34 hours

Application #3

3) The world population in 2005 was 6.2 billion and growing exponentially at a rate of 1.14% per year. The function $P(t) = 6.2(10^{0.005t})$ provides a good model for the population growth pattern.

a. Explain how you can be sure that $P(0) = 6.2$

6.2 is our initial value AND

$P(0) = 6.2(10^{0.005*0}) = 6.2(10^{0}) = 6.2(1) = 6.2$ billion

b. Show that $P(1) = 6.2 + 1.14\%(6.2)$

$P(1) = 6.2(10^{0.005*1}) = 6.2(10^{0.005})10 = 6.27$ billion

c. Find the time when world population would be expected to reach 10 billion if growth continues at the same exponential rate. Explain how to find this time in two ways – one by numerical or graphic estimation and the other by use of logarithms and algebraic reasoning **$10 = 6.2(10^{0.005*t})$ -> solve using logs**

$t = 41.52$ hours

CHECK YOUR UNDERSTANDING:

Use logarithms and other algebraic methods as needed to complete the following tasks.

Solve these equations.

I. $5(10)^x = 450$

$$x = 1.95$$

II. $4(10)^{2x} = 40$

$$x = \frac{1}{2}$$

III. $5(10)^{4x-2} = 500$

$$x = 1$$

IV. $8x^2 + 3 = 35$

$$x = \pm 2$$

Check Your Understanding p 39

I. $5(10)^x = 450$

$\frac{5(10)^x}{5} = \frac{450}{5}$

$10^x = 90$

$\log 10^x = \log 90$

$x \log 10 = \log 90$

$\frac{x \log 10}{\log 10} = \frac{\log 90}{\log 10}$

$x = 1.9542$

II. $4(10)^{2x} = 40$

$10^{2x} = 10$

$10^{2x} = 10^1$

$2x = 1$

$x = 1/2$

OR

$\frac{4(10)^{2x}}{4} = \frac{40}{4}$

$(10)^{2x} = 10$

$\log (10)^{2x} = \log(10)$

$\frac{2x \log 10}{\log 10} = \frac{\log 10}{\log 10}$

$2x = 1$

$x = 1/2$

III. $5(10)^{4x-2} = 500$

$10^{4x-2} = 100$

$10^{4x-2} = 10^2$

$4x - 2 = 2$

$4x = 4$

$x = 1$

OR

$\frac{5(10)^{4x-2}}{5} = \frac{500}{5}$

$10^{4x-2} = 100$

$\log 10^{4x-2} = \log 100$

$\frac{(4x-2)\log 10}{\log 10} = \frac{\log 100}{\log 10}$

$4x - 2 = \frac{\log 100}{\log 10}$

$x = 1$

IV. $8x^2 + 3 = 35$

$8x^2 = 32$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

b. The population of the United States in 2006 was about 300 million and growing exponentially at a rate of about 0.7% per year. If that growth rate continues, the population of the country in year 2006 + t will be given by the function $P(t) = 300(10^{0.003t})$. According to that population model, when is the U.S. population predicted to reach 400 million? Check the reasonableness of your answer with a table or a graph of P(t).

t = 41.65

2006 + 41.65 = 2047.65

In the Year 2047

Review

Find the solution to each equation algebraically.

1) $\sqrt{20x-6}=\sqrt{5x+39}$

2) $2(x-2)^{2/3}-8=192$

3) $(x+7)^{1/2}-x=5$

4) Your new painting is valued at \$2400. It's value depreciates 7% each year. The value is a function of time.

a) Write a recursive (next-now) equation for the situation
b) Write an explicit function for the situation
c) When will the painting be worth \$1000?

5) Graph $y=2^{x+4}-3$. Identify the domain, range, asymptotes, and transformations of the parent function $y=2^x$.

Review ANSWERS

Review

Find the solution to each equation algebraically.

1) $\sqrt{20x-6}=\sqrt{5x+39}$

x = 3

2) $2(x-2)^{2/3}-8=192$

x = 1002

3) $(x+7)^{1/2}-x=5$

x = -3

Review ANSWERS

4) Your new painting is valued at $2400. It's value depreciates 7% each year. The value is a function of time.

a) Write a recursive (next-now) equation for the situation

NEXT = Now * (.93) start = 2400

b) Write an explicit function for the situation

$y = 2400(.93)^x$

c) When will the painting be worth $1000?

During year 12

5) Graph $y = 2^{x+4} - 3$. Identify the domain, range, asymptotes, and transformations of the parent function $y = 2^x$.

Domain: All real #s OR $(-\infty, \infty)$ HA: $y = -3$

Range: $y > -3$ OR $(-3, \infty)$

Transformation: Left 4, down 3

Tonight's Homework

Packet p. 22 - 23 and
Finish Notes p. 38 - 39