Day 3: Finding Extrema of Quadratic Functions

Warm-Up:

3. Factor to solve the following:

a. $x^2 - 5x + 50 = 0$ — prime

b. $x^2 + 3x = 10$ — $x^2+3x-10=0$; $(x+5)(x-2)=0$; $x=-5, 2$

c. $2x^2 + 7x = -3$ — $2x^2+7x+3=0$; $2x^2+6x+1x+3=0$; $2x(x+3)+1(x+3)=0$; $(2x+1)(x+3)=0$; $x=-\frac{1}{2}, -3$

4. Factor to solve the following:

a. $x^2 + 2x - 35 = 0$ — $(x+7)(x-5)=0$; $x=-7, 5$

b. $2x^2 + x = 3$ — $2x^2+x-3=0$; $2x^2+3x-2x-3=0$; $x(2x+3)-1(2x+3)=0$; $(x-1)(2x+3)=0$; $x=1, -\frac{3}{2}$

c. $3x^2 + 10x = 8$ — $3x^2+10x-8=0$; $3x^2+12x-2x-8=0$; $3x(x+4)-2(x+4)=0$; $(3x-2)(x+4)=0$; $x=\frac{2}{3}, -4$

Finding Extrema using Zeros

Given the following trinomials, factor each to find the zeros:

1. $x^2 + 8x + 15 = 0$
2. $x^2 - 13x + 42 = 0$
3. $x^2 + 2x - 24 = 0$

Discuss with your partner what you'd expect the graph to look like based on this information.

Polynomial	Factors	Zeros	Average of the Zeros
$x^2+8x+15=0$	$(x+3)(x+5)=0$	$x=-3,-5$	$(-3+-5)/2=-4$
$x^2-13x+42=0$	$(x-7)(x-6)=0$	$x=7,6$	$(7+6)/2=6.5$
$x^2+2x-24=0$	$(x+6)(x-4)=0$	$x=-6,4$	$(-6+4)/2=-2/2=-1$

Analyze the Data:

1. Graphically inspect each polynomial to find any connections between the zeros and the graph.
   zeros are where graph touches the x-axis
   average of zeros is x-value of vertex
2. What patterns do you see?
3. Given the equation $x^2 - 2x - 35 = 0$, without looking at the graph, where would you expect the minimum to be located?

$(x-7)(x+5)=0$; $x=7,-5$ zeros

$\frac{7+-5}{2}=\frac{2}{2}=1$ x-value of vertex

$1^2-2(1)-35$; $1-2-35=-36$

$(1,-36)$

$x^2 - 2x - 35 = 0$

$(x+5)(x-7) = 0$

$x+5=0 \qquad x-7=0$

$x = -5, x = 7$

① Set expression = 0 first!
② Factor
③ Set each factor = 0 and solve

To find the zeros

$\frac{-5+7}{2} = 1$

$x = 1$

① Average the zeros to find the x - value of the vertex

To find the vertex (the maximum **or** the minimum)

But there are two numbers in an ordered pair. How could you find the y – value for the previous example?

② Substitute the x-value into the original polynomial to find y-value

Our x – value for the minimum was x = 1. So we'll substitute the 1 in for x in our original polynomial.

$(1)^2 - 2(1) - 35 = 1 - 2 - 35 = -36$

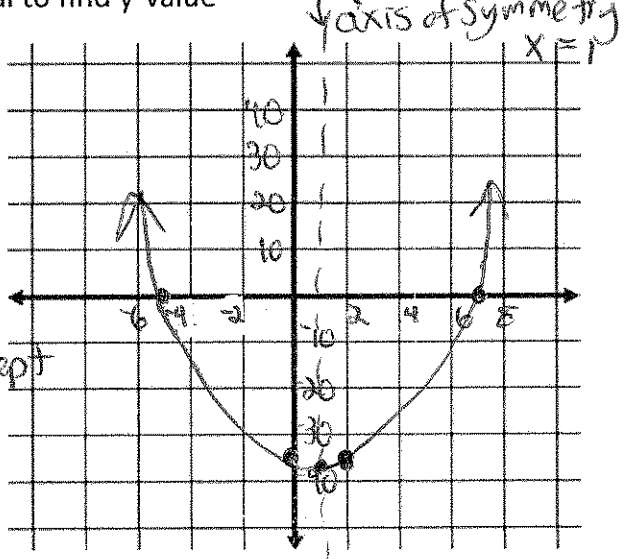
So our vertex, our minimum, is (1, -36)

To find a fourth point, substitute x = 0 into the polynomial. Show your work for this point in the space below.

$(0)^2 - 2(0) - 35 = -35$ (0, -35) "y-intercept"

Graph the four points from above with a smooth curve. Use your fourth point AND your knowledge of reflections & symmetry from Unit 1 for a fifth point.

(2, -35)



What appears to be the line of symmetry on the graph? $x = 1$

*Axis of symmetry: a line that divides the parabola into 2 parts that are mirror images

aka. AOS

*Remember, a minimum is the lowest point on a graph. A maximum is the highest point on a graph.

Without using a calculator, these steps will make sketching a graph much easier! Try graphing the next problem without a calculator.

Let's try another one: $y = x^2 + 2x - 8$

$(x+4)(x-2) = 0$

$x+4=0 \qquad x-2=0$

$x=-4 \qquad x=2$

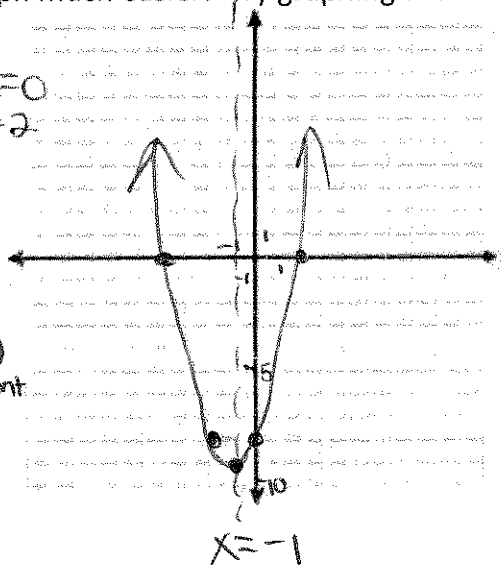
Polynomial	y – intercept	Zeros	Vertex
$x^2 + 2x - 8$	$y = 0^2 + 2(0) - 8$ (0, -8)	x-intercepts (-4, 0) (2, 0)	$x = \frac{-4+2}{2} = \frac{-2}{2} = -1$ $(-1)^2 + 2(-1) - 8$ $1 - 2 - 8 = -9$ (-1, -9)

Is the vertex of $y = x^2 + 2x - 8$ a minimum or maximum? minimum

What is the Axis of Symmetry?

A.O.S. $x = -1$

5th point = y-int mirror point (-2, -8)



To write zeros as x-intercepts, write a coordinate pair.

What should the y-value be for an x-intercept? zero (always!)

For a 4th and 5th point, use the y-intercept and the "y-intercept mirror"
(the reflection of the y-intercept over the axis of symmetry)

Direction of parabolas

Graph the following functions on your calculator. For each function, note whether the parabola is opening up or down.

Function	Parabola opens up or down?
1. $y = x^2 + 3x + 4$	up
2. $y = x^2 - 3x + 4$	up
3. $y = x^2 + 3x - 4$	up
4. $y = -x^2 + 3x + 4$	down
5. $y = x^2 + 4$	up
6. $y = x^2 - 4$	up
7. $y = -x^2 + 4$	down
8. $y = -x^2 - 4$	down
9. $y = x^2 + 3x$	up
10. $y = -x^2 + 3x$	down
11. $y = x^2 - 5x - 2$	up
12. $y = -x^2 - 5x - 2$	down

Make a conjecture: In a quadratic of the form $y = ax^2 + bx + c$, what determines if the parabola opens up or down?

- if $a > 0$ (or "a" is positive) then parabola opens up (like a smile :))
- if $a < 0$ (aka. "a" is negative) then the parabola opens down (like a frown :( )

Make up your own quadratics and test your conjecture with your calculator.

Summary:

If $a > 0$ ("a" is positive) then the parabola opens up. like a smile :)

If $a < 0$ ("a" is negative) then the parabola opens down. like a frown :(

⇒ Look at the "a" value

Musical Chairs

For each graph, solve the function and find the important key points (zeroes, x-intercepts, y-intercepts, maximum or minimum), plot the points on the coordinate plane, and sketch the graph of the function.

You are NOT allowed to use your calculators.

Function 1 $y = 2x^2 + 2x$

Zeroes: $x = 0, -1$

$y = 2x(x+1)$

$0 = 2x(x+1)$

$x = 0, -1$

x-intercepts: $(0,0)(-1,0)$

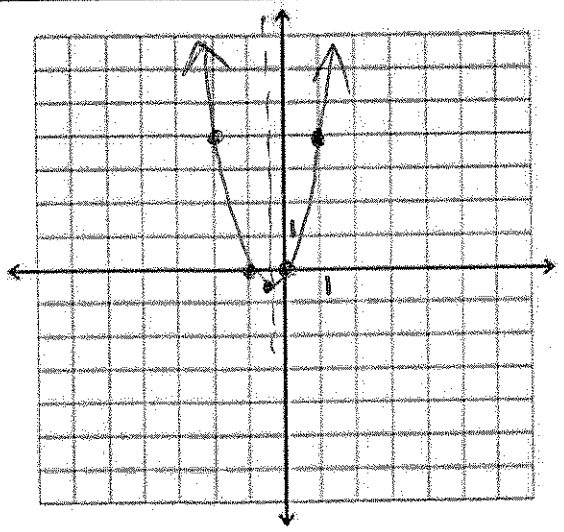
y-intercept: $(0,0)$

$2(-\frac{1}{2})^2 + 2(-\frac{1}{2}) = 2(\frac{1}{4}) - 1$

Location of vertex: $\frac{0+-1}{2} = -\frac{1}{2}$ $(-\frac{1}{2}, -\frac{1}{2})$

Is vertex the minimum or maximum of the function? minimum

* Axis of Symmetry $x = -\frac{1}{2}$



* use $x = 1$ & its mirror to get 4th & 5th points ... since one of your x-intercepts is also a y-intercept you need 2 extra points besides the normal 5 points

Function 2 $y = x^2 + 4x + 3$

Zeroes: $x = -3, -1$

$y = (x+3)(x+1)$

$0 = (x+3)(x+1)$

$x = -3, -1$

x-intercepts: $(-3,0)(-1,0)$

y-intercept: $(0,3)$

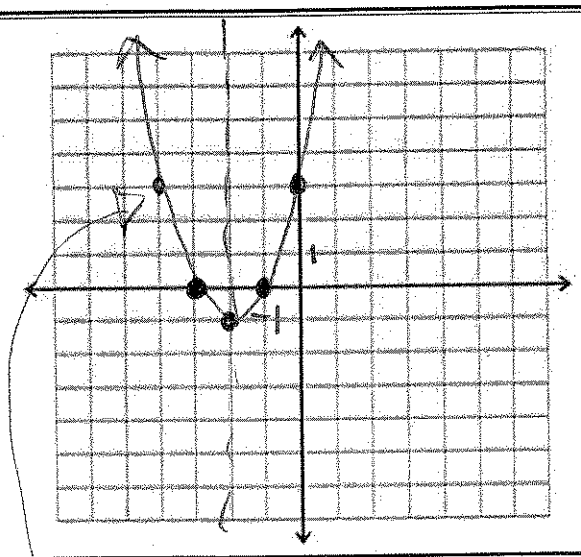
$y = 0^2 + 4(0) + 3$

$y = (-2)^2 + 4(-2) + 3$

$4 - 8 + 3$

Location of vertex: $\frac{-3+-1}{2} = \frac{-4}{2} = -2$ $(-2, -1)$

Is vertex the minimum or maximum of the function? minimum



Axis of symmetry $x = -2$

"y-intercept mirror" point gives 5th point

Function 3 $y = -x^2 - 2x + 8$

$-1(x^2+2x-8)$

$-1(x+4)(x-2)$

Zeroes: $x = -4, 2$

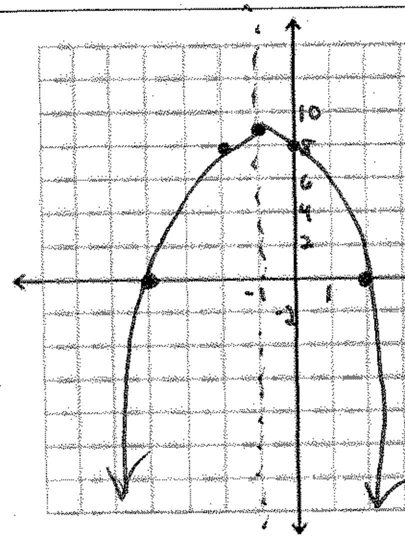
x-intercepts: $(-4,0)$ $(2,0)$

y-intercept: $(0, 8)$ $-0^2 - 2(0) + 8$

Location of vertex: $(-1, 9)$ $x = \frac{-4+2}{2} = \frac{-2}{2} = -1$ $y = -(-1)^2 - 2(-1) + 8$ $-1 + 2 + 8$

Axis of Symmetry: $x = -1$

Is vertex the minimum or maximum of the function? maximum



$x = -1$

*rescale the axes, as needed, to fit your parabola on the graph

Function 4 $y = 2x^2 + 6x + 4$

Zeroes:

$x = -2, -1$

$y = 2(x^2+3x+2)$

$y = 2(x+2)(x+1)$

$0 = 2(x+2)(x+1)$

$-2, -1$

x-intercepts:

$(-2,0)$ $(-1,0)$

y-intercept:

$(0, 4)$ $y = 2(0)^2 + 6(0) + 4$

Location of vertex:

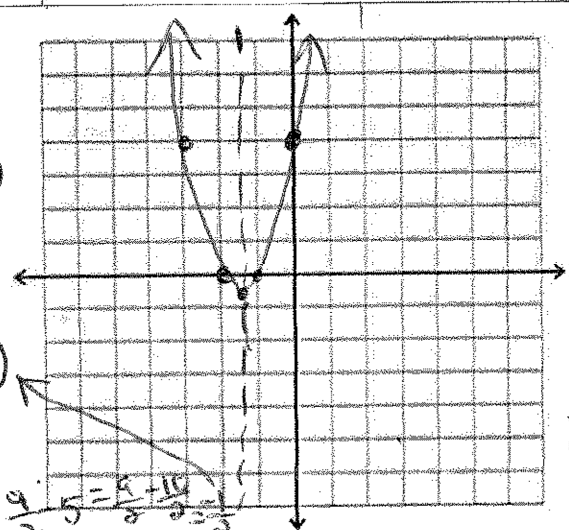
$\frac{-2+-1}{2} = \frac{-3}{2} = -1.5$ $(-1.5, -0.5)$

A.O.S. $x = -1.5$

Is vertex the minimum or maximum of the function? minimum

$y = 2(\frac{-3}{2})^2 + 6(\frac{-3}{2}) + 4$

$y = 2(\frac{9}{4}) + -9 + 4 = \frac{9}{2} - 5 = \frac{9}{2} + \frac{-10}{2} = \frac{-1}{2}$



*Look for a GCF 1st!

Function 5 $y = -2x^2 + 2x + 4$

$-2(x^2 - x - 2)$

$-2(x-2)(x+1)$

Zeroes: $x = 2, -1$

x-intercepts: $(2,0)$ $(-1,0)$

y-intercept: $(0,4)$ $y = -2(0)^2 + 2(0) + 4$

Location of vertex: $(\frac{1}{2}, 4\frac{1}{2})$ $x = \frac{2+-1}{2} = \frac{1}{2}$

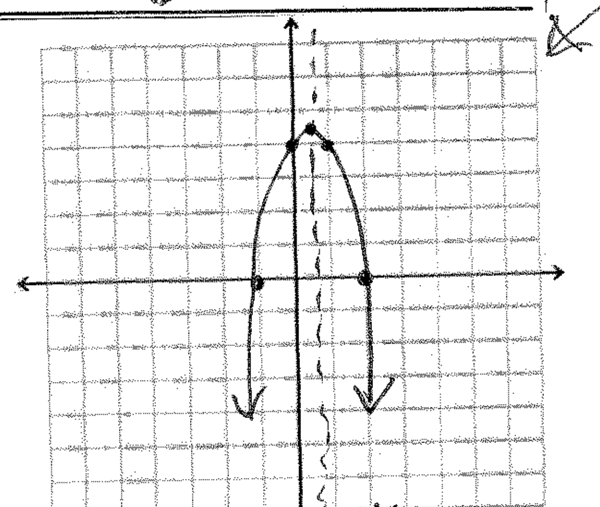
$-2(\frac{1}{2})^2 + 2(\frac{1}{2}) + 4$

$-2(\frac{1}{4}) + 1 + 4$

$-\frac{1}{2} + 5$

Axis of Symmetry: $x = \frac{1}{2}$

Is vertex the minimum or maximum of the function? maximum



$x = \frac{1}{2}$