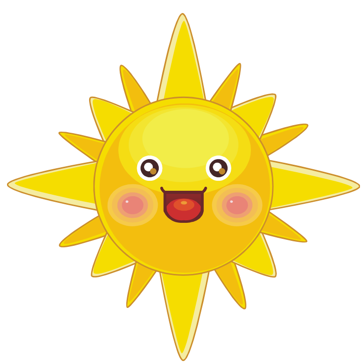
Unit 3 Day 11

More on Inverses of Functions (Logarithms & Exponentials)



Warm Up

Day 10 Warm-up: Inverses…Find the algebraic inverse.

1. $f(x)=15x-1$ y = 1/15x + 1/15	2. $f(x)=\frac{4}{7}x$ y = 7/4x
3. $f(x)=\frac{1}{3}x+7$ y = 3x - 21	4. $f(x)=-5x-11$ y = -1/5x - 11/5
5. $f(x)=(x-2)^2$ y = $x^{1/2}$ + 2	6. $f(x)=\sqrt{x-4}$ y = x^2 + 4

Extra one…. I-123 is used in thyroid scans. 13.2 hours after the formation of a 45 mg sample, only 22.5 mg remain. How much of the I-123 will remain after 66 hours? Show your work algebraically.

Half life in disguise!! ☺

$$y = 45\left(\frac{1}{2}\right)^{\frac{x}{13.2}} \qquad y = 45\left(\frac{1}{2}\right)^{\frac{66}{13.2}}$$

$\boxed{1.41 \text{ mg}}$

Tonight's Homework:

Packet pages 17 & 18 all
&
19-21 odds

Homework Answers for HW p. 19 – 21 evens

p. 19

2. $x = \{-5, 5\}$

4. $x = 169$

6. $x = 9$

8. $x = 7/3$

p. 20

2. $x = 8$

4. $x = -4$

6. $m = 8$

8. $b = -1$

10. $k = 4$

p. 21

12. $b = 1$

14. $p = \{2, 5\}$

16. $v = 4$

18. $x = \{-1, -3\}$

20. $b = 6$

Homework Answers for HW p. 24 – 27 Starred Problems

p. 24

2. $\frac{1}{2}k^{\frac{3}{2}}$ 4. $m\sqrt[6]{m^5}$

6. $8x^3\sqrt{2x}$

8. c. $2^{\frac{5}{2}}x^2$ d. $2^{-1}m^{-3}$

9. c. $k\sqrt[6]{k^5}$ d. $9xy\sqrt{x}$ e. $-6y^2\sqrt[4]{y}$

11. a. x = 27 c. x = 0, -1 e. x = 64 h. no solution

Homework Answers for HW
p. 24 – 27 Starred Problems

p. 25-27

14. a. $x = -5/4$ c. $x = 15$

18. Answer will vary. Example: The price of a new car is $40,000 and its value depreciates 5% every year.

21. a. 1/3 b. 0 d. -.4375 or -7/16

26.

X	Y
-7	4
-4	1
3	0
12	-2

*Inverse notation $g(x)^{-1}$

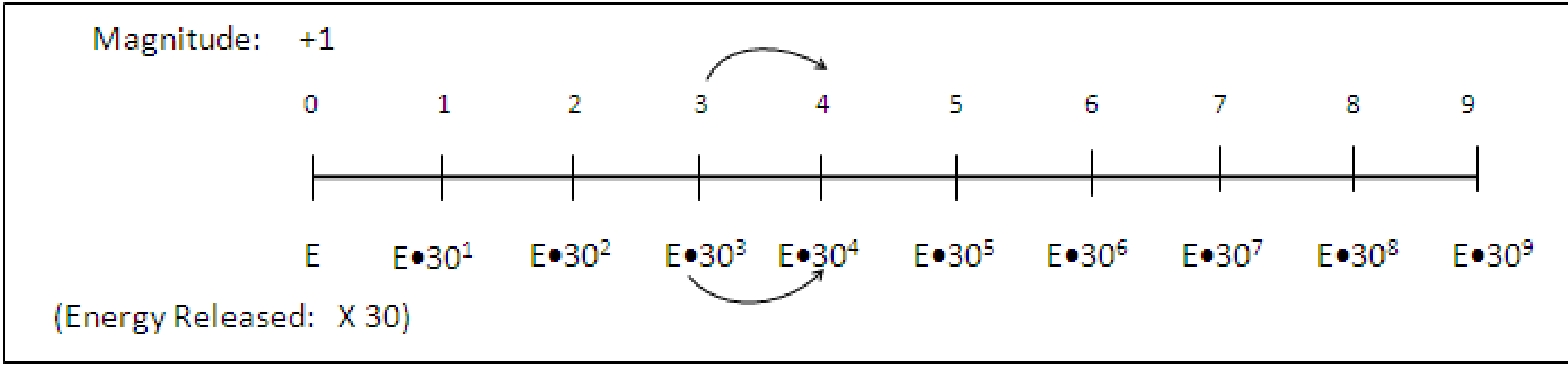
Today's Notes

1st) Logarithm Graphs

2nd) Converting between Logarithms and Exponentials

You need Graph Paper & the printed notes

The Richter Scale



In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. Let's compare the amounts of energy released in the two earthquakes.

For the earthquake in Mexico at 8.0 on the Richter Scale, the energy released is $E \bullet 30^{8}$ and for the earthquake in Washington state, the energy released is $E \bullet 30^{6.8}$. A ratio of the two quakes and using the properties of exponents yields the following:

$$\frac{\text{Mexico Eartquake}}{\text{Washington Earthquake}} = \frac{E \cdot 30^{8}}{E \cdot 30^{6.8}} = \frac{30^{8-6.8}}{} = 30^{1.2} = 59.23$$

This means the earthquake in Mexico released about 59.23 times as much energy as the one in Washington.

The exponents used by the Richter scale in the example are called **logarithms** or logs.

Guided Practice with Logarithmic Functions

In 1812, an earthquake of magnitude 7.9 shook New Madrid, Missouri. Compare the amount of energy released by that earthquake to the amount of energy released by each earthquake below.

1) Magnitude 7.7 in San Francisco, California, in 1906.

$$\boxed{\frac{New\,Madrid}{San\,Francisco} = \frac{E \cdot 30^{7.9}}{E \cdot 30^{7.7}} = 30^{7.9-7.7} = 30^{0.2} = 1.97}$$

New Madrid was 1.97 times stronger.

2) Magnitude 3.2 in Charlottesville, Virginia, in 2001.

$$\boxed{\frac{New\,Madrid}{Charlottesville} = \frac{E \cdot 30^{7.9}}{E \cdot 30^{3.2}} = 30^{7.9-3.2} = 30^{4.7} = 8759310.01}$$

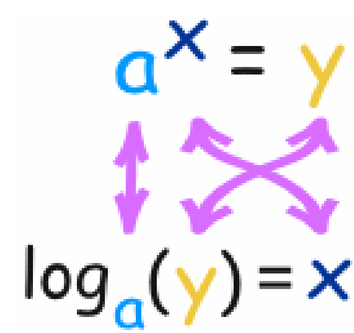
New Madrid was 8759310.01 times stronger.

Logs

The logarithm base *b* of a positive number *y* is defined as follows:

If $y = b^x$, then $\log_b y = x$.

b ≠ 1 and b > 0



So a logarithm answers a question like this:

$$2^{?} = 8$$

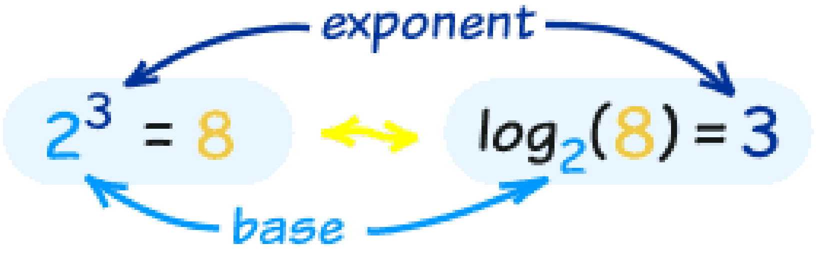
In this way:

$$2^{3} = 8$$

$$\log_2(8) = 3$$

The logarithm tells you what the exponent is!

In that example the "base" is 2 and the "exponent" is 3:



A little experiment…

Partner 1

- Graph $y = 10^x$ on one side of the graph paper.

  (pick a good scale)

- Use this table of points.

x	y
-2	
-1	
0	
1	
2	

Partner 2

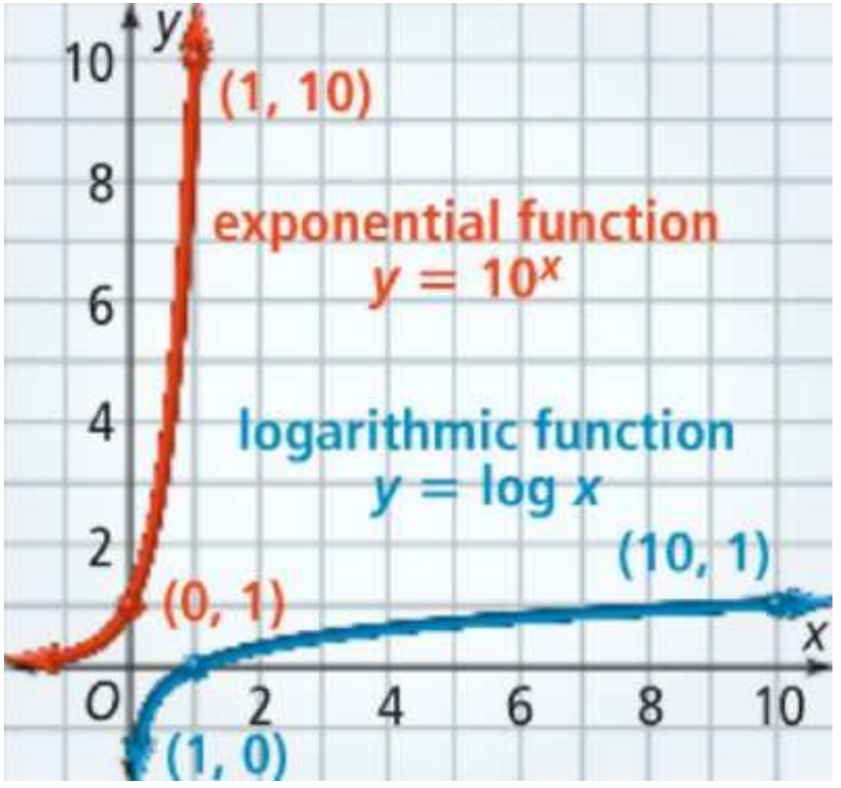
- Help partner 1 with the first graph.

- Make a new table by flipping the x's and y's from the $y = 10^x$ graph.

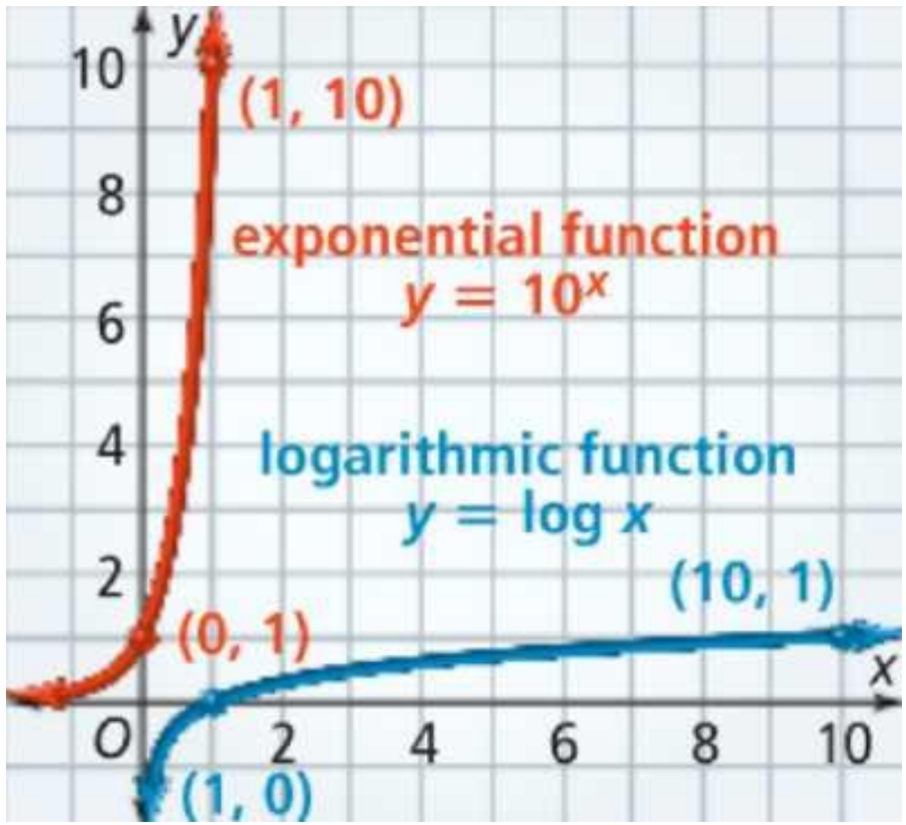
- Graph on the other side of the graph paper. (pick a good scale)

The log function is the inverse of the exponential function!

$y = 10^x$ and $y = \log_{10} x$



Key Features of Exponential and Logarithmic Functions		
Characteristic	Exponential Function $y = b^x$	Logarithmic Function $y = \log_b x$
Asymptote		
Domain		
Range		
Intercept		



Translations of logs are just like other functions!!

Parent Function	$y = \log_b x$
Shift up	$y = \log_b x + k$
Shift down	$y = \log_b x - k$
Shift left	$y = \log_b(x + h)$
Shift right	$y = \log_b(x - h)$
Combination Shift	$y = \log_b(x \pm h) \pm k$
Reflect over the x-axis	$y = -\log_b x$
Stretch vertically	$y = a \log_b x$
Stretch horizontally	$y = \log_b ax$

Remember the two types of functions we have already translated:

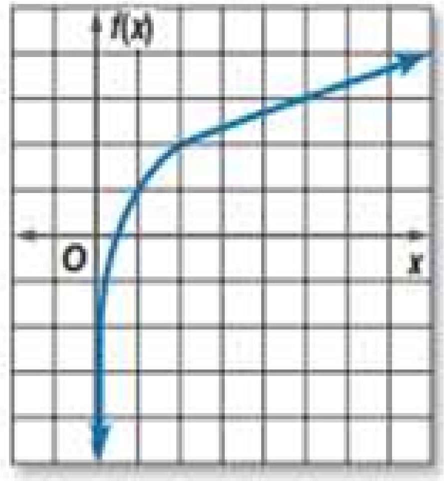
$$y = 2(x - 2)^2 + 3$$

$$y = 3^{x-2} + 3$$

Let's look at the following example.

The graph on the right represents a transformation of the graph of $f(x) = 3\log_{10} x + 1$.

- $|x| = 3$: Stretches the graph vertically.
- $h = 0$: There is no horizontal shift.
- $k = 1$: The graph is translated 1 unit up.



Vertical Asymptote: x = 0

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Foundational Example...

1.) $\log_{10}(x+1)$

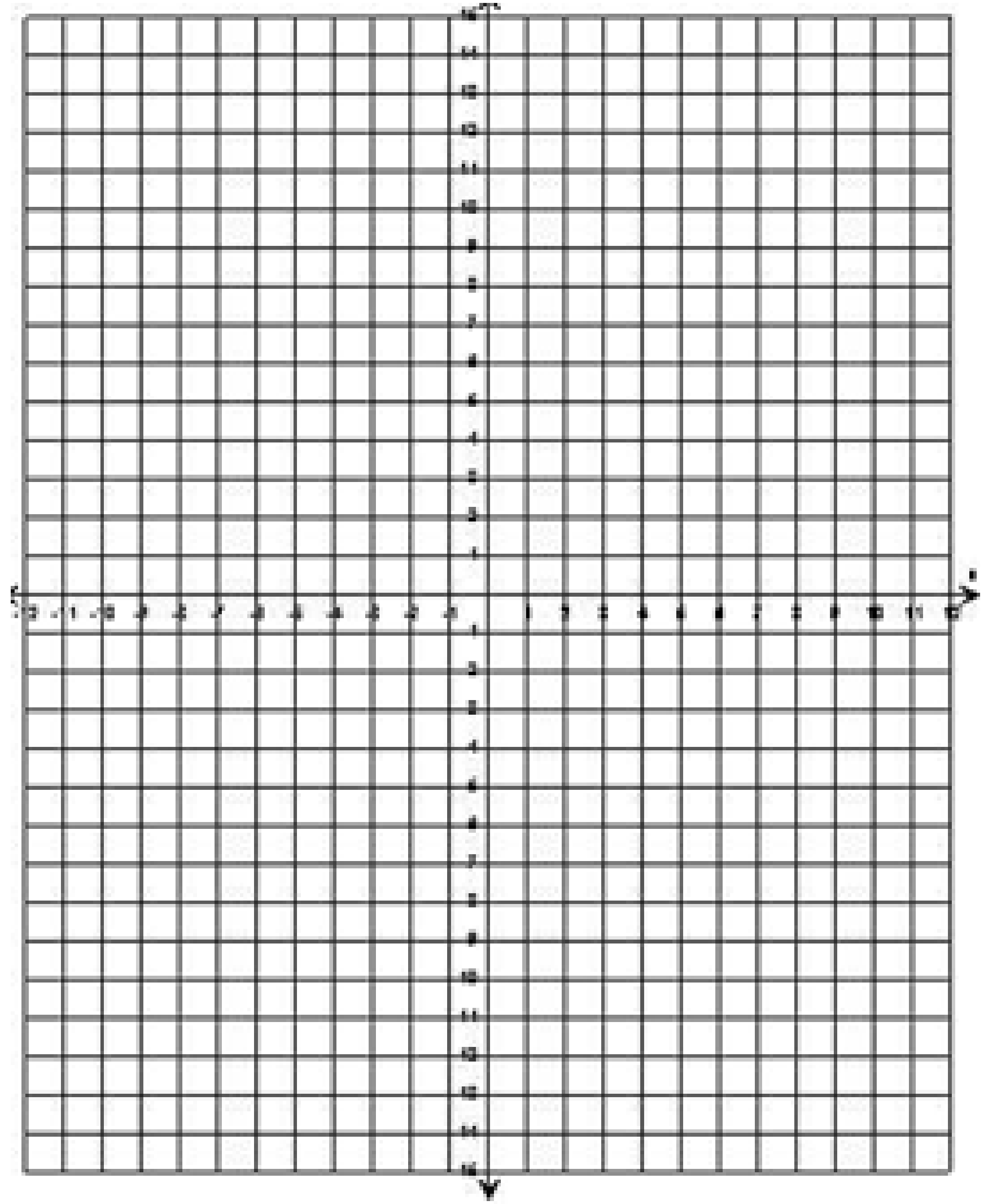
Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$

Asymptotes: **x = -1**

Description: **Left 1**

1. Graph on calc
2. Look at the table for pts.
3. Plot



Another example…

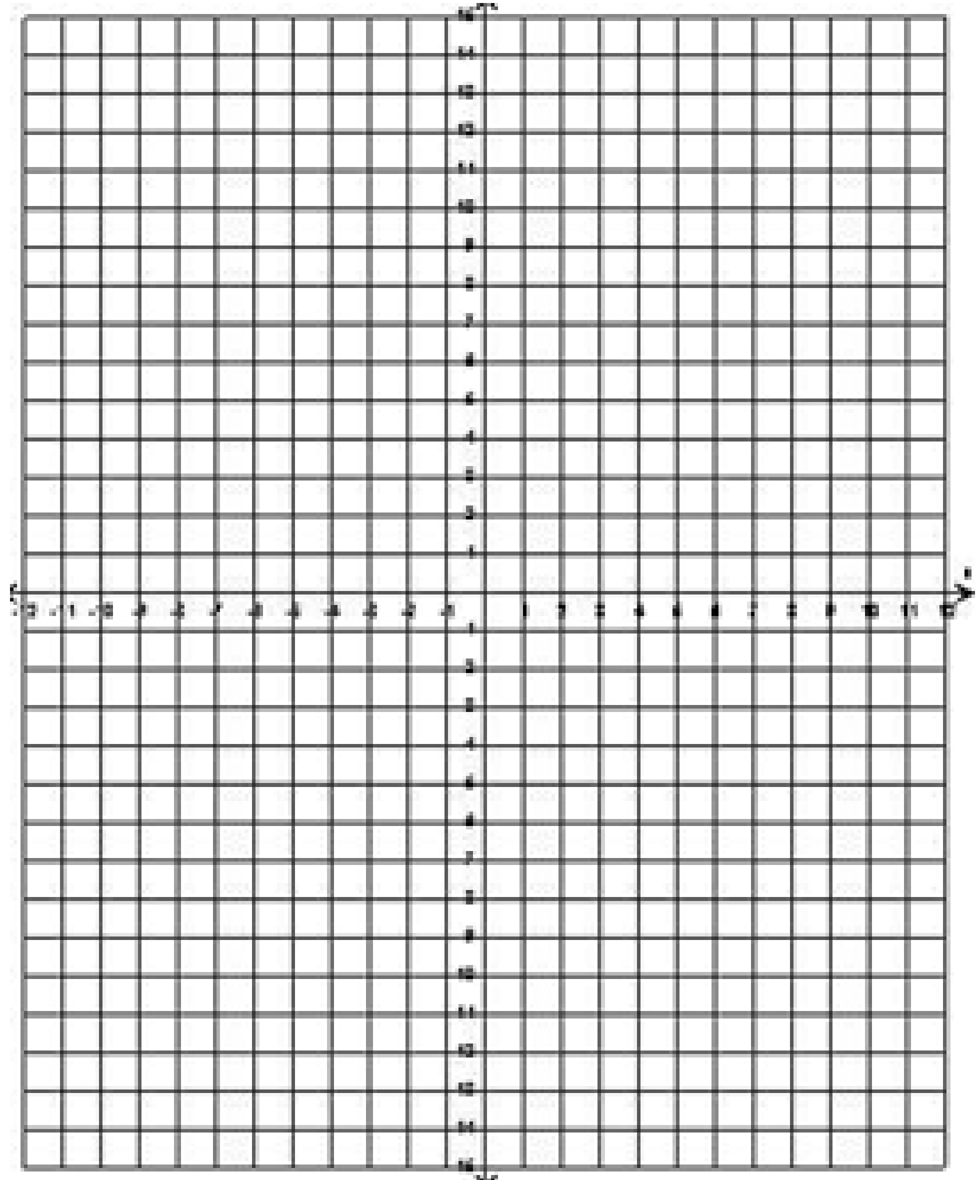
2.) $\log_{10}x + 1$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptotes: **x = 0**

Description: **Up 1**



A tip for tougher problems….

For equations that have transformations, remembering a key feature of logs can help with finding the domain and asymptote.

Remember the domain of logs MUST be positive!

Set the part in the parentheses > 0 and solve to find the domain AND to find the value of the asymptote.

Let's do this one again: $\log_{10}(x+1)$

Domain: **$x + 1 > 0$ so** $(-1, \infty)$

Asymptote: **$x = -1$**

Try Now…one a little tougher…

Graph the following function on the graph at right. Describe each transformation, give the domain and range, and identify any asymptotes.

$F(x) = -2\log_{10}(x + 2) - 4$

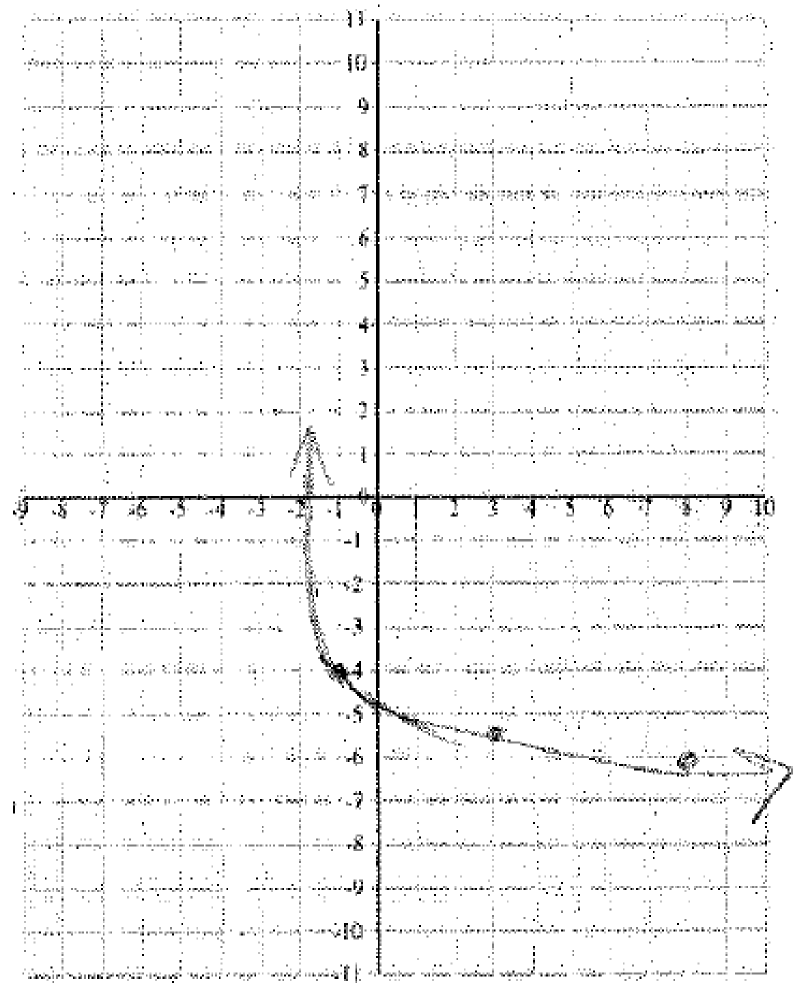
Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: **x = -2**

Description of transformations:

Reflection over x axis, stretched vertically by 2, left 2, down 4



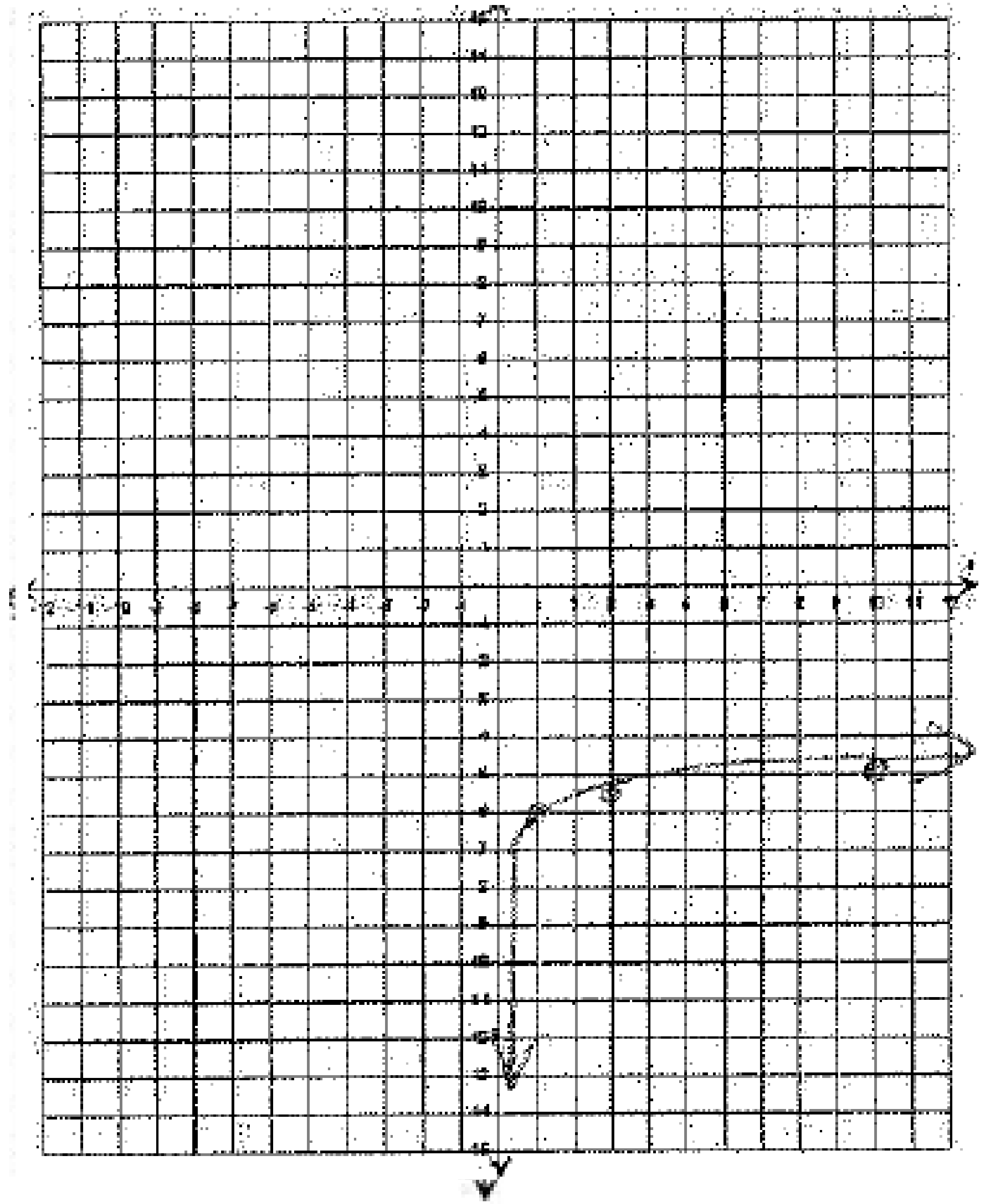
5) $y = \log_{10} x - 6$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptotes: **x = 0**

Description: **Down 6**



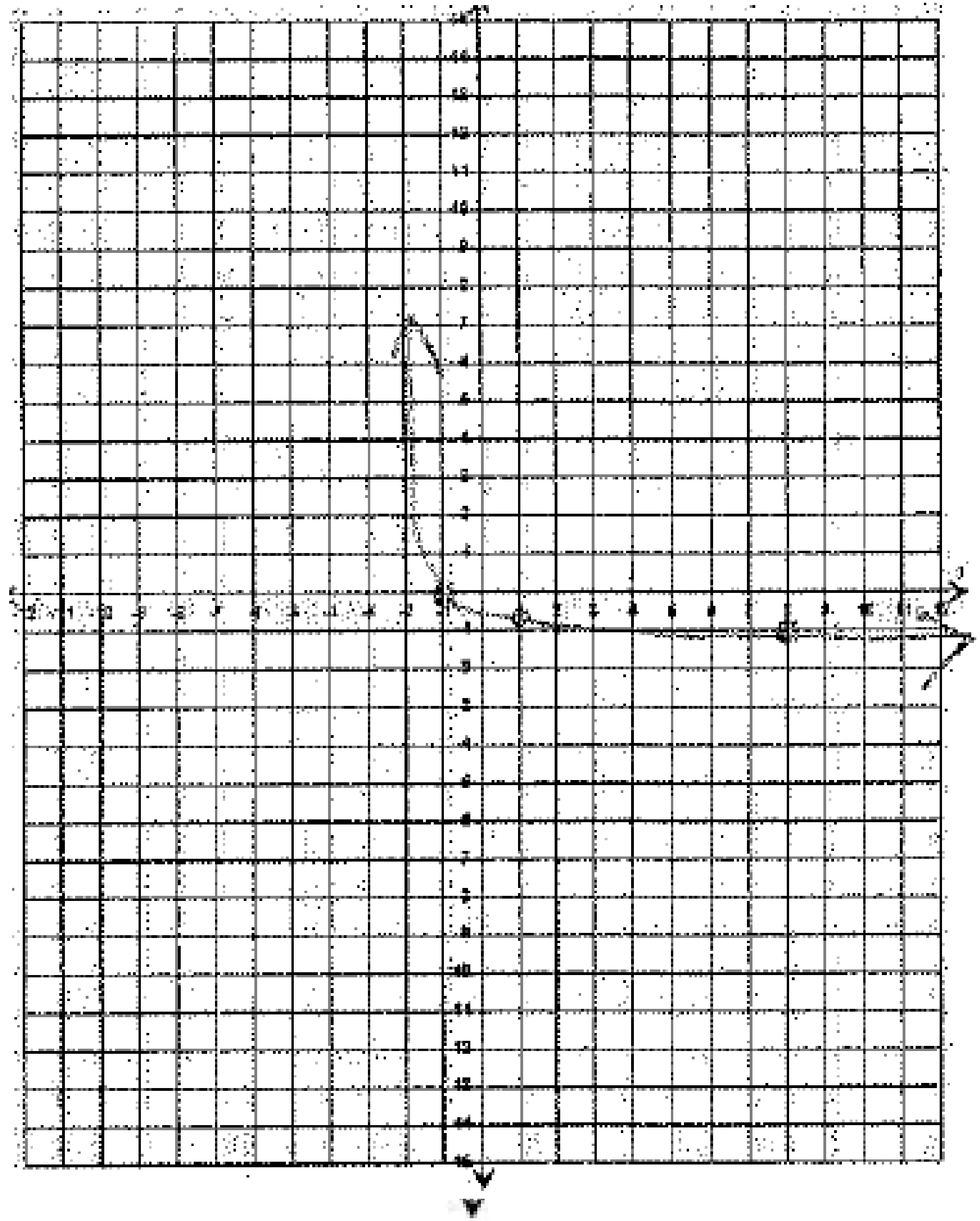
6) $y = -\log_{10}(x + 2)$

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptotes: $x = -2$

Description: Reflected over x-axis and left 2



7) $y = \log_{10} 2x$

Domain: $(0, \infty)$

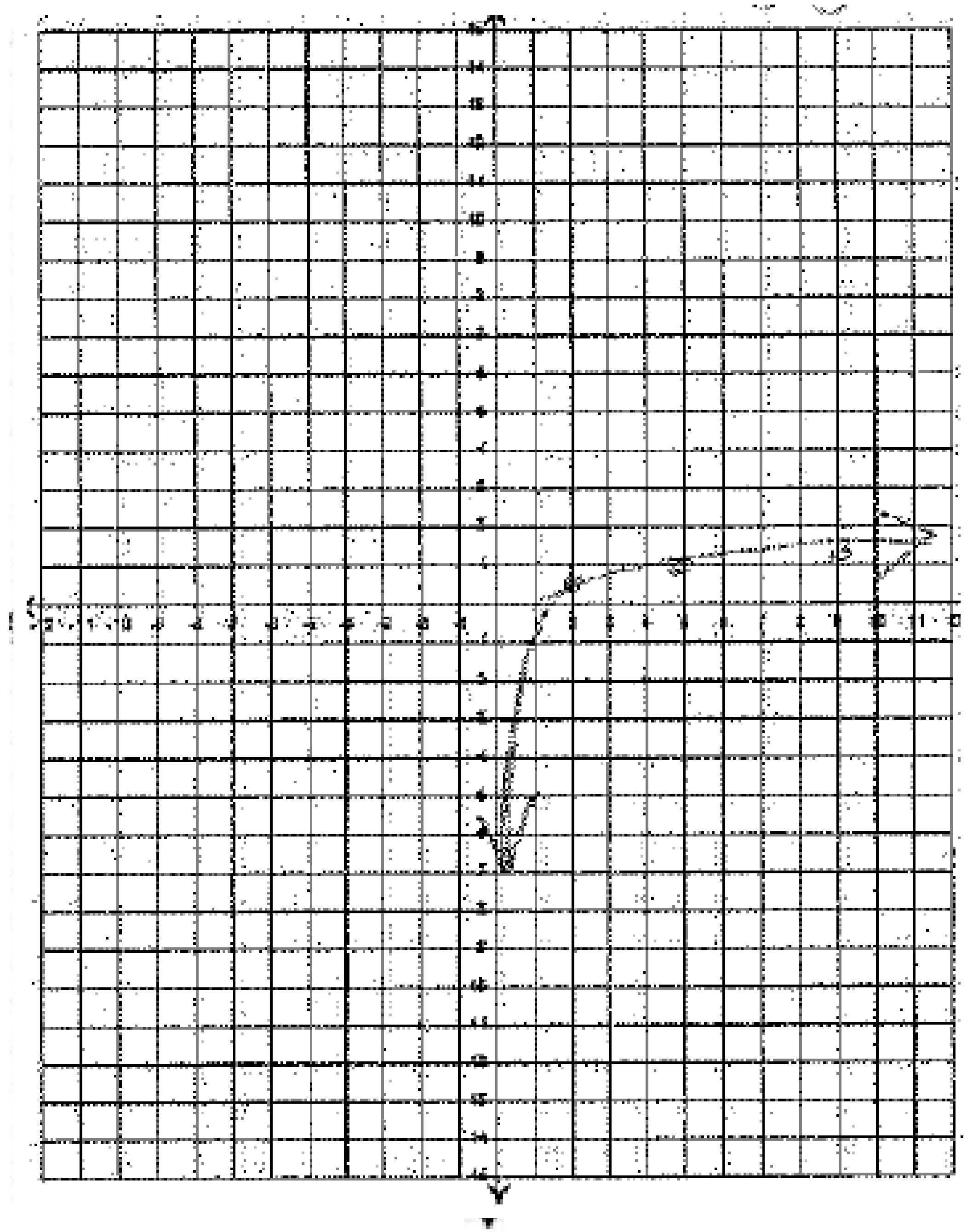
Range: $(-\infty, \infty)$

Asymptotes: **x = 0**

Description:

horizontal stretch by 2

(2 times wider)



Notes Part 2

Logarithms and Exponentials

Problem 1

Express each of the numbers below as accurately as possible as a power of 10.

a. 100 $10^x=100,\ x=2$ 10^{2}	b. 10,000 10^{4}	c. 1,000,000 10^{6}
d. 0.01 10^{-2}	e. -0.001 -10^{-3}	f. 3.45 $10^{0.538}$
g. -34.5 $-10^{-1.538}$	h. 345 $10^{2.538}$	i. 0.0023 $10^{-2.63}$

Common Logarithms

We have already discussed logarithms in the previous lesson, but in this lesson we will solve exponent problems which focus on a base of 10.

When we solve logarithmic problems in base 10, we call them **common logarithms.**

The definition of common logarithms is usually expressed as:
$\log_{10} a = b$ if and only if $10^b = a$

Use the "Log Loop" to convert between forms.

$\text{Log}_{10}\ a$ is pronounced "log base 10 of a".

Because base 10 logarithms are so commonly used, $\log_{10} a$ is often written as log a. Most calculators have a built-in log function that automatically finds the required exponent value.

You Try!

1. Use your calculator to find the following logarithms. Then compare the results with your work on Problem 1

a. log 100 **= 2** because $10^2 = 100$	b. log 10,000 **= 4** $10^4 = 10{,}000$	c. log 1,000,000 **= 6** $10^6 = 1{,}000{,}000$
d. log 0.01 **= -2** $10^{-2} = 0.01$	e. log -0.001 **Non real answer!** Because domain $x > 0$	f. log 3.45 **= 0.538** $10^{0.538} = 3.45$
g. log -34.5 **Non real answer!** Because domain $x > 0$	h. log 345 **= 2.538** $10^{2.538} = 345$	i. log 0.0023 **= -2.638** $10^{-2.638} = 0.0023$

Continue to question # 2 ☺ >

2) What do your results from Problem 2 (especially Parts e and g) suggest about the kinds of numbers that have logarithms? See if you can explain your answer by using the connection between logarithms and the exponential function $y = 10^x$.

Domain of logs must be **positive** because a base that is positive raised to a power can never give you an answer that is a negative number.

Tonight's Homework:

Packet pages 17 & 18 all
&
19-21 odds