Day 7: The Quadratic Formula and Discriminants

Warm-Up:

9. Solve each of the quadratic functions by graphing and algebraic reasoning

a. $x^2 - 3 = 0$ $\sqrt{x^2} = \sqrt{3}$ $x = \pm\sqrt{3}$

b. $x^2 + 5x - 8 = 0$ graph $x = -6.3, 1.2$

c. Explain why having alternative methods of solving is important.

if the problems aren't factorable.

Simplify the radicals

d) $\sqrt{16} = \sqrt{4 \cdot 4} = \boxed{4}$ need group of 2

e) $\sqrt{50} = \sqrt{5 \cdot 5 \cdot 2} = \boxed{5\sqrt{2}}$ need group of 2

f) $\sqrt{80} = \sqrt{4 \cdot 4 \cdot 5} = \boxed{4\sqrt{5}}$ need group of 2 for $\sqrt{\ }$

Day 7: The Quadratic Formula and Discriminants

Standard form of a quadratic *equation*: $y = ax^2 + bx + c$

The solutions of some quadratic equations are not rational, or are too messy to obtain by factoring. For such equations, the most common method of solution is the quadratic formula.

The quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be used to solve for the values of x.

Notice that there is a +/- sign in the formula. There are actually two answers for any quadratic formula.

Solve using the quadratic formula.

Example 1: $x^2 + 9x + 20 = 0$

$x = \dfrac{-9 \pm \sqrt{9^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1} = \dfrac{-9 \pm 1}{2}$

$x = \dfrac{-9+1}{2}, \dfrac{-9-1}{2}$ $x = -4, -5$

Example 2: $x^2 - x = 5x - 9$

$x^2 - 6x + 9 = 0$

$x = \dfrac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$ $x = \dfrac{6 \pm 0}{2}$ $x = 3$

looks like "bounce" on axis

*called a "double root"

$\sqrt{60} = \sqrt{2 \cdot 3 \cdot 2 \cdot 5}$ $2\sqrt{15}$

Example 3: $-x^2 + 2x = 2$ $-x^2 + 2x - 2 = 0$

You Try

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot -1 \cdot -2}}{2 \cdot -1}$ $x = \dfrac{-2 \pm \sqrt{-4}}{-2}$ $x = 1 + \dfrac{\sqrt{-4}}{-2}$

* negative under square root gives 2 "imaginary" roots... more on this later!

Example 4: $7x^2 - 12x + 3 = 0$

$x = \dfrac{+12 \pm \sqrt{12^2 - 4 \cdot 7 \cdot 3}}{2 \cdot 7}$ $x = \dfrac{12 \pm \sqrt{60}}{14}$

$x = \dfrac{12 \pm 2\sqrt{15}}{14}$ $x = \dfrac{6 \pm \sqrt{15}}{7}$

Get GCF of 12, 2, & 14 and factor 2 it out

$\sqrt{45}$ $\sqrt{9}\sqrt{5}$ or $3\sqrt{5}$

$\sqrt{3 \cdot 3 \cdot 5}$ look for pairs

Example 5: $4x^2 + 12x + 9 = 0$

You Try

$x = \dfrac{-12 \pm \sqrt{12^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}$ $x = \dfrac{-12 \pm 0}{8}$

$x = \dfrac{-12}{8}$ $x = \dfrac{-3}{2}$ "double root"

Example 6: $x^2 - 5x - 5 = 0$

$x = \dfrac{5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot -5}}{2}$ $x = \dfrac{5 \pm \sqrt{45}}{2}$

$\sqrt{3 \cdot 3 \cdot 5}$ $3\sqrt{5}$

$x = \dfrac{5 \pm 3\sqrt{5}}{2}$ You Try

Types of Zeroes

Given the following quadratic functions, use the quadratic formula to find the zeros:

1. $x^2 - x - 6 = 0$

$x = \frac{1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot -6}}{2 \cdot 1}$ $\boxed{x = 3, -2}$

$x = \frac{1 \pm \sqrt{25}}{2}$ $x = \frac{1 \pm 5}{2}$

2. $x^2 + 16 = 0$

$x = \frac{0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 16}}{2 \cdot 1}$

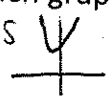
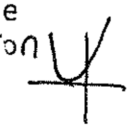
$x = \frac{0 \pm \sqrt{-64}}{2}$ $\boxed{x = \frac{\pm\sqrt{-64}}{2}}$

2 "imaginary" roots

3. $x^2 + 4x + 4 = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

$x = \frac{-4 \pm 0}{2}$ $\boxed{x = -2}$ "double root"

Use your calculator to examine the graphs of each function. How does each graph relate to the number of solutions for that problem? 2 solutions  0 real solutions  1 solution 

– crosses the x-axis that many times

Which function's graph did not touch the x-axis? Which touched the x-axis once?

#2 → 2 "imaginary roots" #3 → "double root"

How was the quadratic formula different when you had two, one, or no zeroes?

1. When can you expect 2 solutions in a quadratic equation? a positive # under $\sqrt{\ }$
2. When can you expect 1 solution in a quadratic equation? a $\emptyset$ under $\sqrt{\ }$ → 1 "double root"
3. When can you expect a quadratic to have no real solutions? a negative # under $\sqrt{\ }$ → 2 "imaginary" roots

Notes:

Recall the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

This part in the square root helps us to determine how many solutions a quadratic will have:

$b^2 - 4ac$

This is called the **Discriminant**. Calculate the discriminant for the 3 introductory problems.

1. $x^2 - x - 6 = 0$

$b^2 - 4ac$

$(-1)^2 - 4 \cdot +1 \cdot -6$

25

2. $x^2 + 16 = 0$

$0^2 - 4 \cdot 1 \cdot 16$

-64

3. $x^2 + 4x + 4 = 0$

$b^2 - 4 \cdot a \cdot c$

$4^2 - 4 \cdot 1 \cdot 4$

0

Types of Quadratic Solutions

Quadratic solutions are either <u>real</u> or <u>imaginary</u>

Real solutions are the solutions you get from factoring, the zeroes on the graph, and when you are able to do the square root in the quadratic formula.

Imaginary solutions do not show up on the graph or when factoring. In fact, quadratics with imaginary solutions cannot be factored.

- If the discriminant is **positive**, the quadratic has two real solutions.
- If the discriminant is **zero**, the quadratic has One real solution.
- If the discriminant is **negative**, the quadratic has two imaginary solutions

Real solutions can also be divided into two types:

- Rational solutions are when the discriminant evaluates to a perfect square
  (can be written as a fraction)
- Irrational solutions are when the discriminant evaluates to NOT a perfect square.
  (can't be written as a fraction)

$b^2 - 4ac$

Practice: Given the following quadratics, calculate the discriminant (b^2-4ac) and determine how many and what types of solutions they will have. For real solutions, determine if they will be rational or irrational.

1. $x^2 - 6x + 11 = 2$ $6^2 - 4 \cdot 1 \cdot 11 = -8$
   - 2 imaginary solutions (imag. b/c negative)

2. $3x^2 + 5x = 12$ ($3x^2+5x-12$) $5^2 - 4 \cdot 3 \cdot -12 = 169$
   - 2 real/rational solutions (rational b/c 169 is perfect square)

3. $3x^2 + 48 = 0$ $0^2 - 4 \cdot 3 \cdot 48 = -576$
   - 2 imaginary solutions

4. $x^2 - 27 = 0$ 108
   - 2 real/irrational solutions (irrational b/c 108 is NOT perfect square)

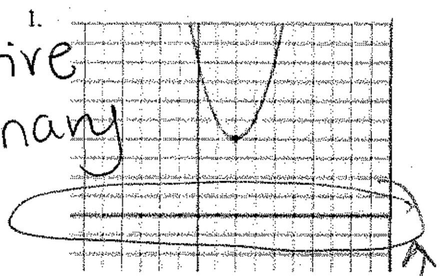
5. $x^2 + x + 1 = 0$ -3
   - 2 imaginary solutions

6. $x^2 + 4x - 1 = 0$ 20
   - 2 real/irrational solutions (irrational b/c 20 is NOT perfect square)

Given the following graphs of quadratic functions:

a) Determine the **sign** of the discriminant and **b)** whether the solutions are **real** or ~~**non-real**~~ imaginary.

1.

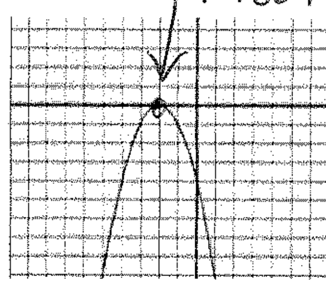


- negative
- imaginary

no x-intercepts

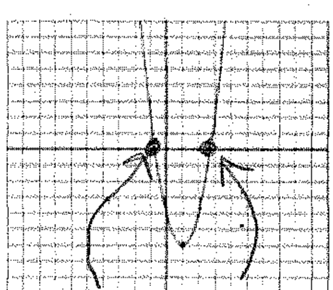
2.

1 root



- positive
  real
  "1 double root"

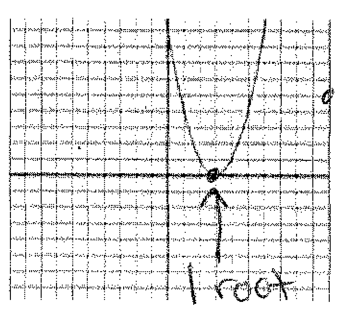
3.



- positive
- real

2 roots

4.



1 root

- positive
  real

Practice

$b^2 - 4ac$

Calculate the discriminant and determine the number and types of solutions.

Function	Discriminant	Number and Type of Solutions
1. $x^2 - 3x - 4 = 0$ $a = 1, b = -3, c = -4$ $(-3)^2 - 4(1)(-4)$ $9 - (-16)$ $9 + 16$	25	real 2 rational solutions
2. $x^2 - 6x + 9 = 0$	0	real 1 rational
3. $x^2 + 6x = -9$ $x^2 + 6x + 9$	0	real 1 rational
4. $x^2 - 6x - 16 = 0$	100	real 2 rational
5. $2x^2 - 6x - 13 = 0$	140	real 2 irrational
6. $-x^2 + 2x - 1 = 0$	0	real 1 rational
7. $2x^2 + 3 = 2x$ $2x^2 - 2x + 3$	-20	2 imaginary
8. $x^2 + 2x + 1 = 0$	0	real 1 rational
9. $x^2 + 2x = -3$ $x^2 + 2x + 3$	-8	2 imaginary
10. $x^2 - 6x + 9 = 0$	0	real 1 rational
11. $x^2 + 5x + 8 = 0$	-7	2 imaginary
12. $2x^2 - 5x + 6 = 0$	-23	2 imaginary
13. $x^2 - 5x = 10$ $x^2 - 5x - 10$	65	real 2 irrational
14. $x^2 - 6x + 3x = 4 - 11$ $x^2 - 3x + 7 = 0$	-19	2 imaginary