Day 1: Factoring Review and Solving For Zeroes Algebraically

Warm-Up:

1. Write an equivalent expression for each of the problems below:

   a. $(x + 2)(x + 4)$

   b. $(x – 5)(x + 8)$

   c. $(x – 9)^2$

   d. $(x + 10)^3$

   e. $(x – 8)(x + 8)$

   f. $(x – 3)(x + 2)(x – 4)$

3. Simplify the following polynomial expressions

a. $(2x^3 + 4x^2 - 3x + 8) – (6x^2 + 5x - 7) + 4x^3$

b. $(5 + 30x – 16x^2) + (4x^3 + 6x^2) – (25x - 7)$

Solving Quadratics Algebraically Investigation

Instructions: Today we will find the relationship between 2 **linear** binomials and their product, which is a **quadratic** expression represented by the form $ax^2 + bx + c$. First we will generate data and the look for patterns.

Part I. Generate Data

1. Use the distributive property to multiply and then simplify the following binomials.

a. $(x+3)(x+5)$

b. $(x+4)(x-2)$

c. $(x-1)(x-2)$

2. Where do you expect each of the above equations to “hit the ground” or “Intersect with the x-axis”?

Part II. Organize Data

Fill in the following chart using the problems from above

FACTORS	PRODUCT ax^2+bx+c	a	b	c
$(x+3)(x+5)$		1	8	15
$(x+4)(x-2)$				
$(x-1)(x-2)$				

Part III. Analyze Data

Answer the following questions given the chart you filled in.

1. Initially, what patterns do you see?

2. How is the value of "**a**" related to the factors you see in each problem?

3. How is the value of "**b**" related to the factors you see in each problem?

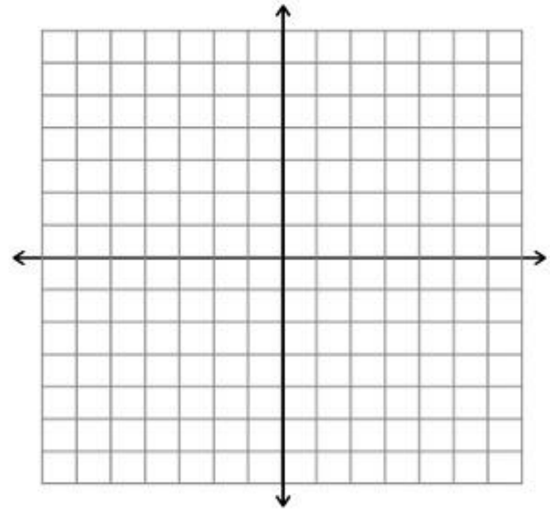
4. How is the value of "**c**" related to the factors you see in each problem?

BEFORE COMPLETING PART IV, DISCUSS WITH THE GROUP YOUR ANSWERS TO PART III

Part IV: Application

Knowing this, fill out the values for a, b, and c in the following chart. Work backwards using your rules from part III to find 2 binomial factors for each product. Put these in the first column.

FACTORS	PRODUCT ax^2+bx+c	a	b	c	Hint: list factors of "c"
$(x+4)(x+\quad)$	x^2+6x+8				
	$x^2+7x+12$				
	$x^2+13x+12$				
	$x^2+3x-10$				
	$x^2-3x-10$				
	$x^2-15x+54$				



For each of the quadratics from the previous chart, use your graphing calculator to inspect where the quadratic “hits the ground”, or touches the x-axis.

1. What do you notice about the relationship between the factors and the x-intercepts?

2. Why is factoring a useful skill to learn?

3. Choose one of the quadratics and create a rough sketch of the graph using all the information.

❖ **Summary: Factoring Polynomials**

- ALWAYS factor out the _______________ _____________ _____________ (______) FIRST!!!
- A polynomial that can not be factored is _____________.
- A polynomial is considered to be completely factored when it is expressed as the product of ___________ polynomials.

A. Factoring out the Greatest Common Factor or GCF:

i. $16m^2n + 12mn^2$

ii. $14a^3b^3c - 21a^2b^4c + 7a^2b^3c$

iii. $-36x^4z^2 - 24x^2zy + 6x^2z^3y$

B. Factor by grouping—for polynomials with 4 or more terms

i. $3x^3 + 2xy - 15x^2 - 10y$

ii. $20ab - 35b - 63 + 36a$

C. Factoring trinomials into the product of two binomials when leading coefficient is one

(On Day 2, we’ll do ones where the leading coefficient is not one)

i. $x^2 + 5x + 4$

ii. $x^2 + 6x - 16$

D. Difference of “Two Squares”

i. $x^2 - 25$

ii. $16x^4 - z^4$

iii. $16x^2 - 36$

* Remember: When an expression does not include an “x” term then it is known to be “0x”. *

Day 1: Factoring Practice – Factor the following on a separate sheet of paper.

1) x^2+4x+4	2) x^2+5x+6
3) x^2-6x+9	4) $x^2-7x+12$
5) $x^2 - 11x + 30$	6) x^2+x-6
7) $x^2 + 3x - 18$	8) $x^2 - 2x - 15$
9) $x^2 - 9$	10) $x^2 - 16$
11) $3x^2+18x+15$	12) $4x^2-24x+20$
13) $3x^2+x$	14) $5x^3-5x$
15) $9x^2-36$	16) x^3-3x^2+2x-6
17) $x^3-5x^2-3x+15$	18) $-20x^3y^2-10xy-25x^2y$

Day 2: Factoring Review and Solving For Zeroes Algebraically

Warm-Up: Factor the following on your own sheet of paper.

1. $x^2 + 13x + 40$
2. $4x^2 - 100$
3. $x^2 + 5x - 6$
4. $x^2 - 5x - 14$
5. $2x^2 - 8x$
6. $-3x^3 + 12x$

Day 2: Solving Quadratics Algebraically

Review from Math 1:
Graph the equation $y = x^2 + 13x + 40$ on your calculator. Use your calculator to find the zeroes: x = ____ and x = ____
From the warm-up, the factors of $y = x^2 + 13x + 40$ are $(x + 5)(x + 8)$.
Set each factor equal to zero and solve for x.

$x + 5 = 0$ $x + 8 = 0$
x = x =

What do you notice about your answers and the zeroes you found earlier on your calculator?

Summary: To Solve a Quadratic with your Calculator (Review From Math 1)

Enter the equation into "y =" and use the "zero" function. When you set y=0 and the other expression equal to y, what are we trying to find?

What does the intersection of two lines mean? How does this connect to factoring?

****You can use a table or a graph (intersection tool) on the calculator to determine an intersection! ****

To solve a quadratic algebraically (NEW From Honors Math 2)

1) Set the equation equal to zero by using the ___________________________
2) Factor the equation
3) Set each factor equal to zero and solve

Example 1: Solve $x^2 + 8x = -12$

Example 2: Solve $5x^2 - 4 = x - 4$

Practice – Solve each quadratic algebraically. Check your answers using the zero function on your calculator.

1. $x^2 + 4x = -4$ **2.** $x^2 + 5x = -6$ **3.** $x^2 + 9 = 6x$ **4.** $3x^2 = -6x$

Day 2: Factoring when a ≠ 1 (Busting the "B")

What if the problem has "**a**" value (a leading coefficient) that is not equal to 1?

For example, $4x^2 + 8x + 3 = 0$ How can we algebraically find where this graph = 0?

The concept of ***un-distributing*** is still the same! First, set to 0 and identify a = ______, b = ______, c = ________

In this case we need to find out what multiplies to give us ***a* • *c*** but adds to give us ***b***.

Let's list all the factors of **4 • 3** or **12**:

1 • ____
2 • ____
3 • ____

Which one of those sets of factors of 12 also add to give us the b value, 8? ______________

Rewrite the original equation using an equivalent structure:

$$4x^2 + 8x + 3 = 0$$

$$4x^2 + 6x + 2x + 3 = 0$$

Remember!! It doesn't matter which order you write the factors in!

$$(4x^2 + 6x) + (2x + 3) = 0$$

Factor By Grouping! Group the first two and the last two!

$$2x(2x + 3) + 1(2x + 3) = 0$$

For each pair, find the GCF to "undistribute" what is common to both terms

$$(2x + 1)(2x + 3) = 0$$

Create binomial factors out of the GCFs (the undistributed factors on the fronts) and out of the repeated binomial factor

Use multiplication (box, distribution, or FOIL) to check that your answer is equal to what you started with!

Day 2: Practice when a ≠ 1 Solve the following quadratics.

1) $2x^2+5x+3=0$	2) $2x^2+9x+10=0$
3) $3x^2+18x+15=0$	4) $3x^2+13x=10$

Solve by taking the Square Root Examples

5. $x^2-144=0$ **6.** $x^2-28=0$ **7.** $2x^2-150=0$ **8.** $3x^2+27=0$

Day 2 Practice – Solve the Quadratics ON A SEPARATE SHEET OF PAPER

#1-7 Solve by Factoring.	
1. $x^2+5x-24=0$	**5.** $4x^2+7x-2=0$
2. $x^2-3x-28=0$	**6.** $9x^2+30x+24=0$
3. $3x^2+16x-12=0$	**7.** $24x^2+132x=0$
4. $4x^2+3x=0$	

#8-14 Solve by taking the Square Root	
8. $x^2=81$	**12.** $6x^2-72=0$
9. $x^2=25$	**13.** $3x^2-9=0$
10. $5x^2-20=0$	**14.** $2x^2+72=0$
11. $5x^2+5=0$	

Day 3: Finding Extrema of Quadratic Functions

Warm-Up:

1. Factor the following. Then solve.

   a. $x^2 - 5x + 50 = 0$ b. $x^2 + 3x = 10$ c. $2x^2 + 7x = -3$

2. Factor to solve the following:

   a. $x^2 + 2x - 35 = 0$ b. $2x^2 + x = 3$ c. $3x^2 + 10x = 8$

Finding Extrema using Zeros

Given the following trinomials, fill in the following table.

Polynomial	Factors of the Polynomial	Zeros	Average of the Zeros
9. $x^2 + 8x + 15 = 0$			
10. $x^2 - 13x + 42 = 0$			
11. $x^2 + 2x - 24 = 0$			

Analyze the Data:

1. Graphically inspect each polynomial for connections between the zeros and the graph. What patterns do you see?

2. Given the equation $x^2 - 2x - 35 = 0$, without looking at the graph, where would you expect the minimum to be located?

$x^2 - 2x - 35 = 0$ Set expression $= 0$ first!

$(x+5)(x-7) = 0$ Factor

$x + 5 = 0 \qquad x - 7 = 0$ Set each factor = 0 and solve

$x = -5, x = 7$

To find the ____________

$$\frac{-5+7}{2} = 1$$

$x = 1$

Average the zeros to find the x - value of the vertex

To find the ____________

(_________ or ________)

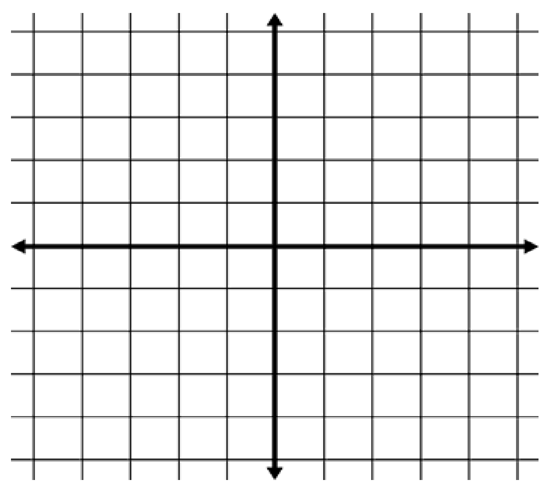
There are two numbers in an ordered pair.
Substitute the x-value into the original polynomial to find y-value

Our x-value for the minimum was x = 1. Substitute the 1 in for x in our original polynomial.

$$(1)^2 - 2(1) - 35 = 1 - 2 - 35 = -36$$

Our vertex, our minimum, is (1, -36)

To find a fourth point, substitute x = 0 into the polynomial. (0, ________).



Graph the four points from above with a smooth curve. Use your fourth point AND your knowledge of reflections & symmetry from Unit 1 for a fifth point.

What appears to be the line of symmetry on the graph? ______________

*Axis of symmetry: ____

*Remember, a minimum is the lowest point on a graph. A maximum is the highest point on a graph.

Tip to find Axis of Symmetry: There is another helpful way to find your Axis of Symmetry!

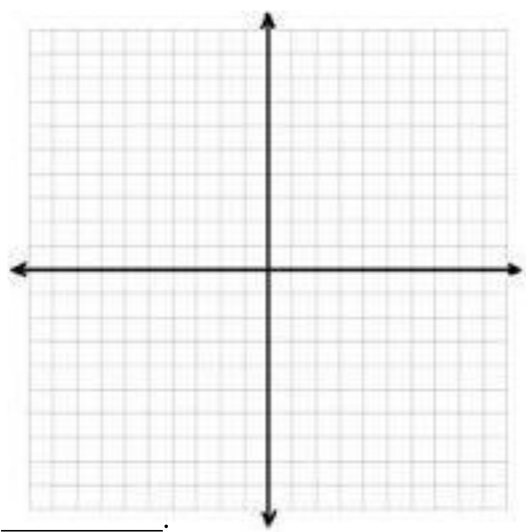
1. Write your equation in **Standard Form**: ________________________

2. Find a, b, and c and use formula: ______________________

Example: $y = x^2 - 2x - 35$, so a = 1, b = -2, and c = -35. Then use $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$.

Therefore the Axis of Symmetry is x = 1.
***Don't forget to substitute this x-value into the original equation to find the y-value of the vertex!**

Without using a calculator, these steps will make sketching a graph much easier!
Try graphing the next problem without a calculator.



Let's try another one: $y = x^2 + 2x - 8$

Polynomial	y – intercept	Zeros	Vertex
$x^2 + 2x - 8$			

Is the vertex of $y = x^2 + 2x - 8$ a minimum or maximum?

What is the Axis of Symmetry?

To write zeros as x-intercepts, write a ______________________________ ___________.

What should the y-value be for an x-intercept? ___________

For a 4th and 5th point, use the y-intercept and the "_____________________ _____________"
(the reflection of the y-intercept over the axis of symmetry)

Direction of parabolas Graph the following functions on your calculator. For each function, note whether the parabola is opening up or down.

Function	Parabola opens up or down?
1. $y = x^2 + 3x + 4$	
2. $y = x^2 + 3x - 4$	
3. $y = -x^2 + 3x + 4$	
4. $y = x^2 - 4$	
5. $y = -x^2 + 4$	
6. $y = -x^2 - 4$	
7. $y = -x^2 + 3x$	
8. $y = x^2 - 5x - 2$	
9. $y = -x^2 - 5x - 2$	

Make a conjecture: In a quadratic of the form $y = ax^2 + bx + c$, what determines if the parabola opens up or down?

Make up your own quadratics and test your conjecture with your calculator.

Summary:
If ___________________ then the parabola opens up. If _____________________ then the parabola opens down.

Musical Chairs – see page 18-19 of this packet

Day 4: Finding Extrema of Quadratic Functions

Warm-Up:

6. For the following two equations, find the following values, showing your work for finding them by hand! Then sketch the graphs on graph paper.

a. $x^2 - x - 20 = 0$

zeros:

vertex:

y-intercept:

Max/min?:

Axis of Symmetry (AoS):

b. $x^2 + 8x + 15 = 0$

zeros:

vertex:

y-intercept:

Max/min?:

Axis of Symmetry (AoS):

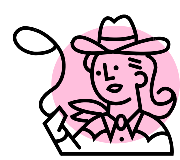
Quadratic Regression Steps: ==============================→

- Stat → Edit then enter the x values into L1 and the y values into L2.
- Stat → Calc → QuadReg
- Do QuadReg L1, L2, Y1 to store the equation in Y1 so that we can make predictions with the equation
  (Press 2nd 1 to get L1, Press 2nd 2 to get L2,
  Press Vars Yvars 1 1 to get Y1)
- Turn on scatter plot with 2nd y= and Enter
- Use Zoom 9 to show your data well on the graph

Maple sap production vs. tree age

Tree age (in years)	Sap production (in ml)
7	200
50	350
10	370
17	380
35	480
8	280
27	420
40	430
12	320
45	360
22	480
42	390
30	430
37	450

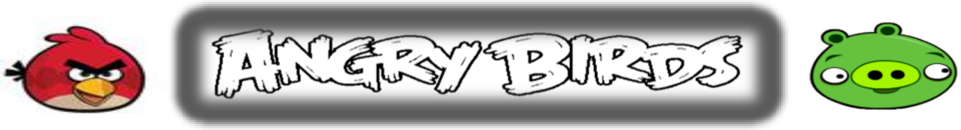
Application:
A rancher is constructing a cattle pen by the river. She has a total of 150 feet of fence and plans to build the pen in the shape of a rectangle. Since the river is very deep, she need only fence 3 sides of the pen. Find the dimensions of the pen so that it encloses the maximum area.



Practice: Factor and solve for #1-3. Factor completely for #4.

1. $4x^2 + 7x = 2$
2. $x^2 - 36 = 0$
3. $4x^2 + 12x - 72 = 0$
4. $20x^2 - 45$

Day 4 Homework Part 1: Discovery of Characteristics of Quadratic Functions



Round 1: Projectiles and Parabolas

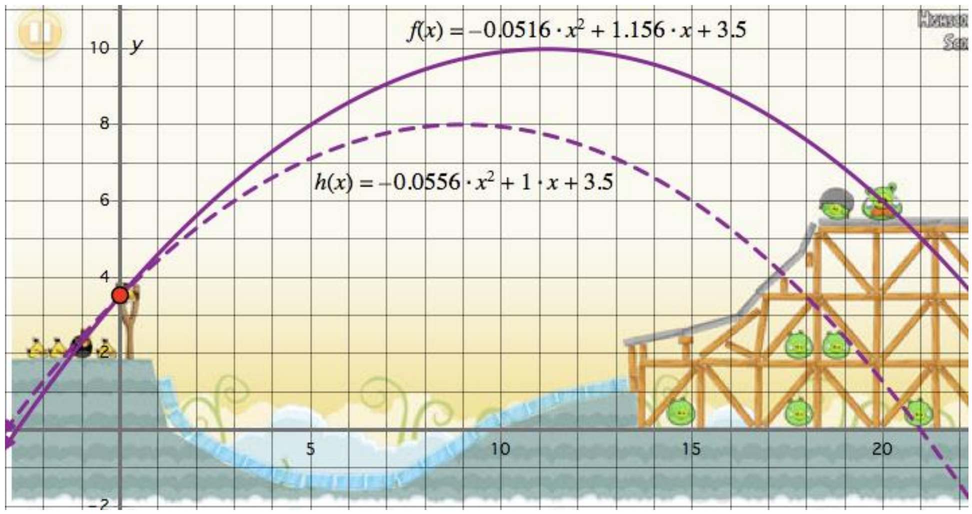
Look at the two trajectories.

1. What is the same about the two equations?

2. What does the y-intercept represent? What part of the equation gives you the y-intercept?

3. What do the x-intercepts represent?

4. The highest part of the bird's flight is represented by what part of the parabola?



5. How far does Angry Bird fly in h(x)? How high does he go? How far away from the catapult is he when he is at his highest? When he is 15 feet away, how high is he flying?

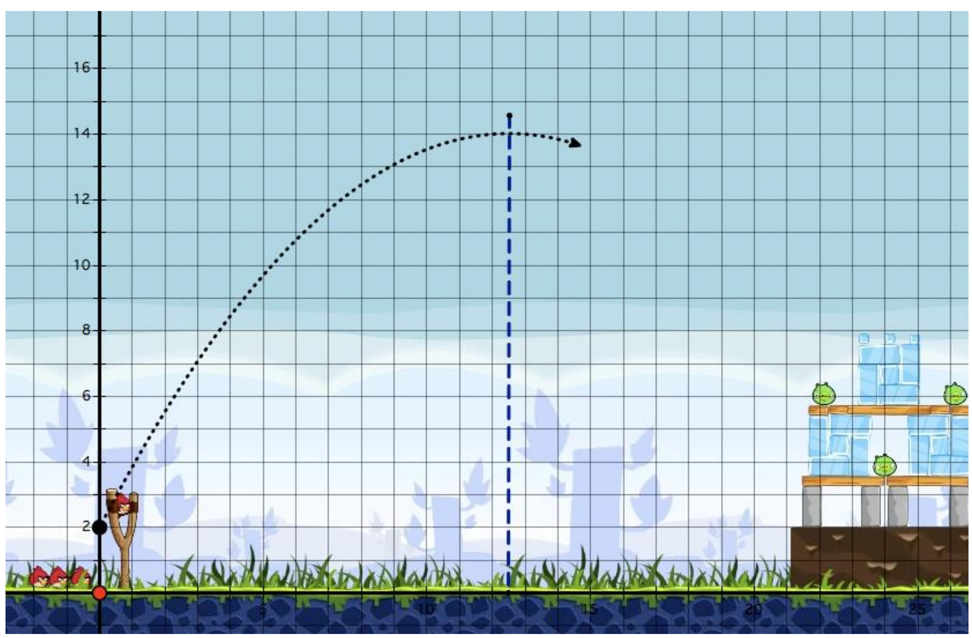
Round 2

1. When Angry Bird is 9 feet away, how high is he flying?

2. The axis of symmetry is provided. What part of the parabola does this pass through? What does this part represent about Angry Bird's flight?

3. How high does the bird fly?

4. Reflect points over the axis of symmetry to complete the parabola. Do you hit any pigs?

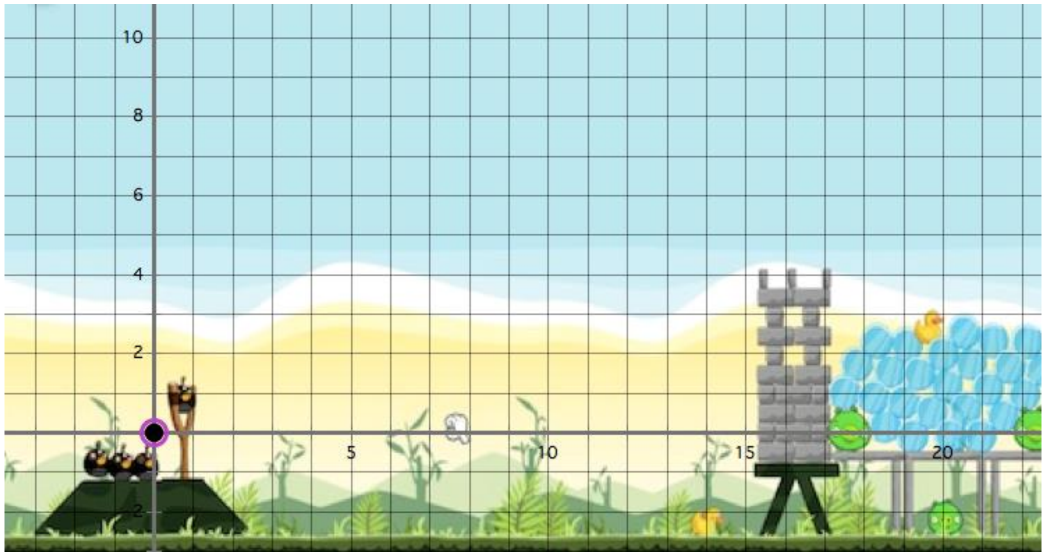


5. How far would Angry Bird fly if he did not hit any obstacles?

6. Without solving for the whole equation, what is "c" value in standard form? Is "a" positive or negative?

Round 3

1. Angry bird and hungry pig are 18 feet away from each other. If angry bird and hungry pig are at the same height (y-value) when angry bird is catapulted, at what distance away is Angry Bird the highest? Think about symmetry.



2. Angry Bird wants to hit the pig on the right.

The equation representing his flight is: $y = -0.083x^2 + 1.82x$

Using the picture, what is the y-intercept?

Using the picture, what are the x-intercepts?

Where is the axis of symmetry?
You may use the picture to visualize, but show your algebraic work using methods from earlier this unit: Round to the nearest integer.

How high does Angry Bird fly (rounded to the nearest integer)? Sketch the graph of Angry Bird's flight.

Day 5: Characteristics of Quadratic Functions

Warm-Up:

1. Jason and Jim jumped off of a cliff into the ocean in Acapulco while vacationing. Jason's height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, while Jim's height could be modeled by $h(t) = -16t^2 + 12t + 480$ where t is the time in seconds and h is the height in feet. Whose jump was higher and by how much? Hint: One of the functions is NOT factorable. So for that one, use your calculator to find the zeros.

2. Find zeros, y-intercept, vertex, one other point and the axis of symmetry, then sketch the graph of
$y = -2x^2 + 4x + 70$

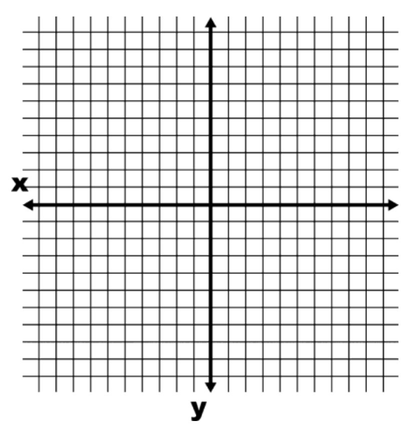
Zeros: ____________________

y-intercept:________________

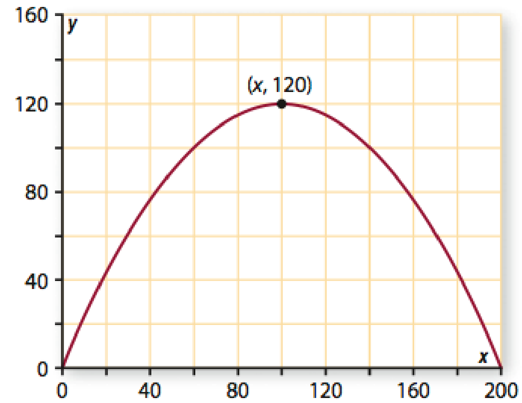
Vertex:____________________

Other Point:________________

Axis of symmetry:___________



Investigation 2: Designing Parabolas



From Core-Plus Mathematics Course 2 Book, p.332-334.

Objective: Determine the equation of a quadratic that guarantees the graph of its parabola will fit given constraints. For example, write an equation of a quadratic with zeroes at x = 0 and x = 200 and a maximum y-value of 120.

1. Using ideas from your earlier study of Quadratic Functions and their graphs, write the rule for a function with a parabolic graph that contains points (0, 0), (200, 0), and a maximum point whose y-coordinate is 120. Use the hints in Parts a-d, below, as needed.

    a. The graph of the desired function has x-intercepts (0, 0) and (200, 0). How do you know that the graph of the function f(x) = x(x - 200) has those same x-intercepts?

    b. What is the x-coordinate of the maximum point on this graph?

    c. Suppose that g(x) has a rule in the form g(x) = k[x(x - 200)], for some particular value of k. What value of k will guarantee that g(x) = 120 at the maximum point of the graph?

    d. Write the rule for g(x) in equivalent expanded form using the k value you found in Part c.

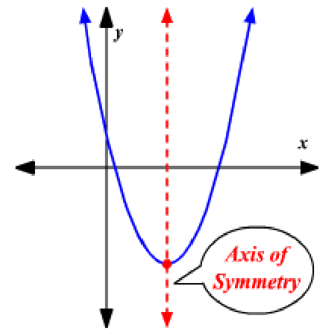
2. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

    a. X-intercepts at (4, 0) and (-1, 0)

    b. X-intercepts at (7, 0) and (1, 0) and minimum point at (4, -10)

    c. X-intercepts at (3, 0) and (-5, 0) and maximum point at (-1, 8)

Some Observations:



- **Axis of symmetry:** a vertical line that divides the parabola into two congruent halves. It always passes through the vertex of the parabola. The x-coordinate of the vertex is the equation of the axis of symmetry

Equation of a Parabola

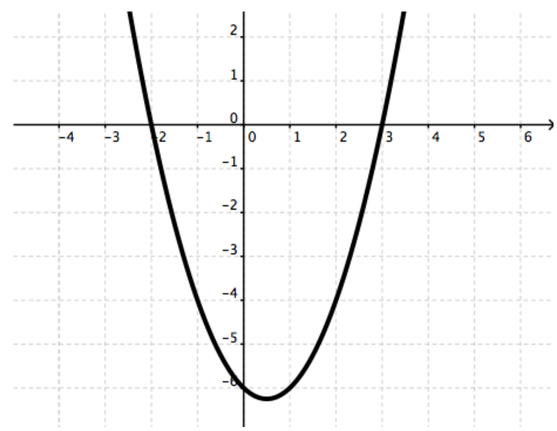
Standard Form: ___________________________

We can also use the zeros to help write an equation using the form:_____________________
Where ***r*** and ***s*** represent the zeros (roots) found on the graph.

How to Create an equation of a parabola from a graph:

1. ____
2. ____
3. ____
4. ____

Example: Given the graph to the right, write an equation.



1. Find the zeros → **(-2, 0) and (3, 0)**
2. Write the zeros as two binomials and multiply them.

→ $\mathbf{(x + 2)(x - 3) = x^2 - 3x + 2x - 6}$
$\mathbf{= x^2 - x - 6}$

3. Substitute a point to find the value of ***a***.
   *Use a point that you can easily find on the graph.
   (Hint: the y-intercept or vertex are typically easy to find)
4. Substitute "a" into the ORIGINAL equation to get Factored Form. Multiply the binomials and distribute to get Standard Form.

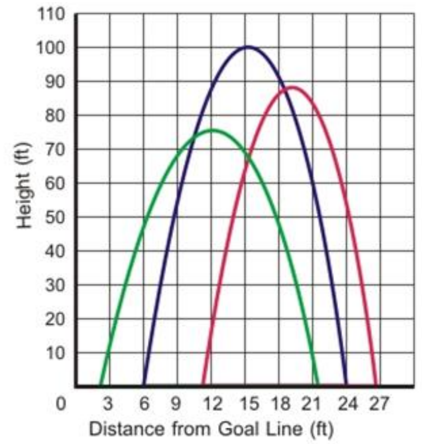
→ **$y = a(x + 2)(x - 3)$ and use y-intercept: (0, -6)**
$-6 = a(0 + 2)(0 - 3)$
$-6 = -6a$
$a = 1$

$y = a(x^2 - x - 6) = 1(x^2 - x - 6)$

$\boxed{y = x^2 - x - 6}$

Practice

For each graph, write an equation based on the characteristics you're given. Then, explain using algebraic reasoning which graph has the greatest maximum and which egg travels the farthest.

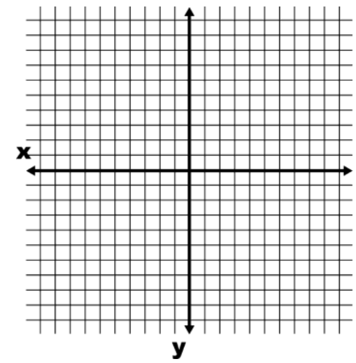


Day 6: Characteristics of Quadratic Functions Continued as well as Even VS. Odd Functions

Warm-Up:

1. Jenna is trying to invest money into the stock exchange. After some research she has narrowed it down to two companies. Company A shows a portfolio value of $v(t) = 800 – 28t +0.25t^2$, and Company B shows a portfolio value of $v(t) = 700 - 65t + 0.3t^2$, where v is the value of the portfolio in hundreds of dollars and t is the time in months. Which company will allow her the peace of mind of having the higher value, even if the stock prices drop to their lowest?

2. Using the following quadratic, find zeros, y-intercept, vertex, one other point, and the Axis of Symmetry then sketch the graph. $y = x^2 + 4x – 5$



Zeros - ________________________

Y-intercept - ___________________

Vertex - _______________________

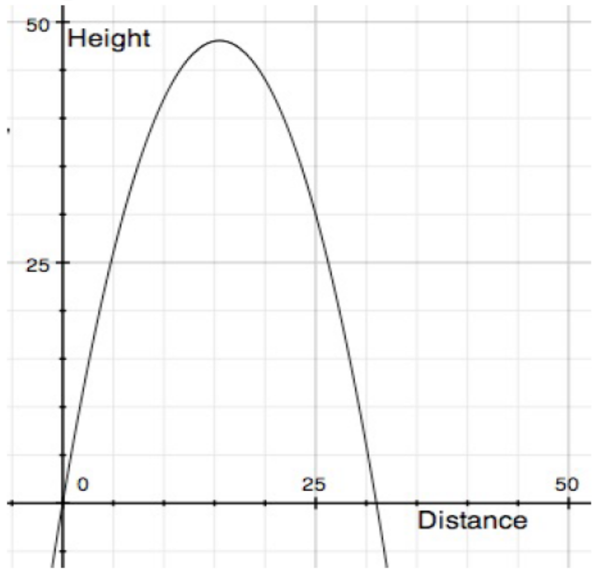
A. O. S. - ______________________

Suppose some very "Angry Birds" are attacking some "pigs" in a castle by using a slingshot to launch themselves at castle walls. Depending on the angle that they are launched at, they will either shoot long and far or high and short. The data about how each slingshot launches each bird is listed below: *Note, how each data is represented in a different way!!*

Slingshot A

Distance the bird is from the slingshot (in meters)	Height of the bird (in meters)
10	20
20	30
30	30
40	20

Slingshot B



Slingshot C

$$y = -0.015x^2 + 0.975x$$

Where x is the distance the bird is from the slingshot and y is the height of the bird.

1. How "far" will each slingshot launch each bird? If the castle is far away, which slingshot should they use and why? If the castle is near, which slingshot should they use and why?

   Slingshot A Launch - _______________
   Slingshot B Launch - _______________
   Slingshot C Launch - _______________

   Far-away Castle:

   Near-by Castle:

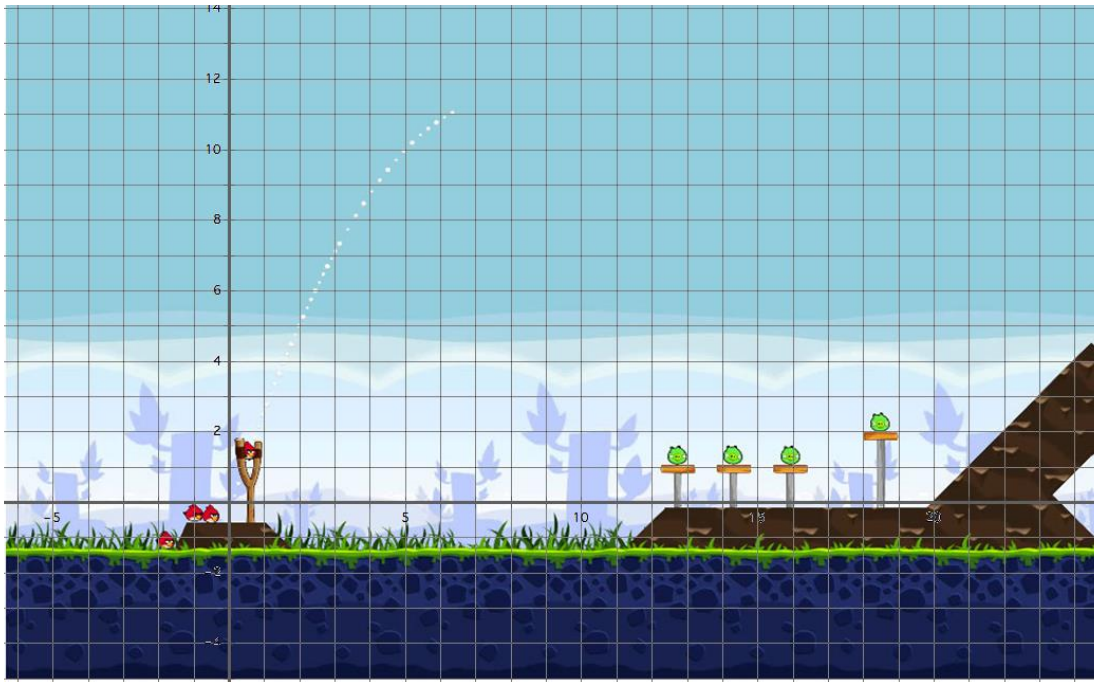
2. Analyze the slingshot data and compare to determine which slingshot shoots the birds the highest. Explain how you know.

3. If the castle walls are 30 feet tall, which slingshot should you use and why?

4. What are the pros and cons of using each Slingshot A, B, or C?

Practice: Can you make the bird hit the pigs?

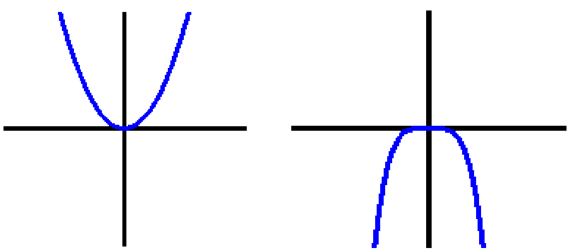
Take a minute to work in small groups with the people around you. Sketch a graph that you think would make you hit the second pig in this picture.

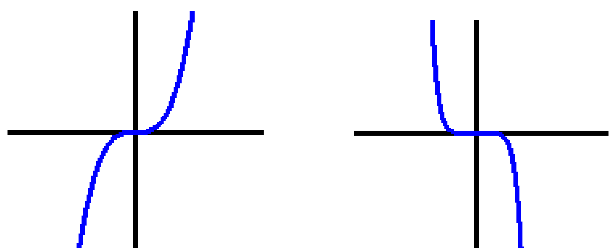


Write the equation of the quadratic that best represents success of hitting the SECOND green pig:

Notes: <u>Functions can be Even, Odd, or Neither</u>

________________ are symmetric across the y-axis. **________________ are symmetric across the origin.**





Functions are neither even nor odd if they do not exhibit one of these types of symmetry.

Discovery Activity: Even VS. ODD Functions

Graph each function on your calculator. Use your graph to fill in the chart.

Graph	Is the Function even, odd, or neither?	Is the leading coefficient positive or negative?	Does the function rise or fall to the left?	Does the function rise or fall to the right?
1. $y = x^2$				
2. $y = x^4$				
3. $y = x^2 + 3$				
4. $y = (x-4)^2$				
5. $y = -x^2$				
6. $y = -x^4$				
7. $y = -x^2 + 3$				
8. $y = x^3$				
9. $y = x^5$				
10. $y = x^3 + 4$				
11. $y = -x^3$				
12. $y = -x^5$				
13. $y = -x^5 - 2$				

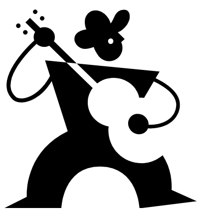
The **Degree** of a polynomial is the highest exponent, when the polynomial is **in standard form**.

SUMMARY: The end behavior of a polynomial depends on:

1. Whether the degree of the polynomial is ___________ or ___________.

2. Whether the leading coefficient is ____________ or ______________.

End Behavior of Polynomial Functions				
	Leading coefficient is **Positive**		Leading coefficient is **Negative**	
	Left	Right	Left	Right
Function is **odd degree**				
Function is **even degree**				



Musical Chairs

Fill in the function. Then for each graph, solve the function and find the important key points (zeroes, x-intercepts, y-intercepts, maximum or minimum), plot the points on the coordinate plane, determine the axis of symmetry, and sketch the graph of the function.

You are NOT allowed to use your calculators.

<u>Function 1:</u>

Zeroes:

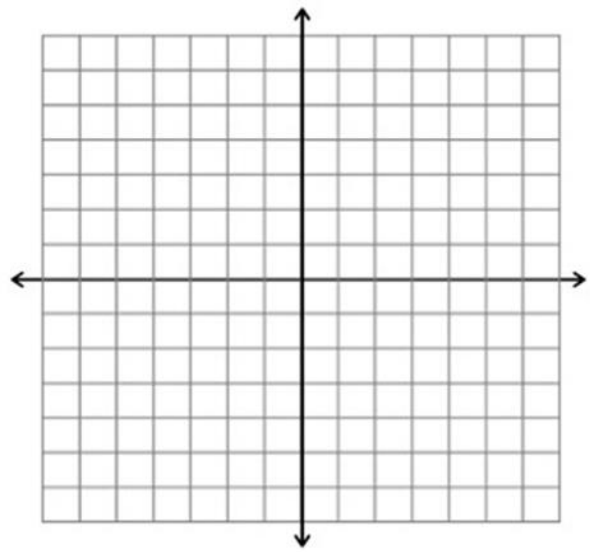
x-intercepts:

y-intercept:

Location of vertex:

Axis of Symmetry:

Is vertex the minimum or maximum of the function?



<u>Function 2:</u>

Zeroes:

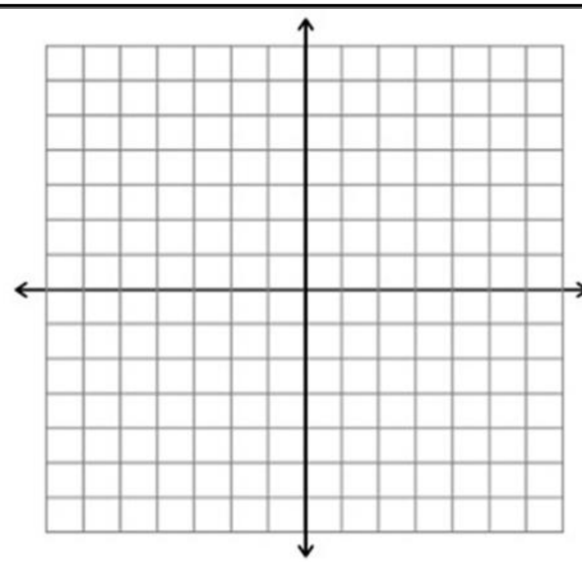
x-intercepts:

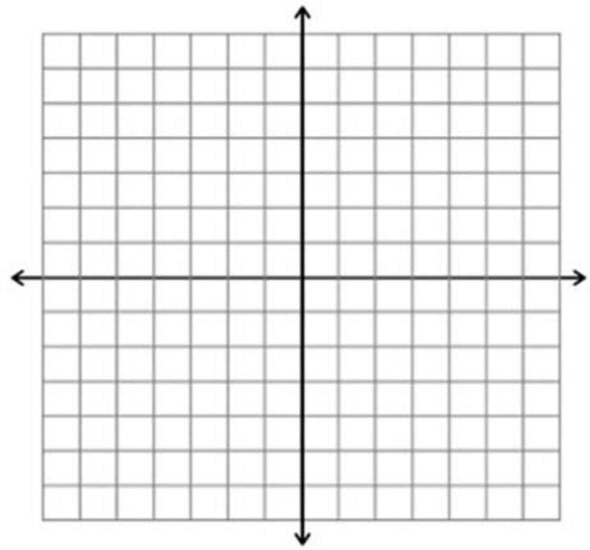
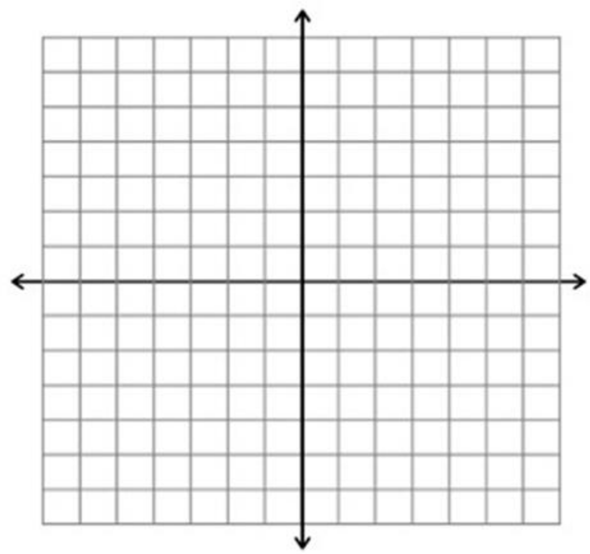
y-intercept:

Location of vertex:

Axis of Symmetry:

Is vertex the minimum or maximum of the function?



<u>Function 3:</u> Zeroes: x-intercepts: y-intercept: Location of vertex: Axis of Symmetry: Is vertex the minimum or maximum of the function?	
<u>Function 4:</u> Zeroes: x-intercepts: y-intercept: Location of vertex: Axis of Symmetry: Is vertex the minimum or maximum of the function?	
<u>Function 5:</u> Zeroes: x-intercepts: y-intercept: Location of vertex: Axis of Symmetry: Is vertex the minimum or maximum of the function?