Unit 2 Day 5

Characteristics of Quadratic Functions

Warm Up

1.) Jason and Jim jumped off a cliff into the ocean in Acapulco while vacationing. Jason's height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, while Jim's height could be modeled by $h(t) = -16t^2 + 12t + 480$ where t is the time in seconds and h is the height in feet. Whose jump was higher and by how much? Hint: One of the functions is NOT factorable. So for that one, use your calculator to find the zeros.

Jason jumped 1.75 ft higher with a maximum at (1/2, 484).
Jim's maximum was (3/8, 482.25).

2.) Using the following quadratic, find zeros, y-intercept, vertex, one other point, and the axis of symmetry, then sketch the graph.

$y = -2x^2 + 4x + 70$

Zeros: (-5,0) (7,0)
y-intercept: (0, 70)
Vertex: (1, 72)
Maximum
A.o.S. $x = 1$

Homework Answers Packet p. 5

Function	Solutions (solve by factoring)	x-intercept locations (x,y)	y- intercept location (x,y)	Vertex location (x,y)	Axis of Symmetry	Is the vertex the maximum or minimum value of the function? Explain why.
1. $y = 15x^2 + x$	x(15x +1) x = 0, -1/15	(0, 0) (-1/15,0)	(0, 0)	(-1/30, -1/60)	x=-1/30	Minimum, a > 0
2. $y = 2x^2 + 3x + 1$	(2x +1)(x + 1) x = -1/2, -1	(-1/2, 0) (-1, 0)	(0, 1)	(-3/4, -1/8)	x=-3/4	Minimum, a > 0
3. $y = 4x^2 - 9$	(2x+3)(2x-3) x = -3/2, 3/2	(-3/2,0) (3/2, 0)	(0,-9)	(0,-9)	x=0	Minimum, a > 0

Homework Answers

4. A smoke jumper jumps from a plane that is 1700 feet above the ground. The function $y = -16x^2 + 1700$ gives a jumper's height y in feet after x seconds.

a. How long is the jumper in free fall if the parachute opens at 1000 ft?

$1000 = -16x^2 + 1700$

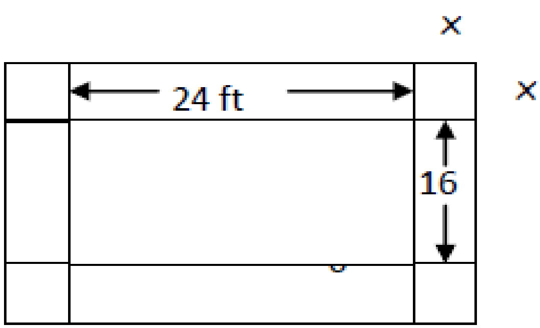
$x = 6.6$ seconds

b. How long is the jumper in free fall if the parachute opens at 940 ft?

$940 = -16x^2 + 1700$

$x = 6.9$ seconds

5. You want to expand the garden below by planting a border of flowers. The border will have the same width around the entire garden. The flowers you bought will fill an area of 276 ft^2. How wide should the border be?



$x = 3$ ft wide

$4x^2 + 80x - 276 = 0$

$4(x^2 + 20x - 69) = 0$

$4(x + 23)(x - 3) = 0$

$x = -23, 3$

Tonight's Homework

Packet p. 7
Finish Packet p. 6

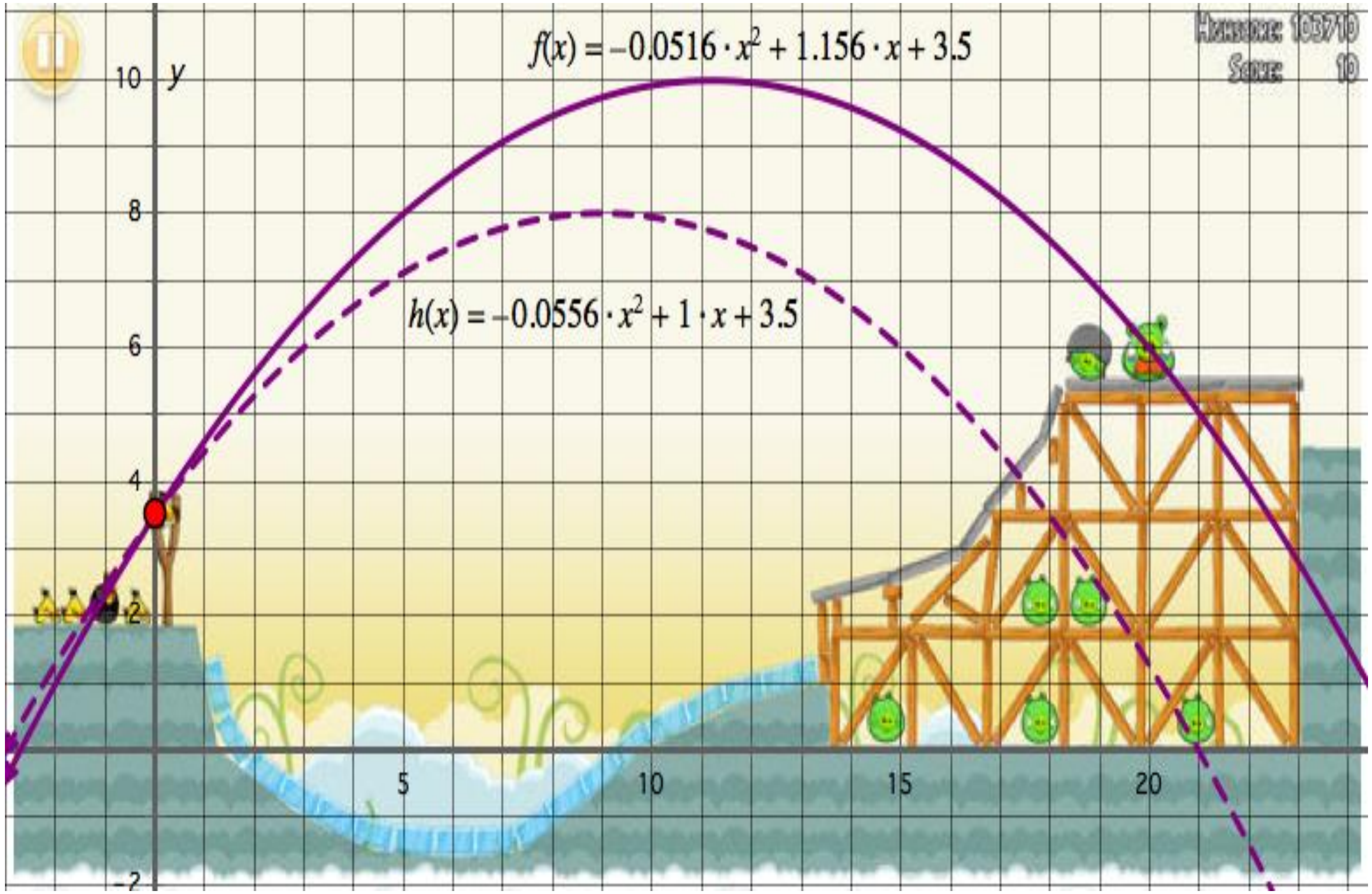
Hint: If you get stuck on pg. 6, look back at the HW assigned for the night of the Unit 1 Test. ☺

Let's Discuss a few questions from Angry Birds:

(Completed for HW)

Show Investigation Notes pages on document camera when reviewing with class.

Angry Birds Round 1



5. How far does Angry Bird fly in h(x)? How high does he go?

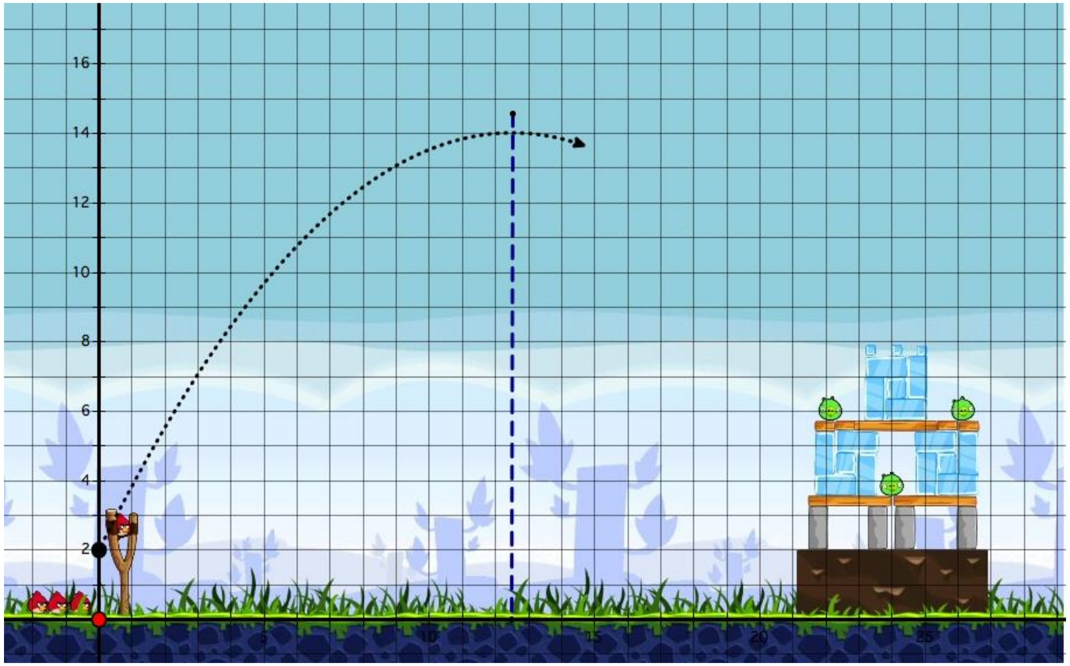
21 units (see x-intercept) 8 units (see y-value of vertex)

How far away from the catapult is he when he is at his highest?

9 units away (see x-value of vertex)

When he is 15 feet away, how high is he flying? 6 units high

Angry Birds Round 2



2. The axis of symmetry is provided. What part of the parabola does this pass through? The vertex

What does this part represent about Angry Bird's flight?

Angry Bird's highest point

6. Without solving for the whole equation, what is "c" value in standard form? Is "a" positive or negative? C is the y-intercept

"a" is negative because the parabola is facing down

Angry Birds Round 3

1. Angry bird and hungry pig are 18 feet away from each other. If angry bird and hungry pig are at the same height (y-value) when angry bird is catapulted, at what distance away is Angry Bird the highest? Think about symmetry.

The x-value of the vertex (0+18)/2 = 9 ft

2. Using the picture, what is the y-intercept? (0, 0)

Using the picture, what are the x-intercepts? (0, 0), (22, 0)

Axis of Symmetry: X = 11

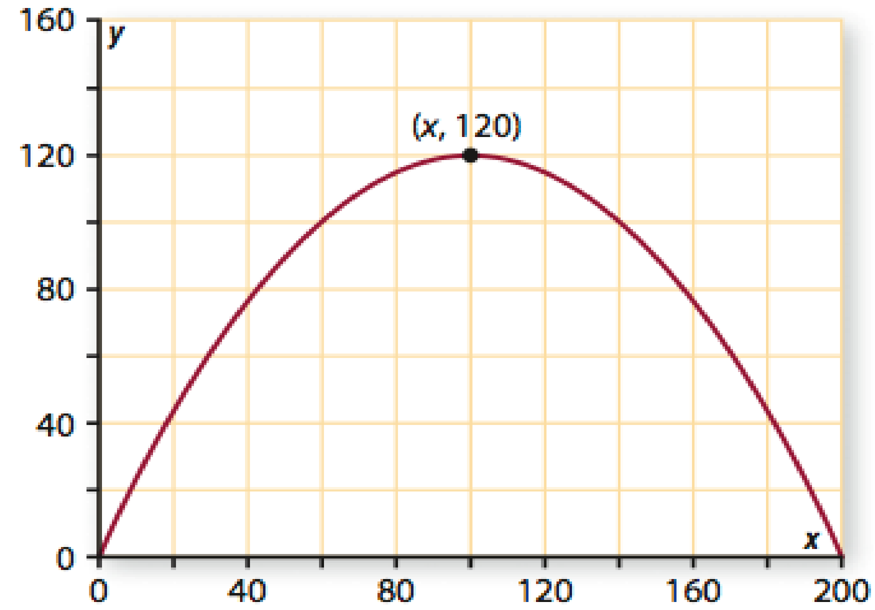
Designing Parabolas

Notes p. 12-13

Guided Practice Notes p. 12

Determine the equation of a quadratic that guarantees the graph of its parabola will fit given constraints.

For example, write an equation of a quadratic with zeroes at $x = 0$ and $x = 200$ and a maximum y-value of 120.



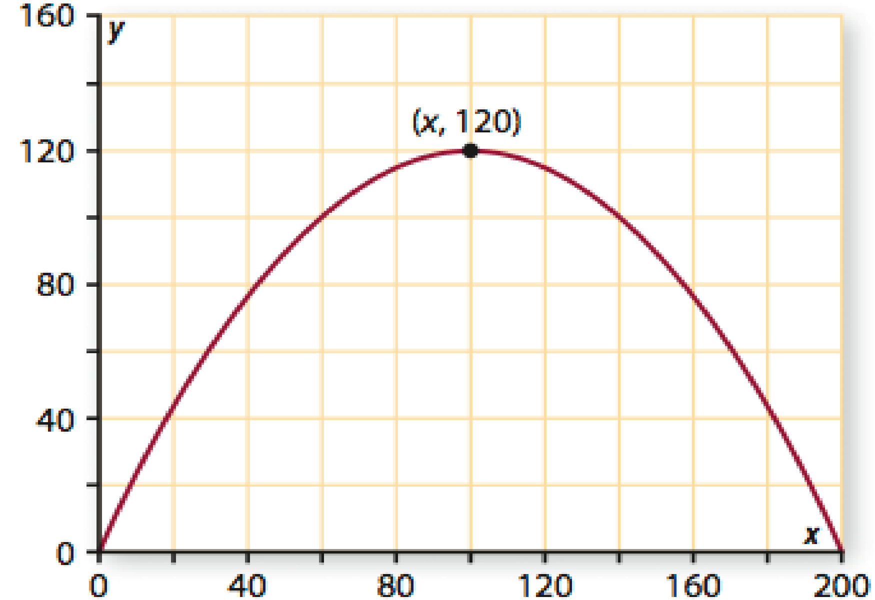
a. The graph of the desired function has x-intercepts (0, 0) and (200, 0). How do you know that the graph of the function $f(x) = x(x - 200)$ has those same x-intercepts?

If you set the factors = 0 and solve, you get the x-values of the x-intercepts.

Problem continued on next slide! ->

Guided Practice Notes p. 12

For example, write an equation of a quadratic with zeroes at x = 0 and x = 200 and a maximum y-value of 120.



We just found that the Vertex x-value is 100.

c. Suppose that $g(x)$ has a rule in the form $g(x) = a[x(x - 200)]$, for some particular value of a. What value of a will guarantee that $g(x) = 120$ at the maximum point of the graph?

Remember that y is another name for $g(x)$ or $f(x)$.

$120 = a(100)(100 - 200)$ Substitute x = 100 & solve for a

$120 = a(-10000)$ $a = 120 / -10000$ **$a = -3/250$**

Problem continued on next slide! ->

Now try the Practice on Notes p. 12

2. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

a. X-intercepts at (4, 0) and (-1, 0)

b. X-intercepts at (7, 0) and (1, 0) and minimum point at (4, -10)

c. X-intercepts at (3, 0) and (-5, 0) and maximum point at (-1, 8)

Practice

2. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

   a. X-intercepts at (4, 0) and (-1, 0)

$y = a(x - \text{root})(x - \text{root})$ **To start the factored Form**

****Write this in your notes!!**

$y = a(x - 4)(x + 1)$ **Factored Form**

Examples:

$y = x^2 - 3x - 4$* **Expanded Form**

$y = 2x^2 - 6x - 8$*

***No point was given besides the x-intercepts, so we can't find just 1 exact k value OR just 1 equation for this problem. ☹**

Practice

2. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

b. X-intercepts at (7, 0) and (1, 0) and minimum point at (4, -10)   Solve $-10 = a(4 - 7)(4 - 1)$ to get $a = 10/9$

$y = 10/9(x - 7)(x - 1)$ **Factored Form**

$y = 10/9x^2 - 80/9x + 70/9$ **Expanded Form**

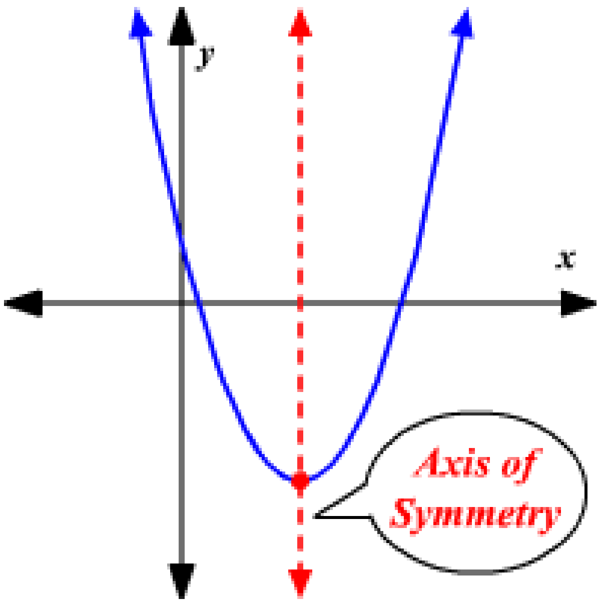
c. X-intercepts at (3, 0) and (-5, 0) and maximum point at (-1, 8)   Solve $8 = a(-1 - 3)(-1 + 5)$ to get $a = -1/2$

$y = -1/2(x - 3)(x + 5)$ **Factored Form**

$y = -1/2x^2 - x + 15/2$ **Expanded Form**

Notes p. 13
Some observations

- **Axis of symmetry:** a vertical line that divides the parabola into two congruent halves. It always passes through the vertex of the parabola. The x-coordinate of the vertex is the equation of the axis of symmetry



Equation of a Parabola

- **Standard Form:** $y = ax^2 + bx + c$

- **We can also use the zeros to help write an equation using the form:** $y = a(x - r)(x - s)$ **where *r* and *s* represent the zeros (roots) found on the graph.**

Summary!

How to create an equation of a parabola from a graph:

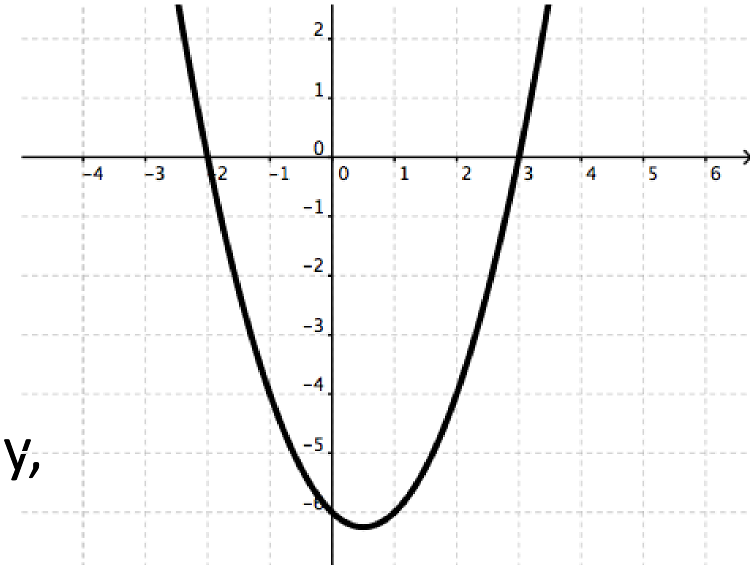
1. Find the zeros.
2. Write the zeros as two binomials.

$$y = a(x - r)(x - s) \text{ } \textit{if you have the roots r, s}$$

3. Substitute a coordinate point for x and y, then solve to find the value “a”.
4. Substitute “a” into the ORIGINAL equation to get Factored Form. Multiply the binomials and distribute to get Standard Form.

Example

On your notes, fix the purple parts!!



1. Find the zeros → (-2, 0) and (3, 0)
2. Write the zeros as two binomials.

$$y = a(x - r)(x - s)$$

→ y = a(x + 2)(x – 3)

Be careful with your signs!!

3. Substitute a coordinate point for x and y, then solve to find the value "a".

Use a point that you can see clearly on the graph.

(Hint: the y-intercept or vertex are typically easy to use)

→ y = a(x + 2)(x – 3) and use <u>y-intercept</u>: (0, -6)

-6 = a(0 + 2)(0 – 3)   -6 = -6a   a = 1

4. Substitute "a" into the ORIGINAL equation to get Factored Form. Multiply the binomials and distribute to get Standard Form.

→ y = a(x + 2)(x – 3) = a(x^2 – x – 6)   *multiplied*

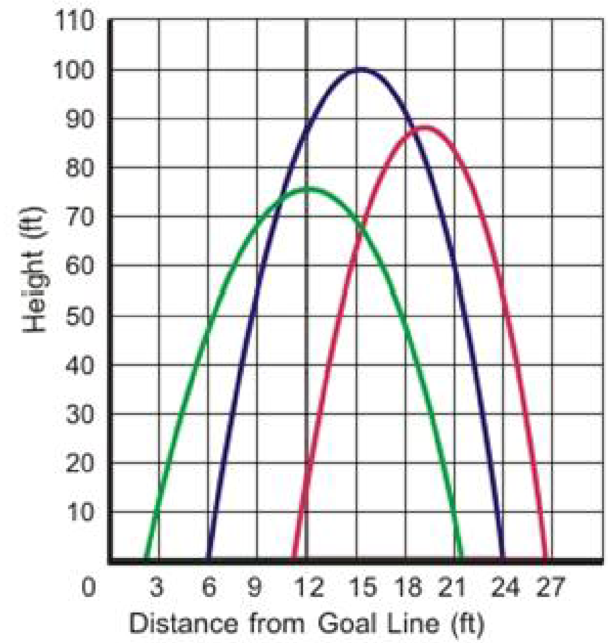
y = 1(x^2 – x – 6)   *substituted in a = 1

$\boxed{y = x^2 - x - 6}$ ***distributed "a" for answer!**

Practice

Practice

For each graph, write an equation based on the characteristics you're given. Then, explain using algebraic reasoning which graph has the greatest maximum and which egg travels the farthest.



***Green points (2, 0), (22, 0), (12, 75)**

***Purple points (6, 0), (24, 0), (15, 100)**

***Red points (11, 0), (26, 0), (24, 50)**

$y = -\frac{3}{4}x^2 + 18x - 33$ *Green points (2, 0), (22, 0), (12, 75)

$y = -100/81x^2 + 1000/27x - 1600/9$ *pts (6, 0), (24, 0), (15, 100)

$y = -25/13x^2 + 925/13x - 550$ *pts (11, 0), (26, 0), (24, 50)

Tonight's Homework

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Finish Packet p. 6

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