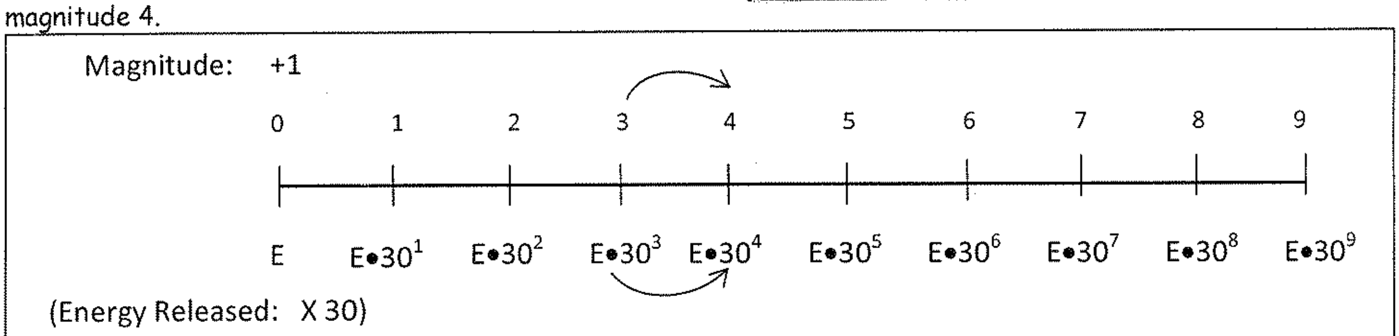
Day ~~8~~ 12: Inverses of Functions (logs and exponentials)

Day 8 Warm-up: Inverses...Find the algebraic inverse.

1. $f(x)=15x-1$ $y=15x-1$ $x=15y-1$ $\frac{x+1}{15}=\frac{15y}{15}$ $\boxed{\frac{1}{15}(x+1)=y}$	2. $f(x)=\frac{4}{7}x$ $y=\frac{4}{7}x$ $x=\frac{4}{7}y$ $\boxed{\frac{7}{4}x=y}$
3. $f(x)=\frac{1}{3}x+7$ $y=\frac{1}{3}x+7$ $x=\frac{1}{3}y+7$ $x-7=\frac{1}{3}y$ $\boxed{3(x-7)=y}$ $3x-21=y$	4. $f(x)=-5x-11$ $y=-5x-11$ $x=-5y-11$ $x+11=-5y$ $\boxed{\frac{-1}{5}(x+11)=y}$
5. $f(x)=(x-2)^2$ $y=(x-2)^2$ $\sqrt{x}=\sqrt{(y-2)^2}$ $\pm\sqrt{x}=y-2$ $\boxed{\pm\sqrt{x}+2=y}$	6. $f(x)=\sqrt{x-4}$ $y=\sqrt{x-4}$ $x=\sqrt{y-4}$ $x^2=y-4$ $\boxed{x^2+4=y}$

Logarithmic Functions: Inverse of an Exponential Function

The Richter Scale: The magnitude of an earthquake is a measure of the amount of energy released at its source. The Richter scale is an exponential measure of earthquake magnitude. A simple way to examine the Richter Scale is shown below. An earthquake of magnitude 5 releases about 30 times as much energy as an earthquake of magnitude 4.



In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. Let's compare the amounts of energy released in the two earthquakes.
For the earthquake in Mexico at 8.0 on the Richter Scale, the energy released is $E \bullet 30^8$ and for the earthquake in Washington state, the energy released is $E \bullet 30^{6.8}$. A ratio of the two quakes and using the properties of exponents yields the following:

Fix

$$\frac{\text{Mexico Earthquake}}{\text{Washington Earthquake}} = \frac{E\cdot 30^8}{E\cdot 30^{6.8}} = 30^{8-6.8} = 30^{1.2} = \boxed{59.23}$$

Mexico's earthquake ~~was~~ had 59.23 times more energy than

This means the earthquake in Mexico released about 59.23 times as much energy as the one in Washington.
The exponents used by the Richter scale in the example are called **logarithms** or logs.
A logarithm is defined as follows:

To convert forms, use loglop
$\log_2 8 = 3$
$2^3 = 8$

> The logarithm base b of a positive number y is defined as follows:
> If $y = b^x$, then $\log_b y = x$. $b \neq 1$ and $b > 0$

$2^3 = 8$ is equivalent to $\log_2 8 = 3$

The exponent x in the exponential expression b^x is the logarithm in the equation $\log_b y = x$. The base b in b^x is the same as the base b in the logarithm. In both cases, $b \neq 1$ and $b > 0$. So what this means is that you use logarithms to undo exponential expressions or equations and you use exponents to undo logarithms, which means that the operations are inverses of each other. Thus, an exponential function is the inverse of a logarithmic function and vice versa.

Partner Graph Activity → See next paper for graphs

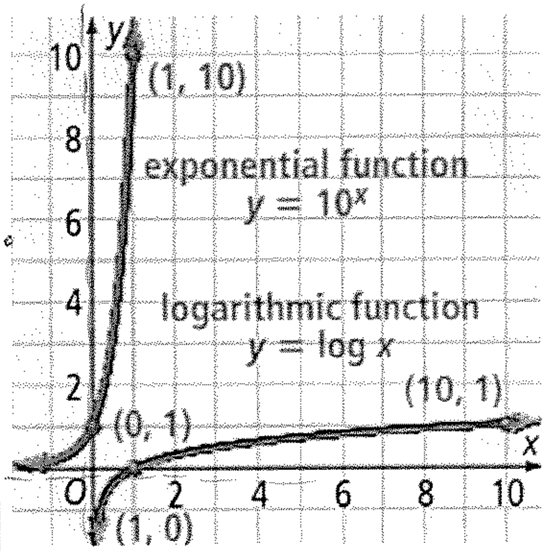
Key Features of Logarithmic Graphs

A **logarithmic function** is the inverse of an exponential function. The graph show $y = 10^x$ and $y = \log x$. Note that (0, 1) and (1,10) lie on the graph of $y = 10^x$ and that (1, 0) and (10, 1) lie on the graph of $y = \log x$, which demonstrates the reflection over the line $y = x$.

*Remember from yesterday, for inverses x's + y's switch.

Since an exponential function $y = b^x$ has an asymptote at $y = 0$, the inverse function $y = \log_b x$ has an asymptote at $x = 0$.

The other key features of exponential and logarithmic functions are summarized in the box below.



y=0

x=0

Key Features of Exponential and Logarithmic Functions		
Characteristic	Exponential Function $y = b^x$	Logarithmic Function $y = \log_b x$
Asymptote	$y = 0$ (horiz. asymp.)	$x = 0$ (vert. asymp.)
Domain	All real numbers	$x > 0$
Range	$y > 0$	All real numbers
Intercept	(0,1)	(1,0)

Translations of logarithmic functions are very similar to those for other functions and are summarized in the table below.

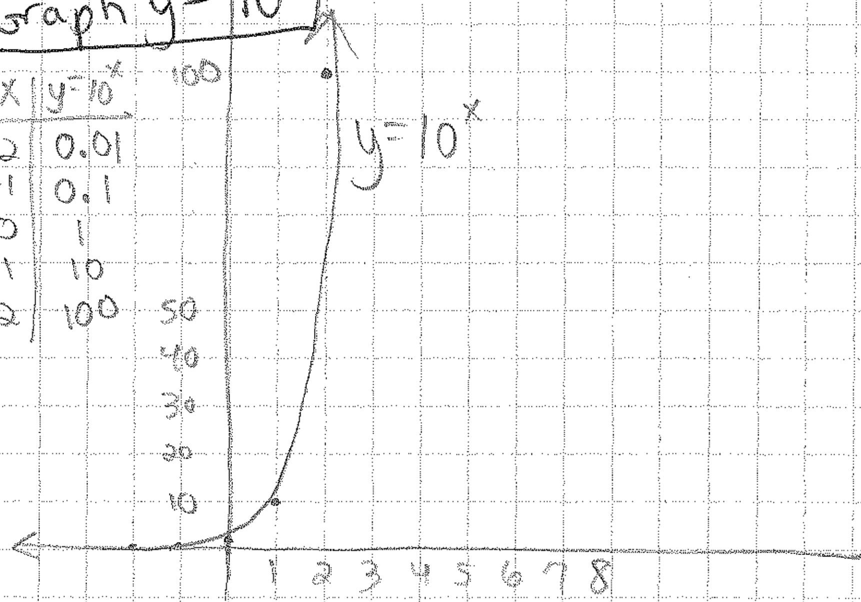
Parent Function	$y = \log_b x$
Shift up	$y = \log_b x + k$
Shift down	$y = \log_b x - k$
Shift left	$y = \log_b(x + h)$
Shift right	$y = \log_b(x - h)$
Combination Shift	$y = \log_b(x \pm h) \pm k$
Reflect over the x-axis	$y = -\log_b x$
Stretch vertically	$y = a \log_b x$
Stretch horizontally	$y = \log_b ax$

Remember the two types of functions we have already translated:
$y = 2(x-2)^2 + 3$
$y = 3^{x-2} + 3$
→ Rules are the same!!

Partner Graph Activity

Graph $y = 10^x$

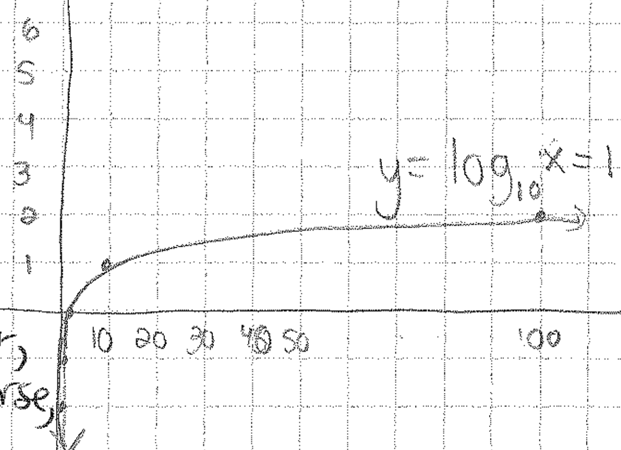
x	$y = 10^x$
-2	0.01
-1	0.1
0	1
1	10
2	100



Graph the inverse of $y = 10^x$ (Graph $y = \log_{10} x$)

$x = 10^y$

0.01	-2
0.1	-1
1	0
10	1
100	2



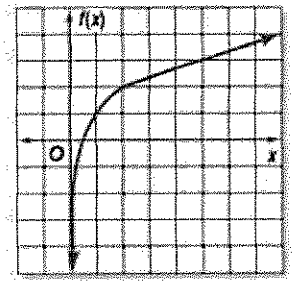
*Remember, to get inverse, switch x's + y's

$y = 10^x$ and $y = \log_{10} x$ are inverses

⇓

they are reflections over line $y = x$

Let's look at the following example.



The graph on the right represents a transformation of the graph of $f(x) = 3\log_{10} x + 1$.

- $|x| = 3$: Stretches the graph vertically.
- $h = 0$: There is no horizontal shift.
- $k = 1$: The graph is translated 1 unit up.

vert. asymptote: $x = 0$ Domain: $x > 0$
Range: All real #s

Now it's your turn to find key features and translate logarithmic functions.

* 2 graphs not in notes... on next paper

TRY NOW

Graph the following function on the graph at right. Describe each transformation, give the domain and range, and identify any asymptotes.

$y = -2\log_{10}(x + 2) - 4$

$y = -2\log(x+2) - 4$ mcalc

-1	-4
8	-6
3	-5.4
4	-5.56
1	-4.95

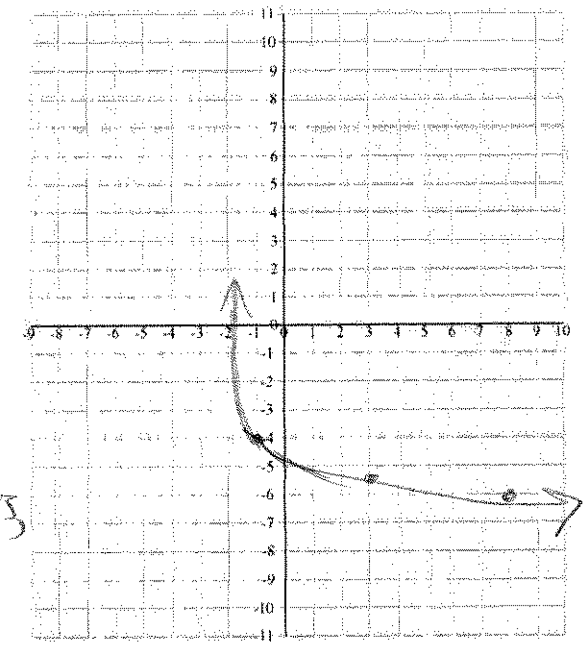
Domain: $x > -2$

Range: all real #s

Asymptote: $x = -2$

Description of transformations:

left 2, down 4, reflected over x-axis, stretched vertically by 2



Guided Practice with Logarithmic Functions

In 1812, an earthquake of magnitude 7.9 shook New Madrid, Missouri. Compare the amount of energy released by that earthquake to the amount of energy released by each earthquake below.

1) Magnitude 7.7 in San Francisco, California, in 1906.

$\frac{\text{New Madrid}}{\text{SanFran}} = \frac{E \cdot 30^{7.9}}{E \cdot 30^{7.7}} = 30^{7.9-7.7} = 30^{0.2}$

NM was 1.97 times stronger

2) Magnitude 3.2 in Charlottesville, Virginia, in 2001.

$\frac{\text{New Madrid}}{\text{charlottesville}} = \frac{E \cdot 30^{7.9}}{E \cdot 30^{3.2}} = 30^{7.9-3.2} = 30^{4.7}$

NM was 8759310.01 times stronger

3) Magnitude 9.5 in Valdivia, Chile, in 1960.

$\frac{\text{N Madrid}}{\text{Valdivia}} = \frac{E \cdot 30^{7.9}}{E \cdot 30^{9.5}} = 30^{7.9-9.5} = 30^{-1.6} = \frac{1}{230.88}$

Valdivia was 230.88 times stronger

4) How much stronger was the earthquake in Valdivia than the earthquake in San Francisco?

$\frac{\text{Valdivia}}{\text{SanFran}} \quad \frac{E \cdot 30^{9.5}}{E \cdot 30^{7.7}} = 30^{9.5-7.7} = 30^{1.8} = 455.85$

Valdivia was 455.85 times stronger.

The location of 1st error typically gives a clue about location of asymptote.

Trace curve on graph to confirm asymptote location.

Here's a basic example…

*Not in Notes… do on graph paper

y= 1.) $\log_{10}(x+1)$

y=log(x+1) in calc.

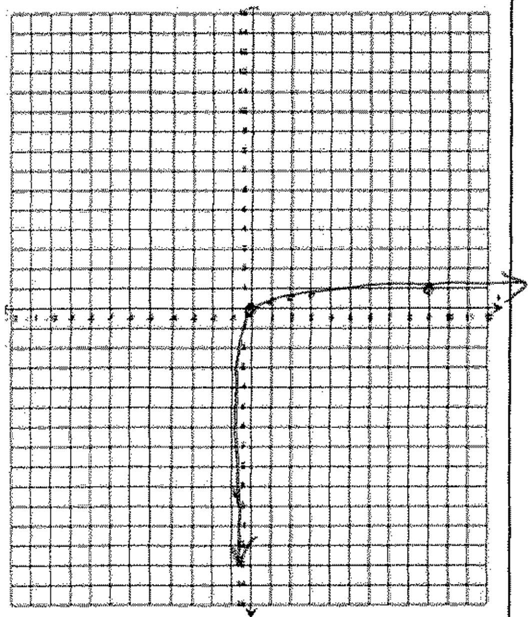
Domain: x > -1

Range: All real #s

Asymptotes: x = -1

Description: Left 1

-1	Error
0	0
1	.30
2	.48
3	.60
9	1



1. Graph on calc
2. Look at the table for pts.
3. Plot

* trace graph where flattens vertically to find asymptote
* looking at first error in calc table can help too ☺

Another example…

*Not in Notes… do on graph paper

2.) $\log_{10}x + 1$

y=log(x) + 1 in calc

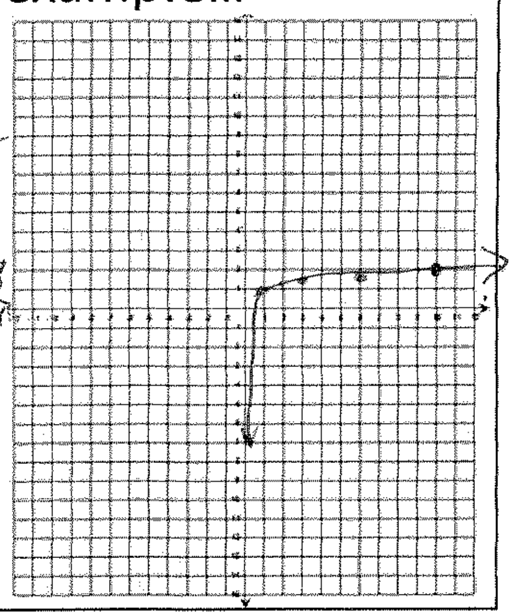
Domain: x > 0

Range: All real #s

Asymptotes: x = 0

Description: Up 1

0	Error
1	1
2	1.30
3	1.48
6	1.78
10	2



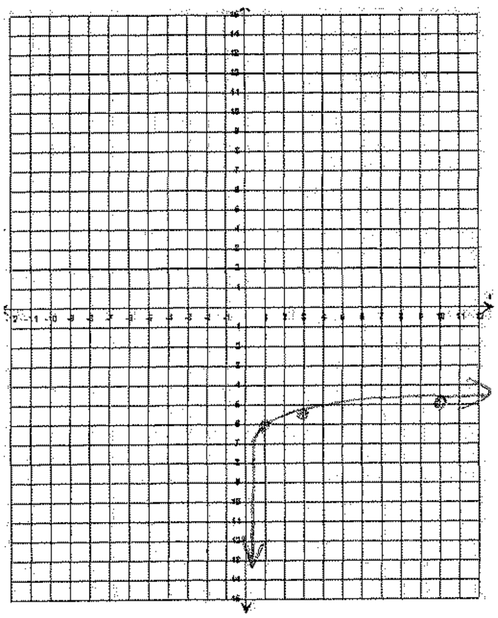
*Note: If no parentheses are in the problem, your parentheses go around x

Graph the following transformations of the function $y = \log_{10} x$ on the coordinate planes. Determine the domain, range, and asymptotes of each transformation. Describe the transformations.

5) $y = \log_{10} x - 6$

3	-5.5
1	-6
10	-5
6	-5.2
5	-5.3

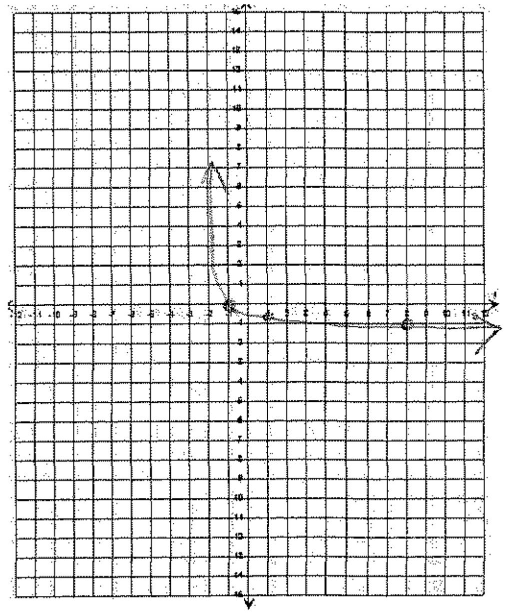
Domain: $x > 0$
Range: all real #s
Asymptotes: $x = 0$
Description: down 6



6) $y = -\log_{10} (x + 2)$

1	-.48
-1	0
8	-1
4	-.78
5	-.8

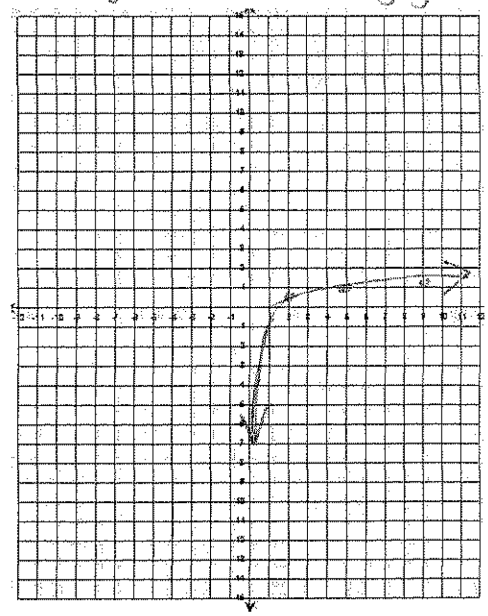
Domain: $x > -2$
Range: all real #s
Asymptotes: $x = -2$
Description: left 2, reflected over x-axis



7) $y = \log_{10} 2x$

5	1
9	1.26
2	0.6
3	.78
1	.3

Domain: $x > 0$
Range: all real #s
Asymptotes: $x = 0$
Description: (2 times wider) Stretched horizontally by 2

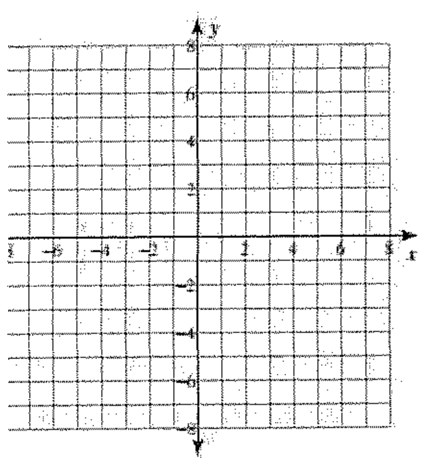


Day ~~9~~ 12: Common Logs: Introduction to Solving Equations and Word Problems

Warm-up: Key Features of Logarithmic Functions

Graph the following function. Identify the domain, range, asymptotes, and transformations of the parent function $y = \log_2 (x)$.

$$y = \log_2 (4x - 11) - 2$$



Domain:

Range:

Asymptote:

Description of transformations: