| Day 9: FRED Functions |
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Warm-Up:

1. An electronics company has a new line of portable radios with CD players. Their research suggests that the daily sales, s, for the new product can be modeled by $s = -p^2 + 120p + 1400$, where p is the price of each unit.
   a. What is the maximum daily sales total for the new product?   Xmax 150, ymax 5000
      2nd Trace → max → (60,5000) $5000 a day is max profit
   b. What price should the company charge to make this profit?
      $60 per portable radio will give max profit

2. The shape of the Gateway Arch in St. Louis is a catenary curve, which closely resembles a parabola. The function
   $y = -\dfrac{2}{315}x^2 + 4x$ closely models the shape of the arch, where y is the height in feet and x is the horizontal
   distance from the base of the left side of the arch in feet.   xmax 1000, ymax 1000
   a. What is the width of the arch at the base?   Find zeros in Calc 2nd Trace 2:zero
      630 feet = distance between X= 0, 630
   b. What is the maximum height of the arch?   two zeros ⤴
      2nd Trace 4:max   (315, 630)   630 feet tall

Day 9: FRED Functions Part 1

To the right is a graph of a **"Fred" function**. We can use Fred functions to explore transformations in the coordinate plane.

I.   Let's review briefly.

1. a. Explain what a function is in your own words.
      A function is a relation in which every element in the domain maps to exactly 1 element of the range
   b. Using the graph, how do we know that Fred is a function?
      • passes vertical line test
      • has one y-value for each x-value

2. a. Explain what we mean by the term domain.
      the set of all inputs (x-values) of a function or relation
   b. Using the graph, what is the domain of Fred?
      $\{x \mid -1 \le x \le 4\}$

3. a. Explain what we mean by the term range.
      the set of all outputs (y-values) of a function or relation
   b. Using the graph, what is the range of Fred?
      $\{y \mid -2 \le y \le 1\}$
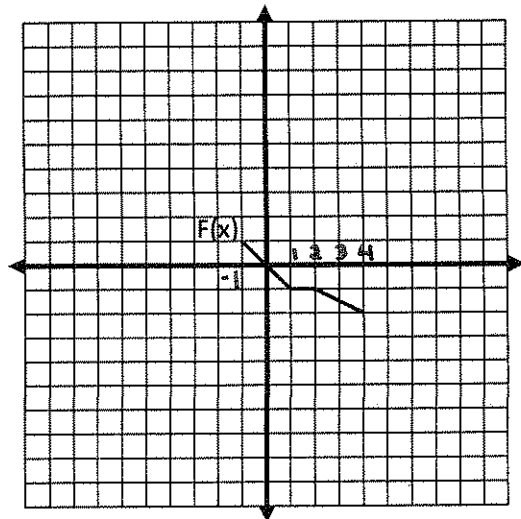
4. Let's explore the points on Fred.

a. How many points lie on Fred? *infinite amount*   Can you list them all? *NO*

b. What are the key points that would help us graph Fred?

$(-1, 1)$ $(1, -1)$ $(2, -1)$ $(4, -2)$

We are going to call these key points **"characteristic" points**. It is important when graphing a function that you are able to identify these characteristic points.

c. Use the graph of graph to evaluate the following.

$F(1) = $ *-1* $F(-1) = $ *1* $F(\underline{4}) = -2$ $F(5) = $ *undefined* ← *our domain doesn't continue to x=5 (graph only goes as far as x=4)*

remember, this means what is the y-value when x=1   *we must find the x-value that goes with a y-value of -2*

II. *Remember that F(x) is another name for the y-values.*

Therefore the equation of Fred is **y = F(x)**.

x	F(x)
−1	1
1	~1
2	~1
4	-2

F(x)+4
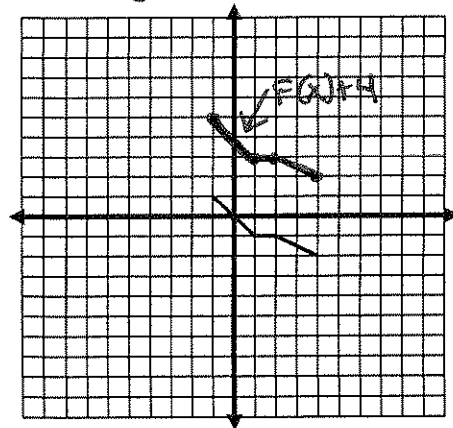
1. Why did we choose those x-values to put in the table?

These are the characteristic points of Fred and are critical to drawing the various parts of the graph

Now let's try graphing Freddie Jr.: **y = F(x) + 4**. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

y = F(x) + 4		
x	y	*y-value of Fred then add 4 to each*
−1	5	*1+4*
1	3	*~1+4*
2	3	*~1+4*
4	2	*-2+4*

y=F(x)+4 means the y-values in the F(x) tables are increased by 4.

★Remember F(x) is another way of writing the y-values for a function F.

2. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) + 4? (Be specific.)

Translation up 4 units   (be sure to tell type of transformation is translation, the amount of change 4, AND the direction up)

3. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

The x-values were NOT changed (compare the tables or characteristic points)

4. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

Each of the y-values for Freddie Jr was the y-value for Fred plus 4

5. In $y = F(x) + 4$, how did the "+4" affect the graph of Fred? Did it affect the domain or the range?

The "+4" translated Fred up 4. The range values were affected.

III.   Suppose Freddie Jr's equation is: $y = F(x) - 3$. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

$y = F(x) - 3$

x	y	
−1	−2	1−3
1	−4	−1−3
2	−4	−1−3
4	−5	−2−3

← F(x)−3
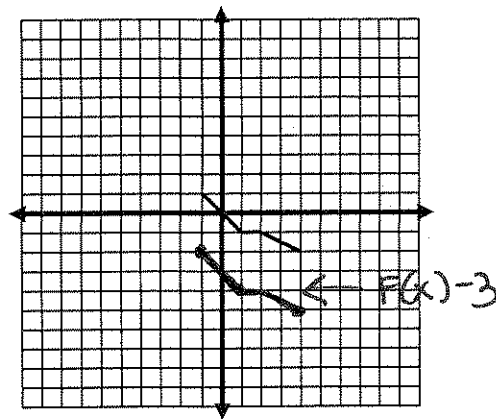
1. What type of transformation maps Fred, F(x), to Freddie Jr., F(x) − 3? Be specific.

Translation down 3, from Fred F(x) to Freddie Jr F(x)−3.

type   direction amount ← *Be sure to include ALL the parts of the description!*

2. How did this transformation affect the x-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

There was NO change to the x-values

3. How did this transformation affect the y-values? *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

Yes. Freddie Jr's y-values are 3 lower than Fred's.

4. In $y = F(x) - 3$, how did the "− 3" affect the graph of Fred? Did it affect the domain or the range?

The "−3" moved Fred down 3. The range values were affected.

IV.   Checkpoint: Using the understanding you have gained so far, describe the affect to Fred for the following functions.

Equation	Effect to Fred's graph
Example: y=F(x) + 18	Translate up 18 units
1. y = F(x) − 100	Translate down 100 units
2. y = F(x) + 73	Translate up 73 units
3. y = F(x) + 32	Translate up 32 units
4. y = F(x) − 521	Translate down 521 units

V.   Suppose Freddie Jr's equation is: y = F(x + 4).

1. Complete the table ✱ See Hint below!

x	x + 4	y
−5	−1	1
-3	1	−1
-2	2	−1
0	4	−2
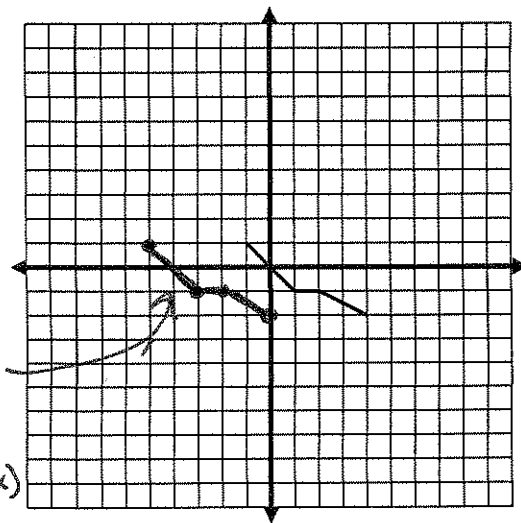
↓ These values came from Fred F(x)

F(x+4)

✱ (Hint: Since, x + 4 = −1, subtract 4 from both sides of the equation, and x = −5. Use a similar method to find the missing x values.)

2. On the coordinate plane above, graph the 4 ordered pairs (x, y). The first point is (−5, 1).
    (-5, 1) (-3, -1) (-2, -1) (0, -2) are the characteristic points

3. What type of transformation maps Fred, F(x), to Freddie Jr., F(x + 4)? (Be specific.)
    Translation left 4 units

4. How did this transformation affect the x-values?   (Hint: Compare the characteristic points of Fred and Freddie Jr.)
    The x-values were decreased 4 units from Fred to Freddie Jr

5. How did this transformation affect the y-values?   (Hint: Compare the characteristic points of Fred and Freddie Jr.)
    The y-values were NOT affected

6. In y = F(x + 4), how did the "+4" affect the graph of Fred?   Did it affect the domain or the range?
    The "+4" translated Fred left 4 units.
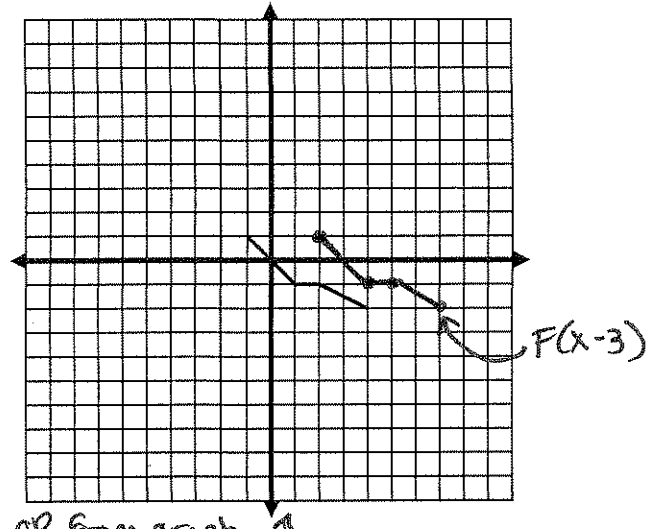   The domain was affected.

VI.   Suppose Freddie Jr's equation is: $y = F(x - 3)$. Complete the table below for this new function and then graph Freddie Jr. on the coordinate plane above.

1.   Complete the table.

☐
Fix
Middle
column

$y = F(x - 3)$

x	x − 3	y
2	−1	1
4	1	−1
5	2	−1
7	4	−2

↑
filled in from earlier
Fred F(x) table OR from graph ↗

F(x-3)

2.   On the coordinate plane above, graph the 4 ordered pairs (x, y). [*Hint: The 1ˢᵗ point should be (2, 1).*]

(2,1) (4,-1) (5,-1) (7,-2)

3.   What type of transformation maps Fred, F(x), to Freddie Jr., F(x − 3)? (Be specific.)

Translation right 3 units

4.   How did this transformation affect the x-values?   *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

From Fred to Freddie Jr the x-values increased 3 units
     F(x)   F(x-3)

5.   How did this transformation affect the y-values?   *(Hint: Compare the characteristic points of Fred and Freddie Jr.)*

The y-values were NOT affected

6.   In $y = F(x - 3)$, how did the " −3" affect the graph of Fred? Did it affect the domain or the range?

The "-3" ~~moved~~ translated the graph right 3

VII.   Checkpoint: Using the understanding you have gained so far, describe the effect to Fred for the following functions.

| Equation	Effect to Fred's graph
Example: y=F(x + 18)	Translate left 18 units
1. y = F(x − 10)	Translate right 10 units
2. y = F(x) + 7	Translate up 7 units
3. y = F(x + 48)	Translate left 48 units
4. y = F(x) − 22	Translate down 22 units
5. y = F(x + 30) + 18	Translate left 30 units and up 18 units

VIII.   Checkpoint: Using the understanding you have gained so far, write the equation that would have the following effect on Fred's graph.

Equation		Effect to Fred's graph
Example:	$y = F(x + 8)$	Translate left 8 units
1.	$y = F(x) + 29$	Translate up 29 units
2.	$y = F(x - 7)$	Translate right 7
3.	$y = F(x + 45)$	Translate left 45
4.	$y = F(x+5) + 14$	Translate left 5 and up 14
5.	$y = F(x-6) - 2$ *	Translate down 2 and right 6

* be careful the order of this saying doesn't have to match the order it goes in on equation necessarily

IX.   Now let's look at a new function.
       Its notation is **H(x)**, and we will call it **Harry**.
       Use Harry to demonstrate what you have learned
       so far about the transformations of functions.
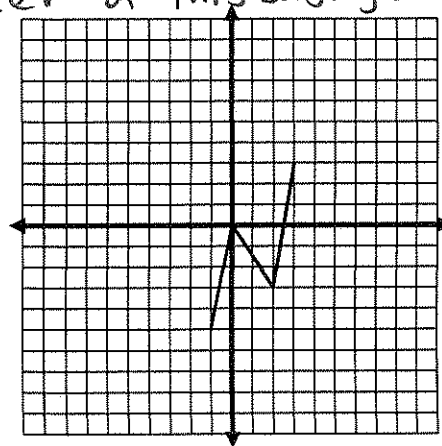
1. What are Harry's characteristic points?

$(-1, 5) (0, 0) (2, -3) (3, 3)$

2. Describe the effect on Harry's graph for each
    of the following.
    a. $H(x - 2)$ Translated right 2 units
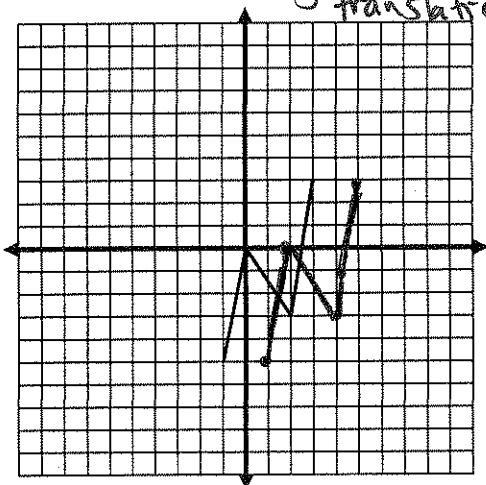   b. $H(x) + 7$ Translated up 7 units
    c. $H(x+2) - 3$ Translated left 2 units and down 3 units

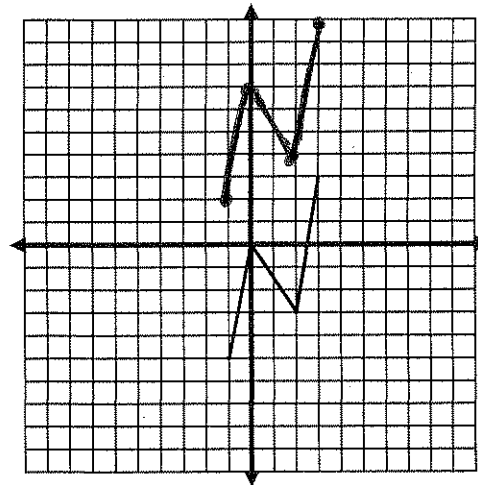3. Use your answers to questions 1 and 2 to help you sketch each graph *without using a table*.
    a. $y = H(x - 2)$ right 2 units translation
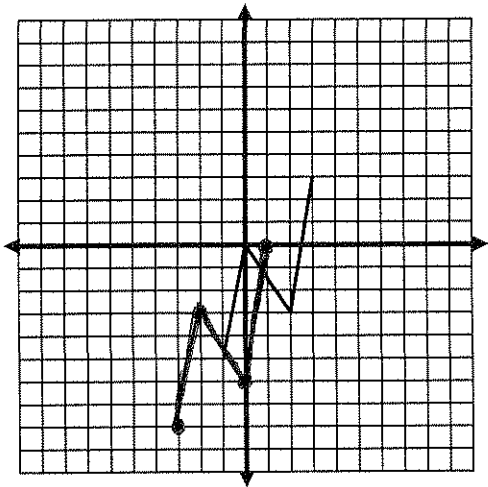   b. $y = H(x) + 7$ translated up 7

c. $y = H(x+2) - 3$ *translated left 2 units & down 3 units*

Day 10: Transformations of Quadractics

Warm-Up:

11. Using the discriminant, determine the amount and type of solutions each equation will have. Then find the exact value of the solutions.

   a. $x^2 + 4x + 5 = 0$

   b. $x^2 - 2x + 1 = 0$

   c. $2x^2 - 3x - 10 = 0$

Day 10: FRED Functions Part 2

I.   Let's suppose that Freddie Jr. is $y = -F(x)$

7. Complete the table.

$y = -F(x)$

x	F(x)	y
−1	1	−1
1		
2		
4		