Day ~~7~~ 11: Transformations of Graphs…kf(x) and f(kx)

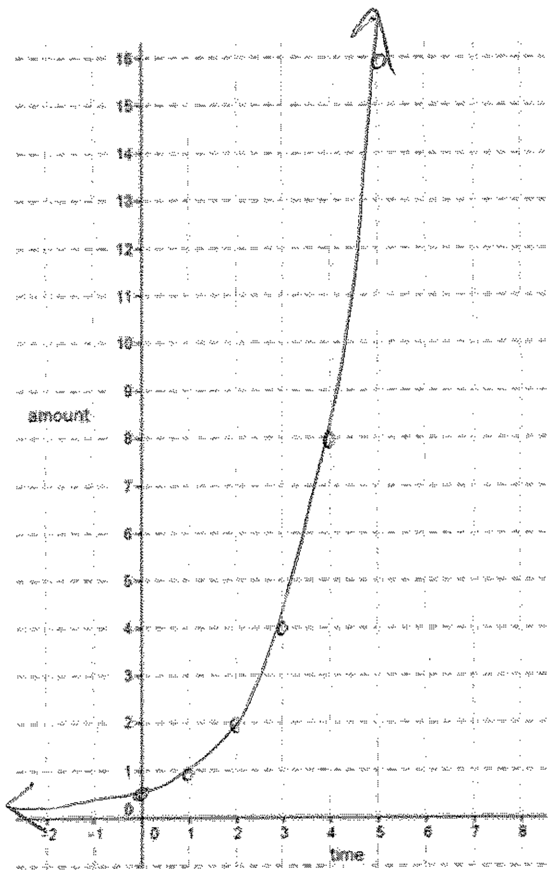
Warm-Up Day 7: Transformations

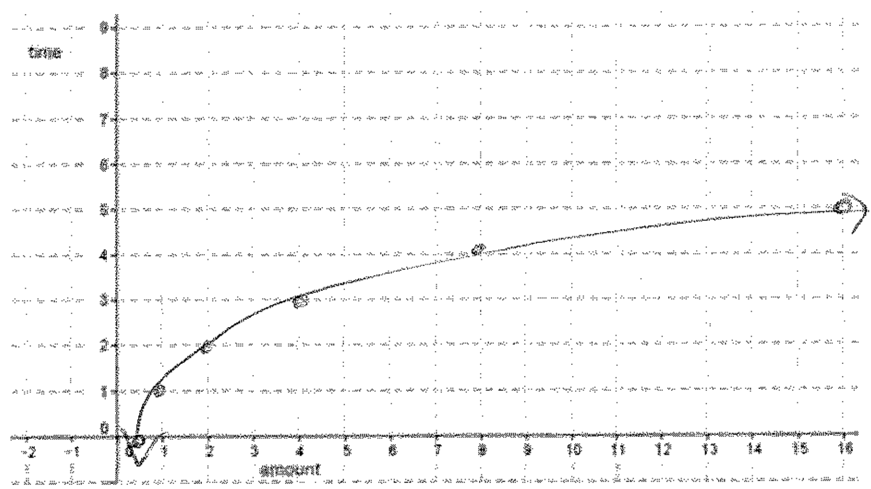
How are the following graphs changed from their parent graph, $y = x^2$? (If you don't remember some of the transformations, graph the equation and the parent in the calculator ☺)

1) $y = (x - 4)^2$ right 4
2) $y = x^2 - 4$ down 4
3) $y = x^2 + 1$ up 1
4) $y = (x + 3)^2$ left 3
5) $y = 4x^2$ 4 times narrower (stretched vertically by 4)
6) $y = \frac{1}{2}x^2$ 2 times wider (stretched horizontally by 2)
7) $y = 3(x - 4)^2$ right 4, 3 times narrower (stretched vertically by 3)
8) $y = 1/4\, x^2 + 3$ up 3, 4 times wider (stretched horizontally by 4)

9) Use the following data to graph on each of the following coordinate planes. Be sure to pay attention to the labels on each axis.

Time	Amount of radioactive material
0	½
1	1
2	2
3	4
4	8
5	16





Explain how the graphs are alike.

have curve shape, level off at one place

Are different.

are reflections over y=x, 1st flattens at y=0, 2nd flattens at x=0
HA: y=0 VA: x=0

The Lesson

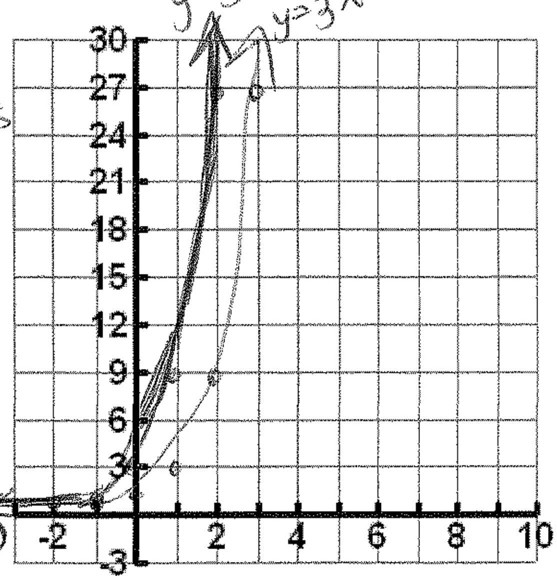
Graph the parent function $y = (3)^x$ using a table of x and y values. Also, CLEARLY INDICATE the horizontal asymptote.

Domain → D: all real #s

x	y
-1	1/3
0	1
1	3
2	9
3	27

HA: $y=0$ R: $y>0$

asymptote: a line that a function approaches, getting closer and closer



On the same grid, graph $y = (3)^{x+1}$ using a different color or mark. Also, CLEARLY INDICATE the horizontal asymptote.

← Be careful in calculator $y=3^\wedge(x+1)$

x	y
-2	1/3
-1	1
0	3
1	9
2	27

↑ these points are left 1 from the top table

HA: $y=0$ Domain: all real #s Range: $y>0$

Explain how the graph shifted from the parent graph:

left 1

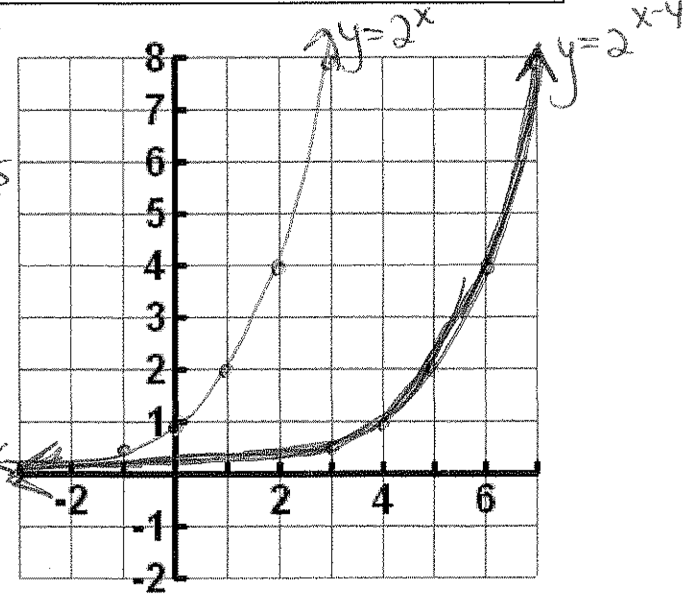
Graph the parent function: $y = (2)^x$

x	y
-1	1/2
0	1
1	2
2	4
3	8

HA: $y=0$

Domain: all real #s

Range: $y>0$



On the same grid, graph $y = (2)^{x-4}$ using a different color or mark.

Domain: all real #s

x	y
3	1/2
4	1
5	2
6	4
7	8

HA: $y=0$ Range: $y>0$

Explain how the graph shifted from the parent graph:

right 4

SUMMARY of Horizontal Translations

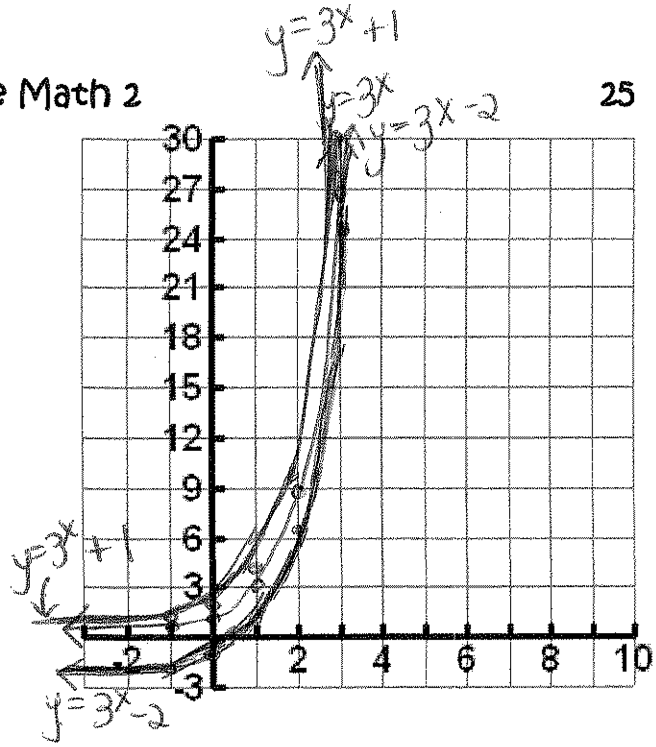
Adding "c" in the exponent shifts the graph to the left c units.

Subtracting "c" in the exponent shifts the graph to the right c units.

Graph the parent function again: $y = (3)^x$

x	y
-1	1/3
0	1
1	3
2	9
3	27

HA: y=0



On the same grid, graph $y = (3)^x + 1$ using a different color or mark.

x	y
-1	1 1/3
0	2
1	4
2	10
3	28

HA: y=1

Explain how the graph shifted from the parent graph: up 1

Range: y>1
Domain: all real #s

On the same grid, graph $y = (3)^x - 2$ using a different color or mark.

x	y
-1	-1 2/3
0	-1
1	1
2	7
3	25

HA: y=-2

Explain how the graph shifted from the parent graph: down 2

Range: y>-2
Domain: all real #s

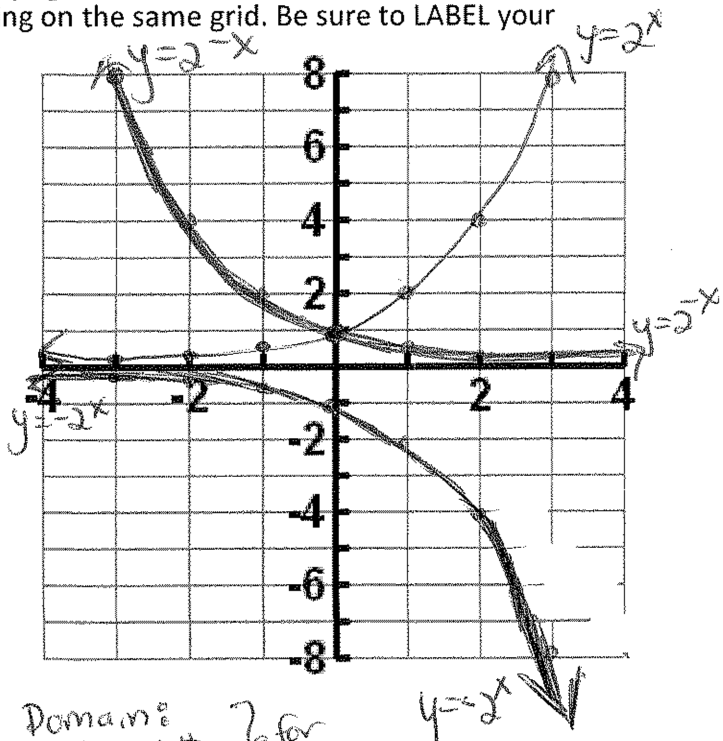
SUMMARY of Vertical Translations

Adding "c" to the whole equation shifts the graph up c units.

Subtracting "c" to the whole equation shifts the graph down c units.

Use a different color or mark to graph each of the following on the same grid. Be sure to LABEL your curves.

x	$y = (2)^x$	$y = (2)^{-x}$	$y = -(2)^x$
-3	1/8=.125	8	-1/8=-.125
-2	1/4=.25	4	-1/4=-.25
-1	1/2=.5	2	-1/2=-.5
0	1	1	-1
1	2	1/2	-2
2	4	1/4	-4
3	8	1/8	-8
HA:	y=0	y=0	y=0
Range:	y>0	y>0	y<0



Domain: all real #s } for all 3 graphs

Explain how the parent graph changed to get the graph of $y = (2)^{-x}$:

Explain how the parent graph changed to get the graph of $y = -(2)^{x}$:

SUMMARY of Reflections:

Negative on x causes the graph to reflect in the opposite direction over the y – axis.

Negative on the front of the equation causes the graph to reflect in the opposite direction over the x – axis.

Day 7 Part 2: Inverses of Functions

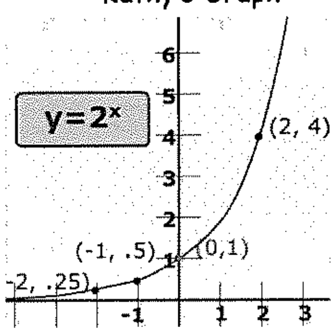
The word "inverse" is used in several different ways in mathematics. For example, we say that -6 is the additive inverse of 6 because -6 + 6 = 0. Essentially adding the inverse of -6 has the effect of undoing the addition of 6. You can think of this property as a way of retrieving the original number.

Similarly, we say that (7) is the multiplicative inverse of (1/7) because (7/1)(1/7) = 1. Multiplying by 7 and then by (1/7) has the effect of undoing the multiplication by 7 and retrieving the original number.

Look at the situation below:

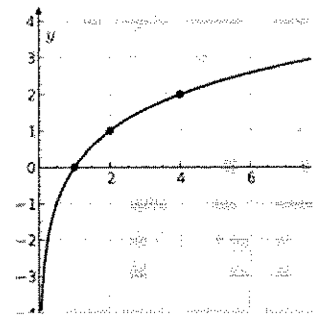
Kathy and Kevin are sharing their graphs for the same set of data. Both students insist that they are correct, but their graphs are different. They have checked and re-checked their data and graphs. Can you explain what might have happened? Has this ever happened to you?

Kathy's Graph



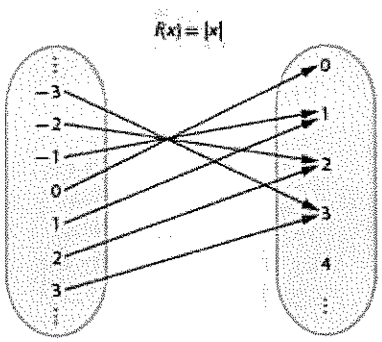
they reversed the x's and y's

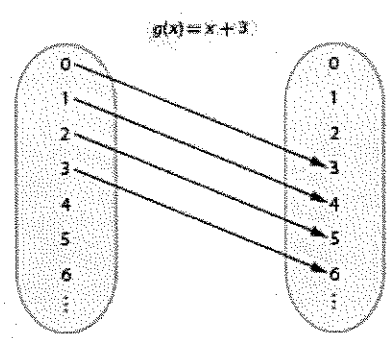
Kevin's Graph



You are correct if you thought the independent and dependent variables were just reversed. In mathematics, when the independent and dependent variables are reversed, we have what is called the **inverse of the function**. If you look at Kathy and Kevin's graph again, you might notice that the ordered pairs have been switched. For example in Kathy's graph the ordered pairs of (0,1) and (2,4) are (1,0) and (4,2) in Kevin's graph.

We talked about inverse operations above and how they help to undo the operation. The inverse of a function helps to get back to an original value of x. Do all functions have inverses? Sometimes this is best explained by looking at a mapping of functions, as shown in the illustration below.





How can you use a mapping diagram like those shown to decide whether a function does or does not have an inverse function?

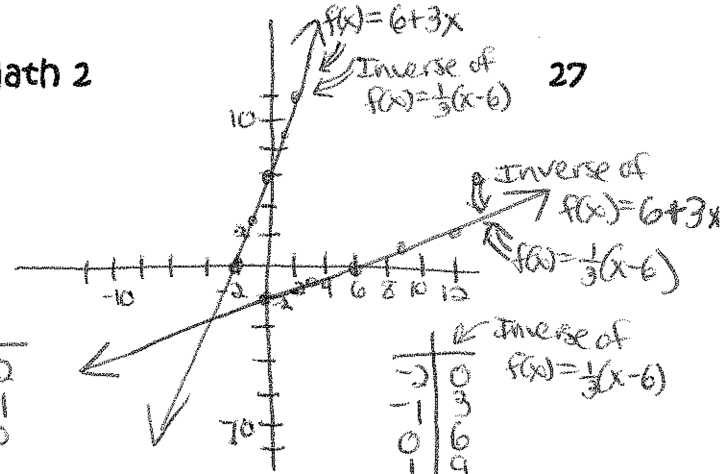
See if mapping from y's to x's is 1-1

Does $f(x) = |x|$ have an inverse?

No. There's not a 1-1 mapping from y's to x's.

Does $g(x) = x + 3$ have an inverse?

Yes. There's a 1-1 mapping back from y's to x's.



Investigation: Inverses

Consider the following functions:

-2	0	0	-2
-1	3	3	-1
0	6	6	0
1	9	9	1
2	12	12	2

Inverse

1) $f(x) = 6 + 3x$ 2) $f(x) = \frac{1}{3}(x-6)$

$y = 6+3x$ $y = \frac{1}{3}(x-6)$

0	-2
3	-1
6	0
9	1
12	2

-2	0
-1	3
0	6
1	9
2	12

Part 1: Graphs of Inverse Functions

For each of the functions above, follow steps 1 – 4.

1) Make a table of 5 values and graph the function on a separate sheet of graph paper.
2) Make another table by switching the x and y values and graph the inverse on the same coordinate plane.
3) What do you notice about the two graphs? the inverse of #1 is graph #2; the inverse of #2 is graph #1
4) What line are the inverses reflected over? $y = x$

Part 2: Equations of Inverse Functions

We saw in part 1 of the investigation that functions 1 and 2 are inverse functions. We also know that we can find inverses of tables by switching the x and y values in a table. So the question we want to explore now is how to find the equation of an inverse function.

5→1) $x = 6+3y$
+6
$\frac{x-6}{3} = \frac{3y}{3}$
$\frac{x-6}{3} = y$
or $\frac{1}{3}(x-6) = y$

5→2) $x = \frac{1}{3}(y-6)$
+6
$3x = y-6$
+6 +6
$3x+6 = y$
or
$6+3x = y$

For each of the functions above, follow steps 5 – 6.

5) Take the function and switch the x and y values.
6) Then solve for y.

The equation for the function 1's inverse should be function 2 and the inverse for function 2 should be the equation for function 1.

$\frac{1}{3}x - 2 = y$

This process will work for any function which has an inverse. So, let's try some different types in the problems below.

NOW TRY

Graph $y = x^2 + 3$ and find the inverse by interchanging the x and y values of several ordered pairs. Is the inverse a function? Check by graphing both $y = x^2 + 3$ and the inverse on the graph on the right.

x	$y=x^2+3$
-2	7
-1	4
0	3
1	4
2	7

$x = y^2+3$
$x-3 = y^2$
$\pm\sqrt{x-3} = y$

*remember to do ± if you put in the square root!

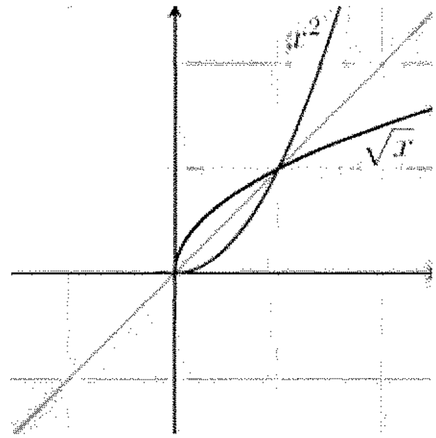
	$y=\sqrt{x-3}$
3	0
4	1
7	2

3	0
4	-1
7	-2

$y = \pm\sqrt{x-3}$ is not a function (Doesn't pass vertical line test; Each x doesn't map to 1 y)

Do Day 8 before new lesson ... reviews getting inverses & VLT

However, if you consider half of the parabola, then...



Let's look at a simpler function $y = x^2$. If you only consider the positive values of x in the original function, the graph is half of a parabola. When you reflect the "half" over the y = x line, it will pass the vertical line test as shown on the right.

Algebraically, let's switch the x and y values and solve for y.

Function: $y = x^2$, where $x \geq 0$

Switch the x and y values: $x = y^2$

$\pm\sqrt{x} = \sqrt{y^2}$

Solve for y: $\pm\sqrt{x} = y$ BUT if $x \geq 0$, then $\sqrt{x} = y$

As shown, ~~the~~ both graphs pass the vertical line tests and are inverse functions of each other. The above example means that if you restrict the domains of some functions, then you will be able to find inverse functions of them.

fix

NOW TRY

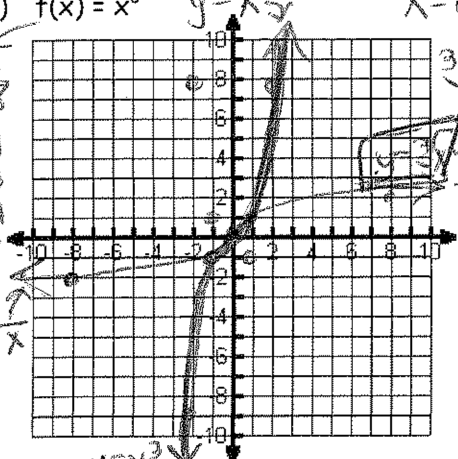
Find the inverses of the functions below. Graph the function and its inverse ~~on graph paper~~ below.

fix

1) $f(x) = x^3$

$y = x^3$ $x = y^3$ $\sqrt[3]{x} = y$

x	y
-2	-8
-1	-1
0	0
1	1
2	8



x	y
-8	-2
-1	-1
0	0
1	1
8	2

2) $y = -3x + 4$

$x = -3y + 4$

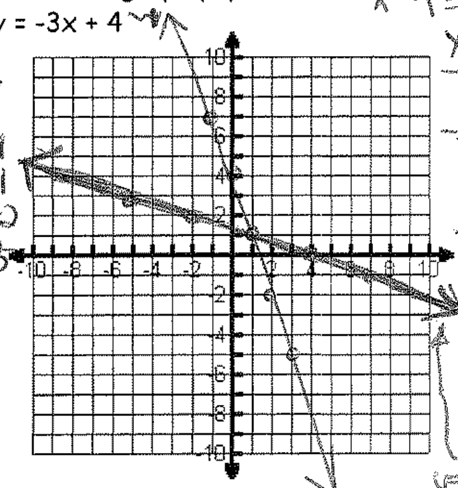
$x - 4 = -3y$

$\frac{x-4}{-3} = y$

$-\frac{1}{3}(x-4) = y$

$-\frac{1}{3}x + \frac{4}{3}$

x	y
-1	7
0	4
1	1
2	-2
3	-5



x	y
-5	3
-2	2
1	1
4	0
7	-1

$y = -\frac{1}{3}(x-4)$

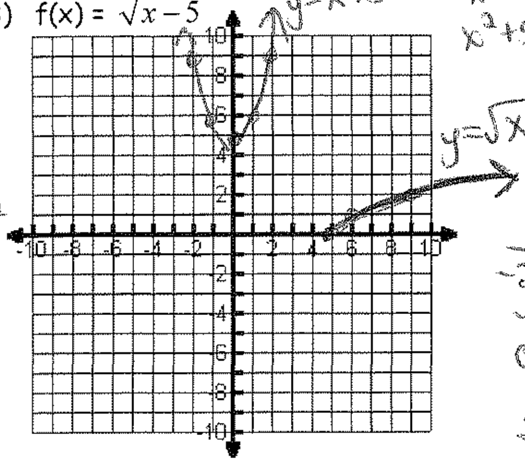
3) $f(x) = \sqrt{x-5}$

$x = \sqrt{y-5}$

$x^2 = y - 5$

$x^2 + 5 = y$

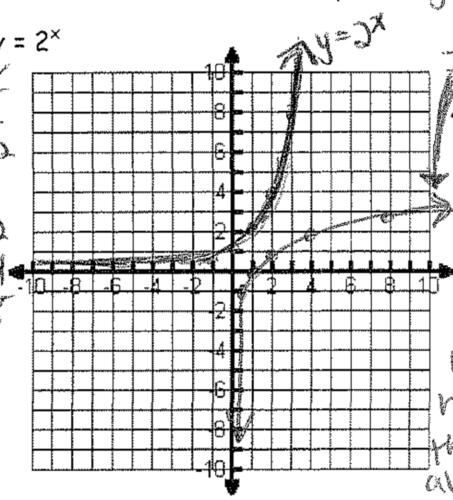
x	y
5	0
6	1
9	2



x	y
-2	9
-1	6
0	5
1	6
2	9

4) $y = 2^x$

x	y
-1	½
0	1
1	2
2	4
3	8



Inverse

x	y
½	-1
1	0
2	1
4	2
8	3

We'll learn how to get the inverse algebraically later