Day 7: The Quadratic Formula and Discriminants

Warm-Up:

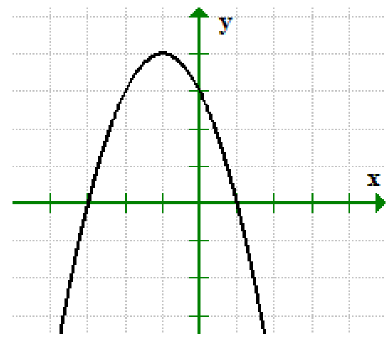
1. Solve each of the quadratic functions by graphing and algebraic reasoning

   a. $x^2 - 3 = 0$ b. $x^2 + 5x - 8 = 0$ c. Why is it important to have alternative methods of solving?

2. Simplify the radicals.

   a. $\sqrt{50}$ b. $3\sqrt{80}$

3. Find the equation of the graph in standard form. Show all work.



Day 7: The Quadratic Formula and Discriminants

Standard form of a quadratic ***equation***: ___________________________

The solutions of some quadratic equations are not rational, or are too messy to obtain by factoring. For such equations, the most common method of solution is the quadratic formula.

The quadratic formula: X = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is used to solve ***ANY*** quadratic function for its zeros (x-intercepts).

Notice that there is a +/- sign in the formula. There are actually ______________________ for any quadratic formula.

X = $\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ AND X = $\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Solve using the quadratic formula.

Example 1: $x^2 + 9x + 20 = 0$

Example 2: $x^2 - x = 5x - 9$

Example 3: $-x^2 + 2x = 2$

Example 4: $7x^2 - 12x + 3 = 0$

Example 5: $4x^2 + 12x + 9 = 0$

Example 6: $x^2 - 5x - 5 = 0$

Types of Zeroes

Given the following quadratic functions, use the quadratic formula to find the zeros:

1. $x^2 - x - 6 = 0$
2. $x^2 + 16 = 0$
3. $x^2 + 4x + 4 = 0$

Use your calculator to examine the graphs of each function. How does each graph relate to the number of solutions for that problem?

Which function's graph did not touch the x-axis? Which touched the x-axis once?

How was the quadratic formula different when you had two, one, or no zeroes?

1. When can you expect 2 solutions in a quadratic equation?
2. When can you expect 1 solution in a quadratic equation?
3. When can you expect a quadratic to have no solutions?

Recall the quadratic formula: $\boxed{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

This part in the square root helps us to determine how many solutions a quadratic will have:

$\boxed{b^2 - 4ac}$

This is called the **Discriminant**. Calculate the discriminant for the 3 introductory problems.

1. $x^2 - x - 6 = 0$
2. $x^2 + 16 = 0$
3. $x^2 + 4x + 4 = 0$

Types of Quadratic Solutions

Quadratic solutions are either ________________ or __________________.

If the discriminant is **positive**, the quadratic has __________________ solutions.

Real solutions are the solutions you get from factoring, the zeroes on the graph, and when you are able to do the square root in the quadratic formula.

- Rational solutions are when the discriminant evaluates to __________________.
  Remember, rational means it can be a ratio…a simplified fraction! ☺
- Irrational solutions are when the discriminant evaluates to _________________.

If the discriminant is **zero**, the quadratic has __________________ solution.

If the discriminant is **negative**, the quadratic has __________________ solutions

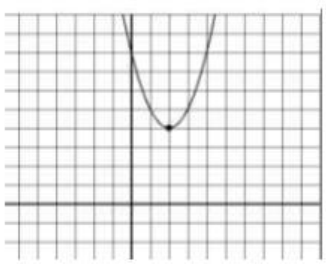
Imaginary solutions do not show up on the graph or when factoring. In fact, quadratics with imaginary solutions cannot be factored.

Practice: Given the following quadratics, calculate the discriminant and determine how many and what types of solutions they will have. For real solutions, determine if they will be rational or irrational.

1. $x^2 - 6x + 11 = 2$__________________
2. $3x^2 + 5x = 12$__________________
3. $3x^2 + 48 = 0$__________________
4. $x^2 - 27 = 0$__________________
5. $x^2 + x + 1 = 0$__________________
6. $x^2 + 4x - 1 = 0$__________________
7. $6x^2 + 12x + 6 = 0$__________________
8. $-3x^2 - 4x - 8 = 0$__________________

Given the following graphs of quadratic functions:

a) Determine the **sign** of the discriminant and **b)** whether the solutions are **real** or **non-real**.

1.


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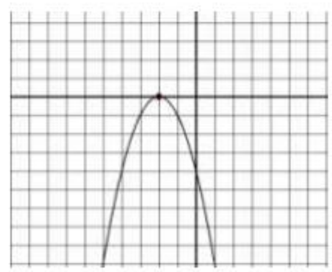

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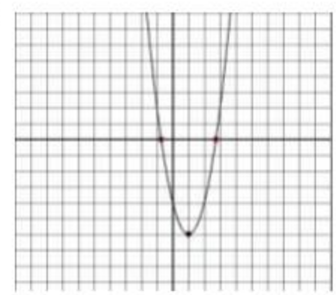


EVERYTHING I NEED TO KNOW ABOUT QUADRATICS

Fill in the following chart based on what you need to know about quadratics

Value of the discriminant ($b^2 - 4ac$)	Number and type of roots	What does the graph look like?
$b^2 - 4ac$ is positive and a **perfect square** $b^2 - 4ac > 0$		
$b^2 - 4ac$ is positive and a **NOT perfect square** $b^2 - 4ac > 0$		
$b^2 - 4ac = 0$		
$b^2 - 4ac$ is negative $b^2 - 4ac < 0$		

Practice

Calculate the discriminant and determine the number and types of solutions.

Function	Discriminant	Number/Type of Solutions
Ex: $x^2 - 3x - 4 = 0$ $a = 1, b = -3, c = -4$ $(-3)^2 - 4(1)(-4)$ $9 - (-16) = 9 + 16 = 25$	25	2 real rational solutions
1. $x^2 - 6x + 9 = 0$		
2. $x^2 + 6x = -9$		
3. $x^2 - 6x - 16 = 0$		
4. $2x^2 - 6x - 13 = 0$		
5. $-x^2 + 2x - 1 = 0$		
6. $2x^2 + 3 = 2x$		
7. $x^2 + 2x + 1 = 0$		
8. $x^2 + 2x = -3$		
9. $x^2 - 6x + 9 = 0$		
10. $x^2 + 5x + 8 = 0$		
11. $2x^2 - 5x + 6 = 0$		
12. $x^2 - 5x = 10$		
13. $x^2 - 6x + 3x = 4 - 11$		

Day 8: Quiz Day

Warm-Up:

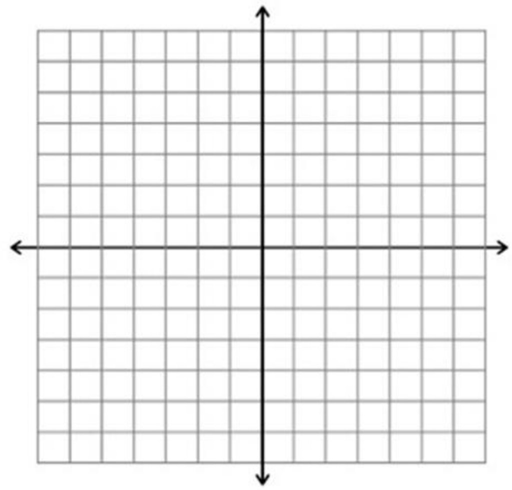
1.Find the zeros of the following. Show all your work using the appropriate method. Leave in simplest radical form when necessary.

a. $x^2 - 9x + 12 = 0$ b. $x^2 - 16 = -4x$ c. $2x^2 + 8x = 13$ d. $x^2 + 3x = 28$

continue on next page

2.Show your work in the boxes to find the requested values of $y = 2x^2 - 5x - 3$ algebraically. Then graph.

Solve by factoring	x-intercepts	Vertex
	y-intercepts	Maximum or minimum?
	Axis of symmetry	



Day 9: FRED Functions

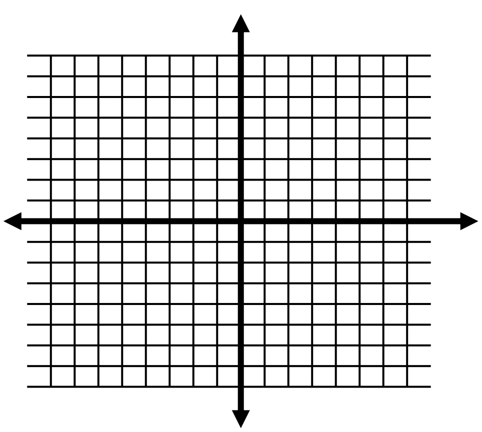
Warm-Up: Graphing Quadratic Inequalities and Applications Practice

STEPS: 1.) Graph the boundary. Determine if it should be solid ($\le$, $\ge$) or dashed ($>$, $<$).

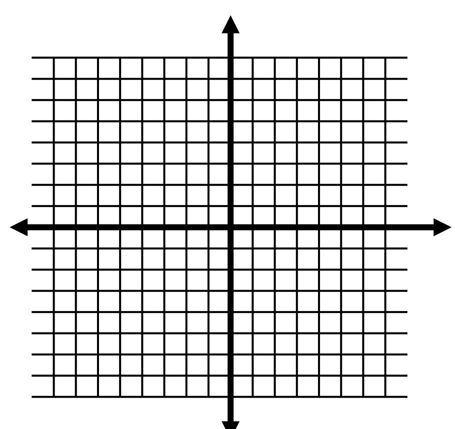
2.) Test a point. (0, 0) is a good point to use IF it's not on the line!

3.) If the point works (ends in a true statement), shade the region where the point lies. If the point does not work (ends in a false statement), shade the opposite region.

1: Graph $y \le x^2 - 2x - 8$



2: Graph $y < -x^2 - 4x + 5$



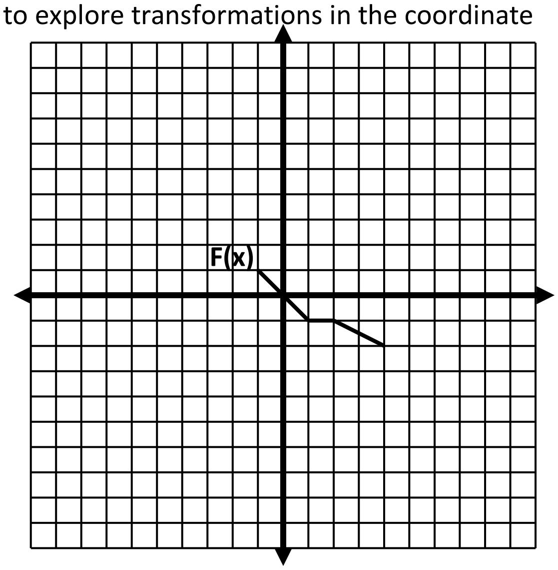
3. Factor Completely a. $4x^2 - 12x + 9$ b. $4x^2 - 36$

4. An electronics company has a new line of portable radios with CD players. Their research suggests that the daily sales, s, for the new product can be modeled by $s = -p^2 + 120p + 1400$, where p is the price of each unit.
   a. What is the maximum daily sales total for the new product?
   b. What price should the company charge to make this profit?

5. The shape of the Gateway Arch in St. Louis is a catenary curve, which closely resembles a parabola. The function $y = -\frac{2}{315}x^2 + 4x$ closely models the shape of the arch, where y is the height in feet and x is the horizontal distance from the base of the left side of the arch in feet.
   a. What is the width of the arch at the base?
   b. What is the maximum height of the arch?

Day 9: FRED Functions Part 1

To the right is a graph of a **"Fred" function**. We can use Fred functions to explore transformations in the coordinate plane.



I. Let's review briefly.

1. a. Explain what a function is in your own words.

   b. Using the graph, how do we know that Fred is a function?

2. a. Explain what we mean by the term domain.

   b. Using the graph, what is the domain of Fred?

3. a. Explain what we mean by the term range.

   b. Using the graph, what is the range of Fred?

4. Let's explore the points on Fred.
   a. How many points lie on Fred? Can you list them all?

   b. What are the key points that would help us graph Fred?

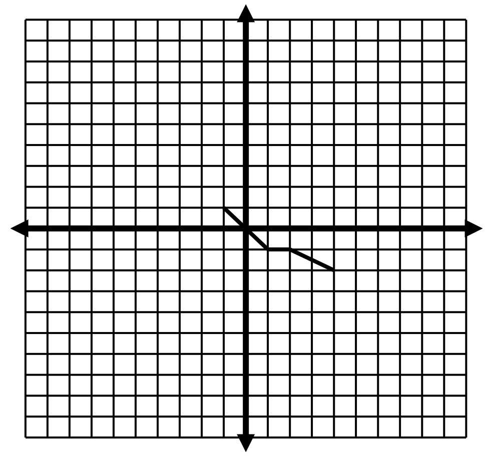
   We are going to call these key points **"characteristic" points**. It is important when graphing a function that you are able to identify these characteristic points.

   c. Use the graph of graph to evaluate the following.

   F(1) = ______ F(–1) = ______ F(______) = –2 F(5) = _______

II. Remember that F(x) is another name for the y-values.

Therefore the equation of Fred is **y = F(x)**.



X	F(x)
–1	
1	
2	
4	

1. Why did we choose those x-values to put in the table?

Now let's try graphing Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) + 4

x	y
–1	
1	
2	
4	

2. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

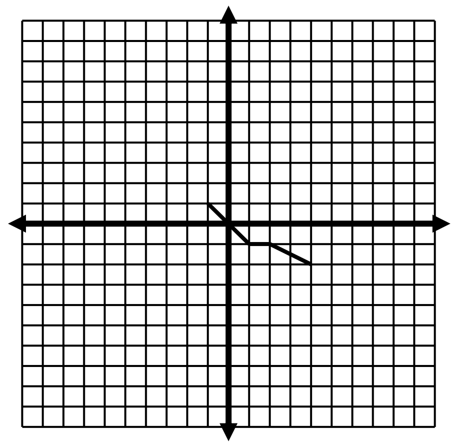
4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. In y = F(x) + 4, how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

III. Suppose Freddie Jr's equation is: **y = F(x) – 3**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) – 3

x	y
–1	
1	
2	
4	



1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) – 3? Be specific.

2. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

3. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. In y = F(x) – 3, how did the "– 3" affect the graph of Fred? Did it affect the domain or the range?

IV. Checkpoint: Using the understanding you have gained so far, describe the affect to Fred for the following functions.

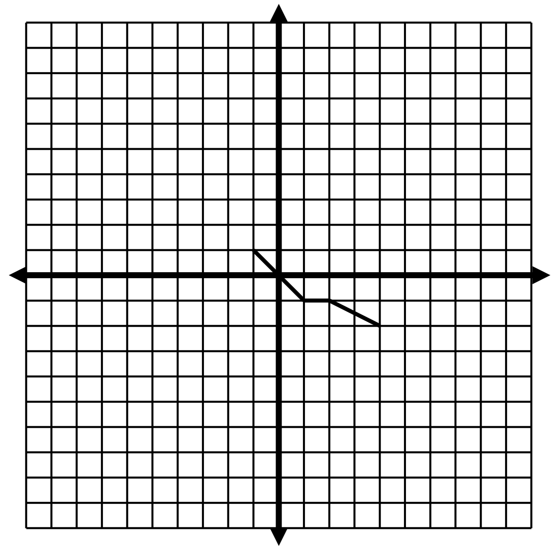
Equation	**Effect to Fred's graph**
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) – 100	
2. y = F(x) + 73	
3. y = F(x) + 32	
4. y = F(x) – 521	

V. Suppose Freddie Jr's equation is: **y = F(x + 4)**.

1. Complete the table.

x	X + 4	y
–5	–1	1
	1	–1
	2	–1
	4	–2

*(**Hint**: Since, x + 4 = –1, subtract 4 from both sides of the equation, and x = –5. Use a similar method to find the missing x values.)*



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (–5, 1).

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

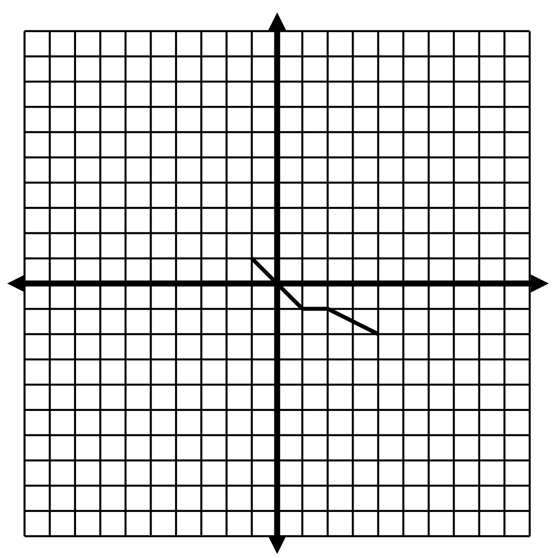
6. In y = F(x + 4), how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

VI. Suppose Freddie Jr's equation is: **y = F(x – 3)**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

1. Complete the table.

y = F(x – 3)

x	x – 3	Y
	–1	
	1	
	2	
	4	



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (2, 1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x – 3)? (Be specific.)

4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

6. In y = F(x – 3), how did the " –3" affect the graph of Fred? Did it affect the domain or the range?

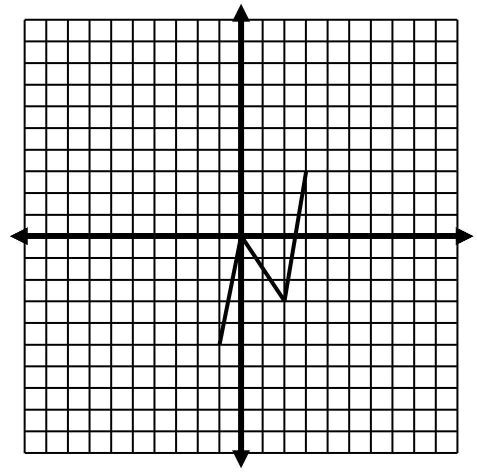
VII. Checkpoint: Using the understanding you have gained so far, describe the effect to Fred for the following functions.

Equation	**Effect to Fred's graph**
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x – 10)	
2. y = F(x) + 7	
3. y = F(x + 48)	
4. y = F(x) – 22	
5. y = F(x + 30) + 18	

VIII. Checkpoint: Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation	**Effect to Fred's graph**
Example: y=F(x + 8)	Translate left 8 units
1.	Translate up 29 units
2.	Translate right 7
3.	Translate left 45
4.	Translate left 5 and up 14
5.	Translate down 2 and right 6

IX. Now let's look at a new function.
Its notation is **H(x)**, and we will call it **Harry**.
Use Harry to demonstrate what you have learned so far about the transformations of functions.

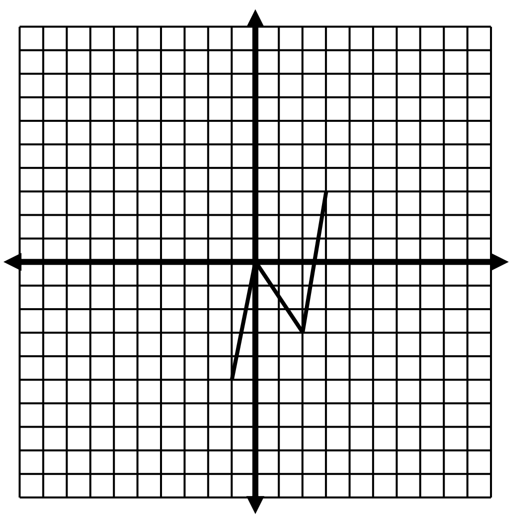


1. What are Harry's characteristic points?

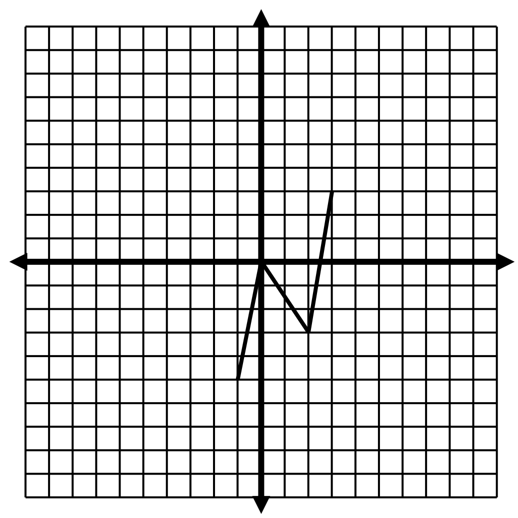
____

2. Describe the effect on Harry's graph for each of the following.
   a. H(x – 2) ____
   b. H(x) + 7 ____
   c. H(x+2) – 3 ____

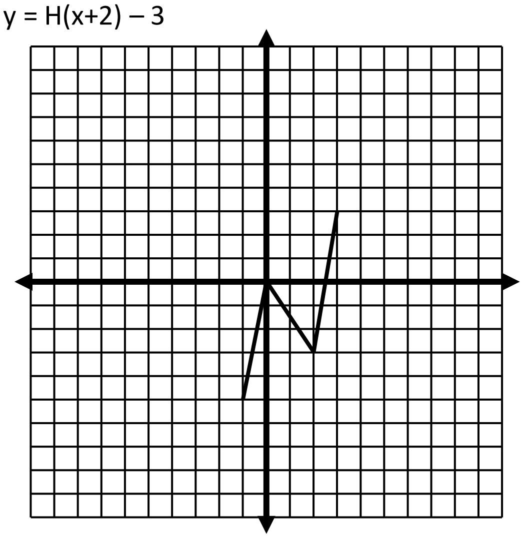
3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.
   a. y = H(x – 2)



   b. y = H(x) + 7



c. y = H(x+2) – 3

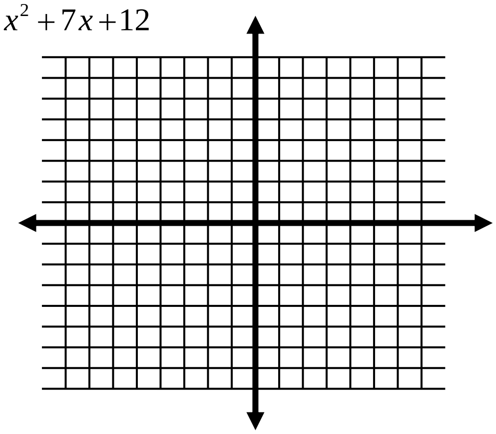


Day 10: Transformations of Quadractics

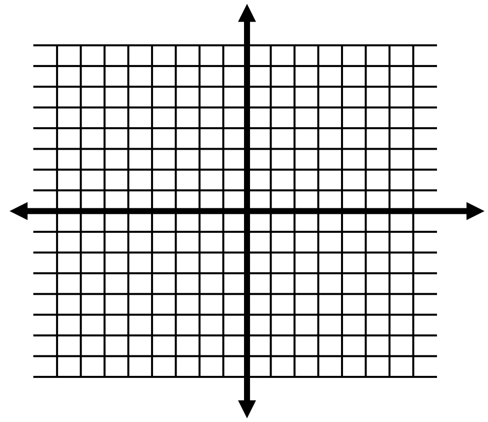
Warm-Up:

Graphing quadratic systems -> Graph both quadratics, then darkly shade the area of overlap.

1) $y \le -x^2 - x + 12$
$y \ge x^2 + 7x + 12$



2) $y < -x^2 + 4x - 3$
$y > x^2 + 6x + 8$



Factor completely

3) $81x^4 - 16$

4) $12x^2 + 26x - 10$

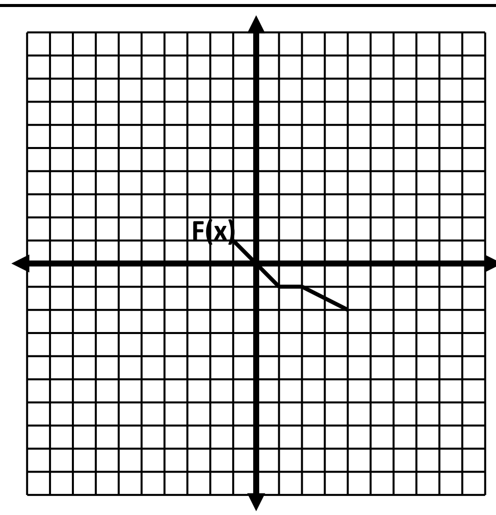
Day 10: FRED Functions Part 2

I. Let's suppose that Freddie Jr. is **y = – F(x)**

7. Complete the table.

y = – F(x)

x	F(x)	y
–1	1	–1
1		
2		
4		



8. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (–1, –1).*]

9. What type of transformation maps Fred, F(x), to Freddie Jr., –F(x)? (Be specific.)

10. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

11. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

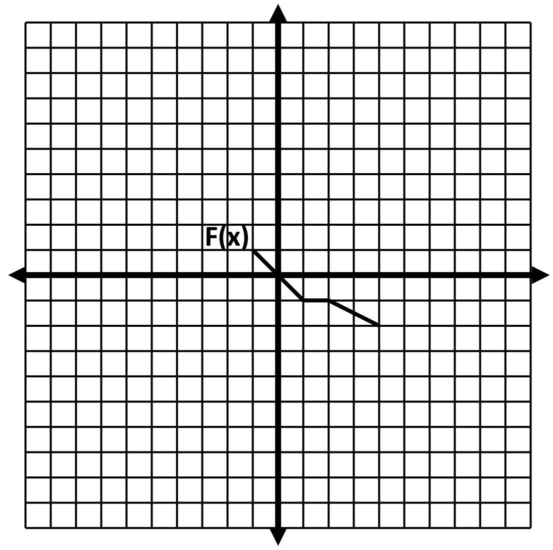
12. In y = – F(x), how did the negative coefficient of "F(x)" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

II. Now let's suppose that Freddie Jr. is **y = F(–x)**

1. Complete the table.

y = F(–x)

x	**–x**	**y**
	–1	
	1	
	2	
	4	



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st point should be (1, 1).*]

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(–x)? (Be specific.)

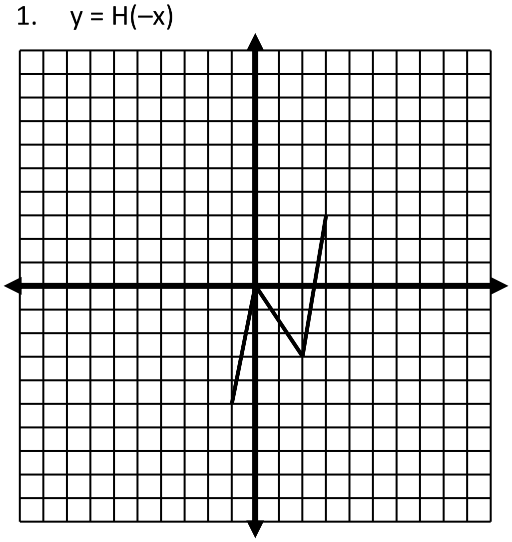
4. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

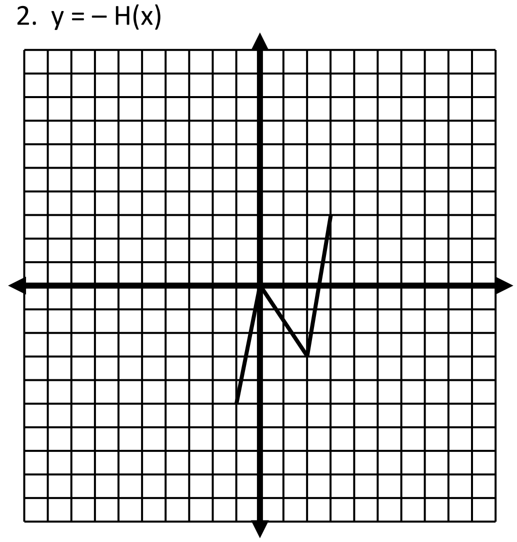
6. In y = F(–x), how did the negative coefficient of "x" affect the graph of Fred? How does this relate to our study of transformations earlier this semester?

III. **Checkpoint: Harry is H(x) and is shown on each grid. Use Harry's characteristic points to graph Harry's children without making a table.**

1. y = H(–x)

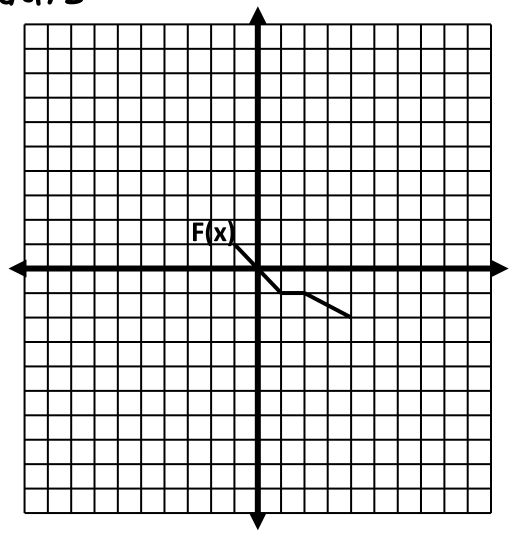


2. y = – H(x)



IV. Let's suppose that Freddie Jr. is **y = 4 F(x)**

1. Complete the table.



y = 4 F(x)

x	F(x)	y
–1		
1		
2		
4		

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, 4).*]

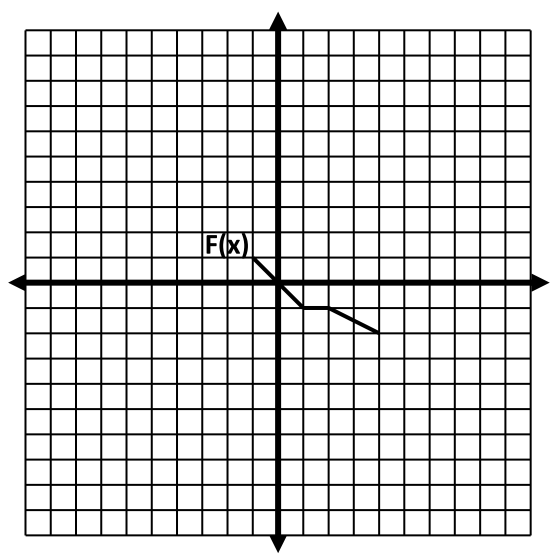
3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. In y = 4 F(x), the coefficient of "F(x)" is 4. How did that affect the graph of Fred? Is this one of the transformations we studied? If so, which one? If not, explain.

V. Now let's suppose that Freddie Jr. is **y = ½ F(x)**.

1. Complete the table.



y = ½ F(x)

x	F(x)	y
–1		
1		
2		
4		

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, ½).*]

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

5. In y = ½ F(x), the coefficient of "F(x)" is ½. How did that affect the graph of Fred? How is this different from the graph of y = 4 F(x) on the previous page?

VI. Checkpoint:

1. Complete each chart below. Each chart starts with the characteristic points of Fred.

X	F(x)	3 F(x)
–1	1	
1	–1	
2	–1	
4	–2	

x	F(x)	¼ F(x)
–1	1	
1	–1	
2	–1	
4	–2	

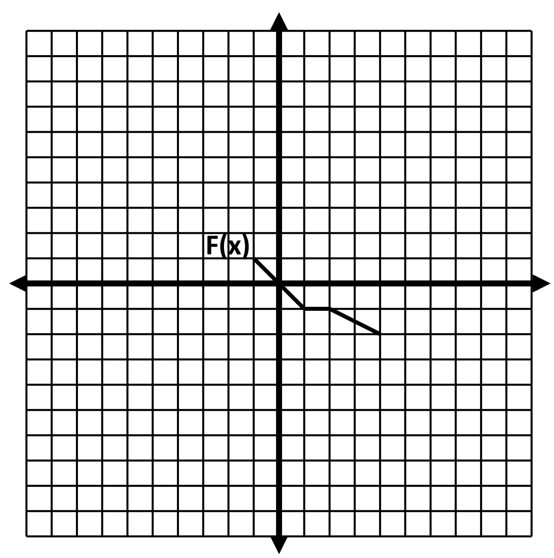
2. Compare the 2nd and 3rd columns of each chart above. The 2nd column is the y-value for Fred. Can you make a conjecture about how a coefficient changes the parent graph?

VII. Now let's suppose that Freddie Jr. is **y = –3 F(x)**.

1. Complete the table.

y = –3 F(x)

x	F(x)	y
–1		
1		
2		
4		



2. On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1st one should be (–1, –3).*]

3. Reread the conjecture you made in part VI. Does it hold true or do you need to refine it?
If it does need some work, restate it more correctly here.

VIII. Checkpoint: Let's revisit Harry, H(x).

4) Describe the effect on Harry's graph for each of the following.

Example: –5H(x) ___Each point is reflected in the x-axis and is 5 times as far from the x-axis.

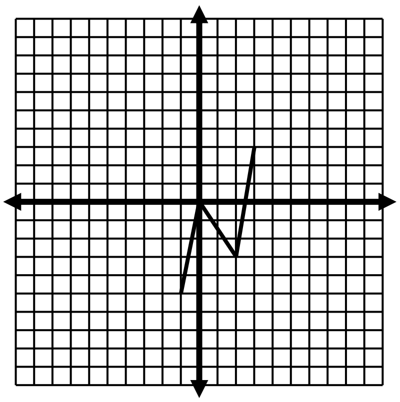
d. 3H(x) ______________________________

e. –2H(x) ______________________________

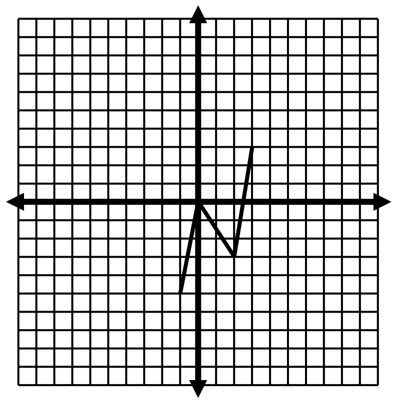
f. $\frac{1}{2}$H(x) ______________________________

5) Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.

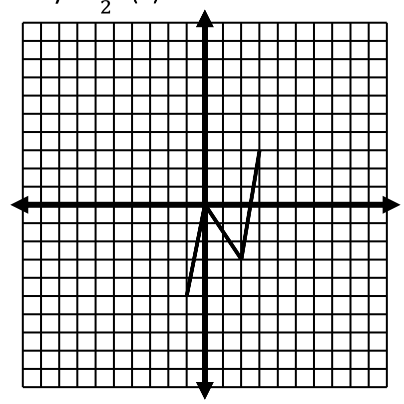
a. y = 3H(x)



b. y = –2H(x)



c. y = $\frac{1}{2}$H(x)



Quadratic Graphs

In your graphing calculator, graph the function y = x^2 (the quadratic parent function). Then graph each function below. Compare the new graph to the parent graph and write your observations about the location of the vertex, the overall shape, and the slope of the sides of the new graph in the blanks at the right.

Part A: The Effect of *a*

Function	Vertex	Shape or Shift Change?	Describe the change
1. $y = 4x^2$			
2. $y = \frac{1}{4}x^2$			
3. $y = -4x^2$			
4. $y = -\frac{1}{4}x^2$			

Part B: The Effect of *h*

Function	Vertex	Shape or Shift Change?	Describe the change
5. $y=(x+2)^2$			
6. $y=(x-4)^2$			
7. $y=-(x+5)^2$			
8. $y=-(x-6)^2$			

Part C: The Effect of *k*

Function	Vertex	Shape or Shift Change?	Describe the change
9. $y=x^2+1$			
10. $y=x^2-2$			
11. $y=-x^2+7$			
12. $y=-x^2-10$			

Day 11: Quadratic Systems

Warm-Up:

12. Given the following functions, specifically describe the transformation from the identity function $y = x^2$

 a. $y = (x+3)^2 - 7$ b. $y = 5x^2 + 12$ c. $y = ½(x - 2)^2 + 4$

1. Application Practice: The sum of two numbers is 21. The sum of the squares of the numbers is 305. What is the product of the two numbers?

2. A Science Olympiad team is having a competition for who can pick an item that will hit the ground the fastest from a 45 ft building. Jenny picked a hockey puck creating a function of $y = -16x^2 + 45$. How many seconds has the hockey puck been in the air when it is 1 foot away from hitting the ground?

Factor completely then solve:

15) $32x^4 - 162 = 0$ 16) $6x^3 - 33x^2 = 18x$

Day 11: Solving and Graphing Quadratic Inequalities and Systems (Algebra 2 Text p. 269)

Solving Quadratic Inequalities

Solve :

1) $0 > x^2 - 6x - 7$

Steps:

1) Solve to find ___________
2) Place zeros on a number line
   < or > means ___________________ point
   ≤ or ≥ means ___________________ point
3) Test a point in each area to determine shading
4) Write answer in ____________ notation! ☺

2) $x^2 + 9x + 14 < 0$

3) $x^2 - x - 12 \geq 0$

NOTE : Interval Notation

For < or > use (

For ≤ or ≥ use [

4) $b^2 \geq 10b - 25$

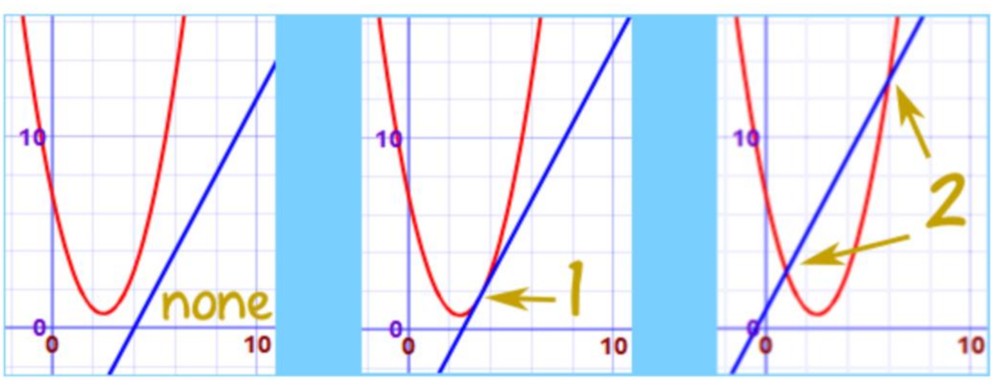
5) $2x^2 + 5x < 12$

6) $n^2 \leq 3$

Solving Linear-Quadratic Systems:

With a Linear-Quadratic System, there are three possible cases:

- _______ real solution (when the line and the quadratic ______________________________)
- _______ real solution (happens when ______________________ just touches the quadratic)
- _______ real solutions (happens when the line and the quadratic intersect ______________)



Solve the System Algebraically:

1) $y = x^2 + 5x + 6$
$y - 6 = x$

2) $y = x^2 - x - 6$
$y - 2x = -2$

3) $x^2 + y^2 = 25$
$4y = 3x$

4) $x^2 + y^2 = 26$
$x - y = 6$

<u>Steps:</u>

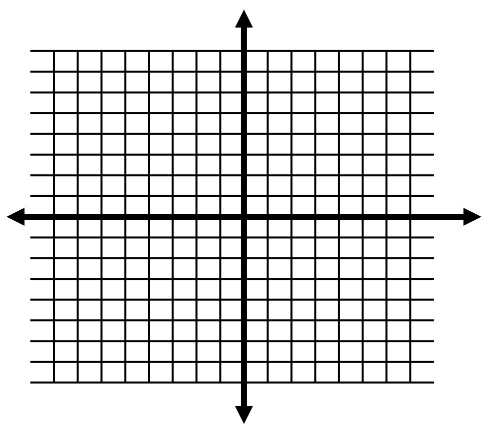
1) Set the equation = y (if this is not done for you already)
2) Since both equations = y, substitute one equation into the other. Then solve for x (you may need to factor).
3) Substitute the x-values back in to find the y-values
4) Your solutions are coordinate points! ☺

There are more ways to solve as well!!

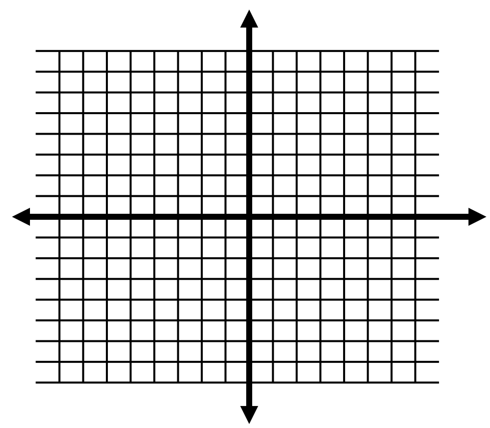
Practice with Solving Quadratics using Various Methods:

Solve by graphing:

1. $y \geq 2x^2 - 2x - 4$



2. $y \leq -x^2 + 2x + 8$



Solve the inequality algebraically:

3. $x^2 - 3x - 10 < 0$

4. $x^2 + x \geq 8$

Practice: Musical Chairs Activity

Day 12: Unit Review

Warm-Up:

13. The cost of an advertisement in a magazine is a function of its size.
    - A company wants its advertisement to have a height that is twice its width.
    - The magazine charges a flat rate of $60 plus an additional $10 per square inch.
    - The company can spend at most $2,060 on the advertisement.

    What is the maximum height that the company can afford for its advertisement?

Factor Completely. For #14, also find the solutions.

14. $3x^2 - 16x = 12$

15. $48x^8 - 3$