Day 11: Quadratic Systems

Warm-Up:

12. Given the following functions, specifically describe the transformation from the identity function $y = x^2$

a. $y = (x+3)^2 - 7$ translated left 3, down 7

b. $y = 5x^2 + 12$ vertically stretched by 5 + translated up 12

c. $y = \frac{1}{2}(x-2)^2 + 4$ vertically compressed by 1/2, translated right 2, and up 4

Application Practice: The sum of two numbers is 21. The sum of the squares of the numbers is 305. What is the product of the two numbers?

one number = x
other number = y

$x + y = 21 \rightarrow y_1 = 21 - x$

$x^2 + y^2 = 305 \rightarrow y_2 = \pm\sqrt{305 - x^2}$

17 + 4 are the numbers

$17 \cdot 4 = \boxed{68}$

Day 11: Solving and Graphing Quadratic Inequalities and Systems (Algebra 2 Text p. 269)

Solving Quadratic Inequalities

Steps:

1) Solve to find zeros
2) Place zeros on a number line
   - < or > means non-included point → open circle
   - ≤ or ≥ means included point → closed circle
3) Test a point in each area to determine shading
4) Write answer in set notation! ☺

$\{x \mid -2 \le x < 5\}$

NOTE : Interval Notation

For < or > use (

For ≤ or ≥ use [

* we'll discuss this notation another day!

Solve :

> (not ≥) so non included point ⇒ open circles at roots on number line

1) $0 > x^2 - 6x - 7$

$0 > (x-7)(x+1)$

$x = 7, -1$

try -2, try 0, try 8

Try $x = -2$: $0 > (-2)^2 - 6(-2) - 7$, $0 > 9 \rightarrow$ False ✗

Try 0: $0 > 0^2 - 6(0) - 7$, $0 > -7$ true ✓

Try 8: $0 > 8^2 - 6(8) - 7$, $0 > 9$ False ✗

$\boxed{\{x \mid -1 < x < 7\}}$

2) $x^2 + 9x + 14 < 0$

$(x+2)(x+7) < 0$

-7, -2

try -8 ✗, try -4 ✓, try 0 ✗

$\boxed{\{x \mid -7 < x < -2\}}$

3) $x^2 - x - 12 \ge 0$

$(x-4)(x+3) \ge 0$

closed circles because "= to"

-3, 4

try -4 ✓, try 0 ✗, try 5 ✓

$\boxed{\{x \mid x \le -3 \text{ or } x \ge 4\}}$

4) $b^2 \ge 10b - 25$

$b^2 - 10b + 25 \ge 0$ ← * it's easier to factor if you "move" parts to keep squared term positive!

$(b-5)(b-5) \ge 0$

5

try 0 ✓, try 10 ✓

$\boxed{\text{All real \#s}}$

5) $2x^2 + 5x < 12$

$2x^2 + 5x - 12 < 0$

$(2x-3)(x+4) < 0$

-4, 3/2

try -5 ✗, try 0 ✓, try 3 ✗

$\boxed{\{x \mid -4 < x < 3/2\}}$

6) $n^2 \le 3$

$\sqrt{n^2} \le \sqrt{3}$

$n \le \pm\sqrt{3}$

$-\sqrt{3}$, $\sqrt{3}$

try -3 ✗, try 0 ✓, try 3 ✗

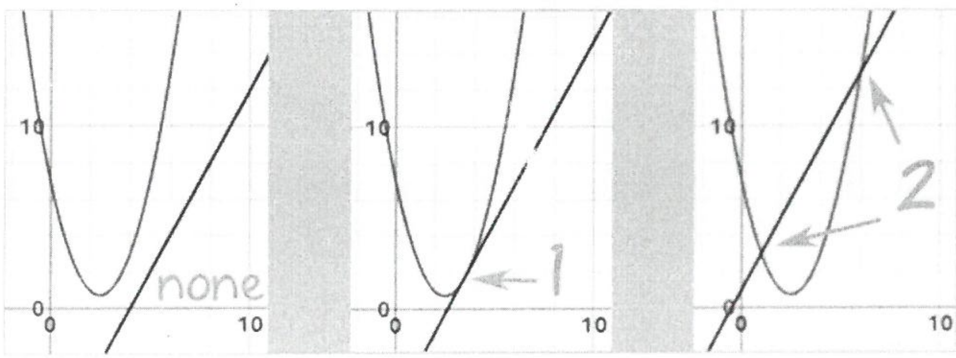
$\boxed{\{x \mid -\sqrt{3} \le n \le \sqrt{3}\}}$

Practice:

Solving Linear-Quadratic Systems:

With a Linear-Quadratic System, there are three possible cases:

- No real solution (when the line and the quadratic never intersect)
- One real solution (happens when the line just touches the quadratic)
- Two real solutions (happens when the line and the quadratic intersect twice)



Solve the System Algebraically: → ✱ Solve by graphing work shown on next page

1) $y = x^2 + 5x + 6$
$y - 6 = x$ $+6$ ... $+6$

① Solve linear equation for y: $y = x + 6$

② Substitute one into the other
$x + 6 = x^2 + 5x + 6$
$-x \ -6 \qquad -x \ -6$
$0 = x^2 + 4x$
$0 = x(x+4)$
$x = 0, -4$

③ Substitute x-values in to find the y-values
$y = 0 + 6$, $y = 6$ ④ $\boxed{(0,6)}$
$y = -4 + 6$, $y = 2$ ④ $\boxed{(-4,2)}$

2) $y = x^2 - x - 6$
$y - 2x = -2$

① Solve linear one for y: $y = 2x - 2$

② Substitute one equation into the other
$2x - 2 = x^2 - x - 6$
$-2x + 2 \qquad -2x \ +2$
$0 = x^2 - 3x - 4$
$0 = (x-4)(x+1)$
$x = 4, -1$

③ Substitute x-values in to find the y-values
$y = 2(4) - 2$, $y = 6$ ④ $\boxed{(4,6)}$
$y = 2(-1) - 2$, $y = -4$ $\boxed{(-1,-4)}$

Steps: → In Honors, you should know solving by hand!
1) Set the linear equation = y (if this is not done for you already)
2) Since both equations are = y, substitute one equation into the other. Then solve for x.
3) Substitute the x-values back in to find the y-values
4) Your solutions are coordinate points! ☺

There are more ways to solve as well!!

3) $x^2 + y^2 = 25$
$4y = 3x$

① $y = \frac{3}{4}x$ Solve linear equation for y

② Substitute one into the other
$x^2 + \left(\frac{3}{4}x\right)^2 = 25$
$x^2 + \frac{9}{16}x^2 = 25$
$\frac{16}{16}x^2 + \frac{9}{16}x^2 = 25$
$\frac{16}{25}\left(\frac{25}{16}x^2\right) = (25)\frac{16}{25}$
$x^2 = 16$
$\sqrt{x^2} = \sqrt{16}$
$x = \pm 4$

③ Substitute x-values in to find y-values
$\frac{4y}{4} = \frac{3(4)}{4}$, $y = 3$
$\frac{4y}{4} = \frac{3(-4)}{4}$, $y = -3$

④ $\boxed{(4,3)}$ $\boxed{(-4,-3)}$

4) $x^2 + y^2 = 26$
$x - y = 6$

① $-y = -x + 6$
$y = x - 6$ Solve linear equation for y

② Substitute one into the other
$x^2 + (x-6)^2 = 26$
$x^2 + (x-6)(x-6) = 26$
$x^2 + x^2 - 12x + 36 = 26$
$2x^2 - 12x + 10 = 0$
$2(x^2 - 6x + 5) = 0$
$2(x-5)(x-1) = 0$
$x = 5, 1$

③ Substitute x-values in to find values

Solving by Hand Steps for Graphs of Systems

Alg 2 Text p577

① For inequalities, graph + shade both lines → $y \ge (x+4)(x+3)$
x-int $(-4,0)(-3,0)$
vertex $(-3\frac{1}{2}, -\frac{1}{4})$

Sp'14

② intersection is overlap → make DARK!

Solve a linear and Quadratic System by Graphing

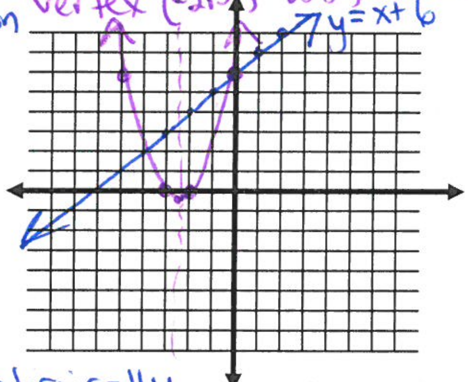
2 WAYS!

Solve:

1) $y = x^2 + 5x + 6$
$y = x + 6$ $m = 1$, y-int $(0,6)$

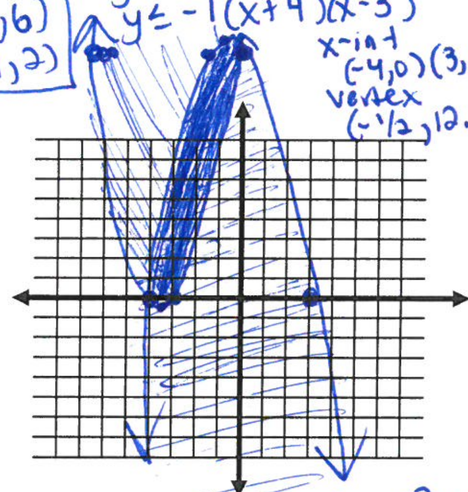
① by graphing → find intersection
$y = x^2 + 5x + 6 = (x+3)(x+2)$
x-int $(-3,0)(-2,0)$
y-int $(0,6)$
vertex $(-2.5, -.25)$

$\boxed{(0,6)\ (-4,2)}$



2) $y \le -x^2 - x + 12$
$y \ge x^2 + 7x + 12$

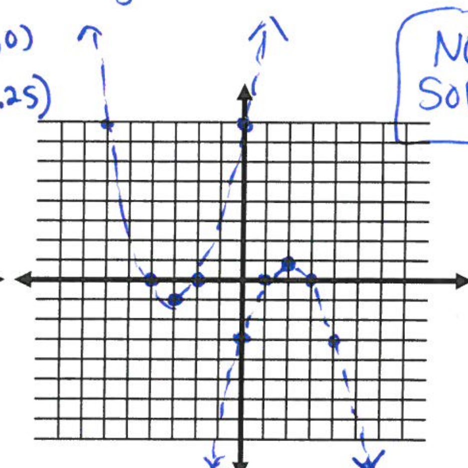
$y \le -1(x^2 + x - 12)$
$y \le -1(x+4)(x-3)$
x-int $(-4,0)(3,0)$
vertex $(-\frac{1}{2}, 12.25)$



3) $y < -x^2 + 4x - 3$
$y > x^2 + 6x + 8$

$y < -1(x^2 - 4x + 3)$
$y < -1(x-3)(x-1)$
Dotted because NOT "= to"
$y > (x+2)(x+4)$

$\boxed{\text{NO Solution}}$



② Algebraically
Since both equations $= y$, substitute one into the other so they = each other then solve
$x + 6 = x^2 + 5x + 6$

$0 = x^2 + 4x$
$0 = x(x+4)$
$x = 0, -4$
then substitute into find y

$y = x + 6$ $y = 0 + 6 = 6$ $\boxed{(0,6)}$
$y = x + 6$ $y = -4 + 6 = 2$ $\boxed{(-4,2)}$

Practice: Musical Chairs activity

Practice with Solving Quadratics using Various Methods:

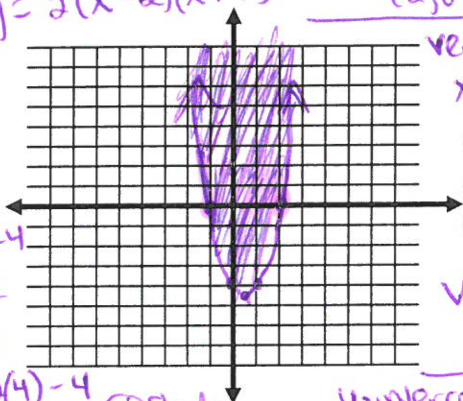
Solve by graphing:

1. $y \geq 2x^2 - 2x - 4$

① Graph
$y \geq 2(x^2 - x - 2)$
$y \geq 2(x-2)(x+1)$

zeros $x = 2, -1$
x-intercepts $(2,0)$ $(-1,0)$

vertex
$x = \frac{2+-1}{2}$
$x = 1/2$
$y = 2(1/2)^2 - 2(1/2) - 4$
$y = -4.5$
Vertex $(1/2, -9/2)$ or $(.5, -4.5)$



② Test $(0,0)$
$0 \geq 2(0)^2 - 2(0) - 4$
$0 \geq -4$ True

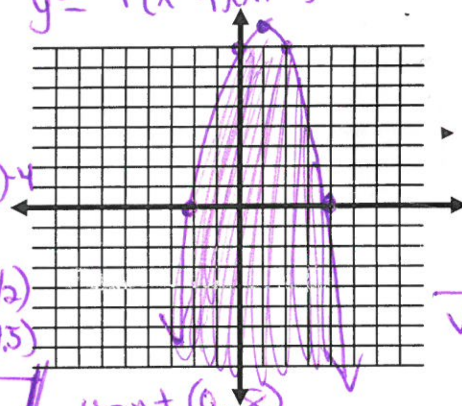
Test $(4,0)$
$0 \geq 2(4)^2 - 2(4) - 4$
$0 \geq 20$ False

③ Shade to include "true" point

y-intercept $(0,-4)$

2. $y \leq -x^2 + 2x + 8$

$y \leq -1(x^2 - 2x - 8)$
$y \leq -1(x-4)(x+2)$



x-intercepts $(4,0)$ $(-2,0)$

Vertex
$x = \frac{4+-2}{2} = \frac{2}{2} = 1$
$y = -(1)^2 + 2(1) + 8$
$y = 9$
$V(1,9)$

y-int $(0,8)$

Test $(0,0)$
$0 \leq -(0)^2 + 2(0) + 8$
$0 \leq 8$ true

Test $(5,0)$
$0 \leq -(5)^2 + 2(5) + 8$
$0 \leq -7$ False

Solve the inequality algebraically:

3. $x^2 - 3x - 10 < 0$

$(x-5)(x+2) < 0$

-2 5

try -5 ✗ try 0 ✓ try 10 ✗

$\boxed{\{x \mid -2 < x < 5\}}$

4. $x^2 + 2x \geq 8$

$x^2 + 2x - 8 \geq 0$
$(x+4)(x-2) \geq 0$

-4 2

try -5 ✓ try 0 ✗ try 5 ✓

$\boxed{\{x \mid x \leq -4 \text{ or } x \geq 2\}}$

Warm-Up:

13. Each year, a local school's Rock the Vote committee organizes a public rally. Based on previous years, the organizers decided that the Income from ticket sales, I(t) is related to ticket price *t* by the equation *I(t) = 400t – 40t²*. *Cost* C(t) of operating the public event is also related to ticket price *t* by the equation *C(t) = 400 – 40t.*

a. What ticket price(s) would generate the greatest income? What is the greatest income possible? Explain how you obtained the value you got.

Ticket price(s) _______________ Income _________________

b. For what ticket price(s) would the operating costs be equal to the income from ticket sales? Explain how you obtained the answer.

c. Which of the following rules would give the predicted profit *P(t)* as a function of the ticket price?

i. $P(t) = -40t^2 + 440t - 400$
ii. $P(t) = -40t^2 - 440t - 400$
iii. $P(t) = -40t^2 - 360t + 400$
iv. $P(t) = -40t^2 - 360t - 400$
v. $P(t) = 40t^2 - 440t + 400$

14. Factor Completely, then find the solutions of $3x^2 - 16x = 12$.