Function 5:

Zeroes:

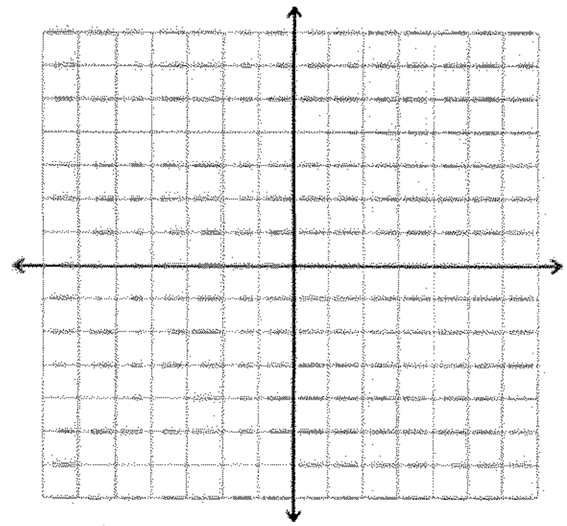
x-intercepts:

y-intercept:

Location of vertex:

Axis of Symmetry:

Is vertex the minimum or maximum of the function?



Day 4: Finding Extrema of Quadratic Functions

Warm-Up:

6. For the following two equations, find the following values, showing your work for finding them by hand! Then sketch the graphs on graph paper.

a. $x^2 - x - 20 = 0$ $(x-5)(x+4)=0$

zeros: $(5,0)$ $(-4,0)$ $\frac{5+-4}{2} = \frac{1}{2} =$ x of vertex

vertex: $(\frac{1}{2}, -20\frac{1}{4})$ $y=(\frac{1}{2})^2 - \frac{1}{2} - 20 = -20\frac{1}{4} =$ y of vertex

y-intercept: $(0,-20)$

Max/min?: min $a>0$ so "smile" parabola

Axis of Symmetry (AoS):

$x = \frac{1}{2}$

b. $x^2 + 8x + 15 = 0$ $(x+5)(x+3)=0$

zeros: $(-5,0)$ $(-3,0)$ $\frac{-5+-3}{2} = \frac{-8}{2} = -4 =$ x of vertex

vertex: $(-4,-1)$ $y=(-4)^2+8(-4)+15 = 16-32+15 = -1 =$ y of vertex

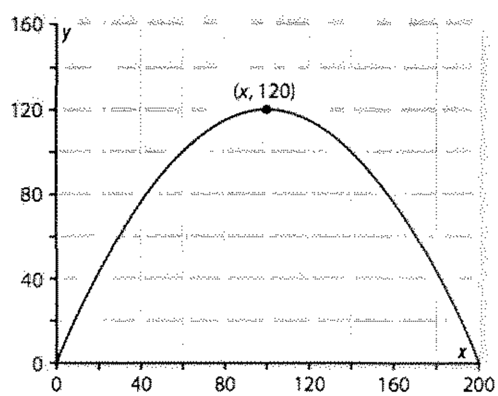
y-intercept: $(0,15)$

Max/min?: min $a>0$ so "smile" parabola

Axis of Symmetry (AoS):

$x = -4$

Investigation 2: Designing Parabolas



Details on writing equation on next page

From Core-Plus Mathematics Course 2 Book, p.332-334.

Objective: Determine the equation of a quadratic that guarantees the graph of its parabola will fit given constraints. For example, write an equation of a quadratic with zeroes at x = 0 and x = 200 and a maximum y-value of 120.

1. Using ideas from your earlier study of Quadratic Functions and their graphs, write the rule for a function with a parabolic graph that contains points (0, 0), (200, 0), and a maximum point whose y-coordinate is 120. Use the hints in Parts a-d, below, as needed.

   a. The graph of the desired function has x-intercepts (0, 0) and (200, 0). How do you know that the graph of the function f(x) = x(x - 200) has those same x-intercepts?

   If set factors = 0 + solve, you get the x-values of the x-intercepts.

   b. What is the x-coordinate of the maximum point on this graph?

   x of vertex $= \frac{0+200}{2} = 100$ (using yesterday's graphing knowledge)

   $\boxed{x \text{ of vertex} = 100}$

   c. Suppose that g(x) has a rule in the form g(x) = k[x(x - 200)], for some particular value of k. What value of k will guarantee that g(x) = 120 at the maximum point of the graph?

   $120 = k(100)(100-200)$ substitute x=100 of vertex from part b

   $\frac{120}{-10000} = \frac{k(-10000)}{-10000}$ $\boxed{k = -3/250}$

   d. Write the rule for g(x) in equivalent expanded form using the k value you found in Part c.

   $g(x) = -3/250\,(x)(x-200)$ substitute k into equation

   $\boxed{g(x) = -3/250\,x^2 + 12/5\,x}$

2. Write rules for quadratic functions whose graphs have the following properties. If possible, write more than one function rule that meets the given conditions.

   a. X-intercepts at (4, 0) and (-1, 0)

   $g(x) = k(x-4)(x+1)$

   Examples $\boxed{y = x^2 - 3x - 4}$ (if k=1) $\boxed{y = 2x^2 - 6x - 8}$ (if k=2)

   b. X-intercepts at (7, 0) and (1, 0) and minimum point at (4, -10)

   $g(x) = k(x-7)(x-1)$

   $-10 = k(4-7)(4-1)$ → $-10 = k(-3)(3)$

   $\frac{-10}{-9} = k$ $k = \frac{10}{9}$

   $y = \frac{10}{9}(x-7)(x-1)$

   $y = \frac{10}{9}(x^2 - 8x + 7)$

   $\boxed{y = \frac{10}{9}x^2 - \frac{80}{9}x + \frac{70}{9}}$

   c. X-intercepts at (3, 0) and (-5, 0) and maximum point at (-1, 8)

   $g(x) = k(x-3)(x+5)$

   $8 = k(-1-3)(-1+5)$

   $8 = k(-4)(4)$ → $8 = k(-16)$ $8/-16 = k$

   $k = -1/2$

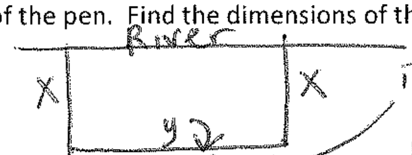
   $y = -1/2(x-3)(x+5)$

   $-1/2(x^2 + 2x - 15)$

   $\boxed{y = -1/2x^2 - x + 15/2}$

Application:

A rancher is constructing a cattle pen by the river. She has a total of 150 feet of fence and plans to build the pen in the shape of a rectangle. Since the river is very deep, she need only fence 3 sides of the pen. Find the dimensions of the pen so that it encloses the maximum area.



if 150 ft of fence and 2 sides are x

perimeter = x + x + y

150 = 2x + y

→ y = 150 - 2x

Area = b·h (or l·w)

= x(150 - 2x)

→ x = 0, x = 75 are intercepts

Vertex gives maximum

$x = \frac{0+75}{2} = 37.5$

150 - 2(37.5)

$\boxed{37.5 \text{ ft by } 75 \text{ ft}}$

Practice: Factor and solve.

1. $4x^2 + 7x = 2$

   $8 \cdot -1 = -8$, $8 + -1 = 7$

   $4x^2 + 7x - 2 = 0$

   $4x^2 + 8x - 1x - 2 = 0$

   $4x(x+2) - 1(x+2) = 0$

   $(4x-1)(x+2) = 0$

   $\boxed{x = 1/4, -2}$

2. $x^2 - 36 = 0$

   $(x+6)(x-6) = 0$

   $\boxed{x = -6, 6}$

3. $4x^2 + 12x - 72 = 0$

   $4(x^2 + 3x - 18) = 0$

   $4(x+6)(x-3) = 0$

   $\boxed{x = -6, 3}$