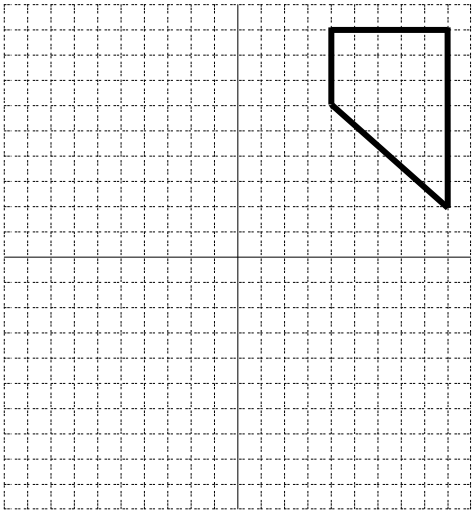
Day 8: Triangle Congruence

Warm-Up:

1. Create two figures congruent to the one given using **two different** transformations and describe them.



2. Solve for x.

$$\frac{x}{x+3} = \frac{x+4}{x-1}$$

3. If ΔCAT ~ ΔDOG, find x and y.

$m\angle C = 3x - 14,$

$m\angle D = 2x + 2y,$

$m\angle T = 3y - 2x,$

$m\angle G = 9 - 3x$

Triangle Congruence notes…

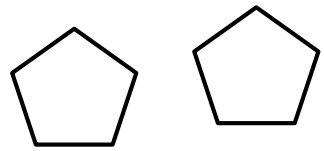
Congruent Figures

Definition

Characteristics

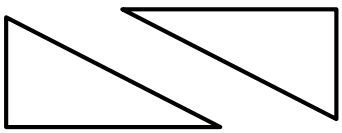
Examples

1.



2.

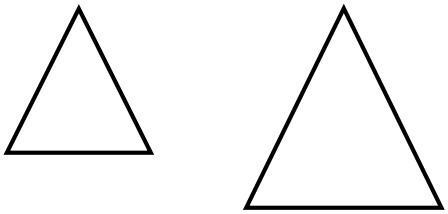
3.



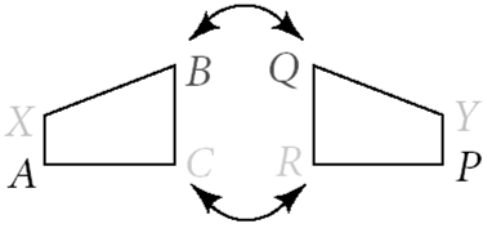
Non-Examples

1.

2.



_____ have congruent corresponding ("matching") parts – their matching sides and angles.



C corresponds to R.
$\angle B$ corresponds to $\angle Q$.
$\overline{AX}$ corresponds to $\overline{PY}$.
$ACBX \cong PRQY$

Naming Congruent Figures

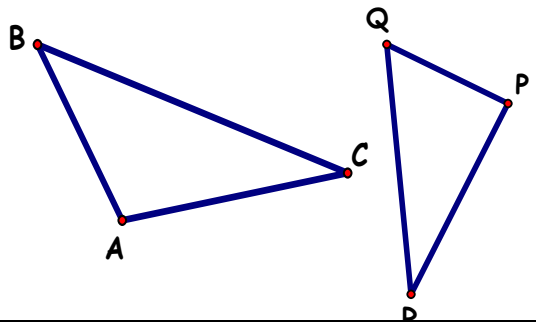
a) Points can be named in any _____ order.

b) Each corresponding vertex must be in the same order for each figure.

Ex. Figure BCAX is congruent to _____.

Example: **ΔABC ≅ ΔPQR**. List the congruent corresponding parts.

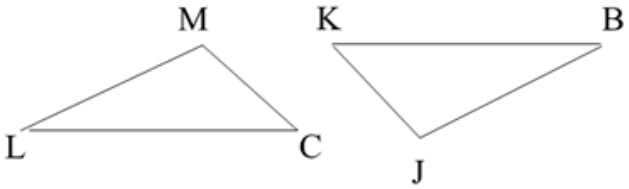
(List the corresponding vertices in the same order. List the corresponding sides in the same order.)



Corresponding Angles	Corresponding Sides

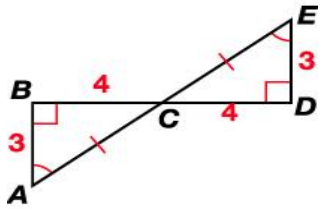
Be careful! The pictures don't always line up corresponding parts! The congruence statement MUST have them in order! ☺

You Try
ΔLMC ≅ ΔBJK
Complete the congruence statements. (Name all congruent angles. Name all congruent sides.)



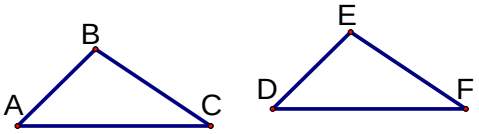
3. $\overline{LC} \cong$ _____
4. $\overline{KJ} \cong$ _____
5. $\overline{JB} \cong$ _____
6. $\angle L \cong$ _____
7. $\angle K \cong$ _____
8. $\angle M \cong$ _____
9. $\Delta CML \cong$ _____
10. $\Delta KBJ \cong$ _____

Can you conclude that $\Delta ABC \cong \Delta CDE$ in the figure below?



Side – Side – Side Postulate (or SSS Postulate)

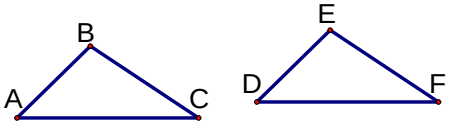
If _____ of one triangle are congruent to _____ of another triangle, then the 2 triangles are _____.



If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\Delta ABC \cong$ _____

Side – Angle – Side Postulate (or SAS Postulate)

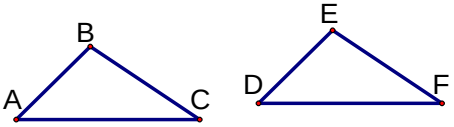
If _____ and the _____ _____ of one triangle are congruent to _____ and the _____ _____ of another triangle, then the 2 triangles are congruent.



If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\Delta BAC \cong$ _____

Angle – Side - Angle Postulate (or ASA Postulate):

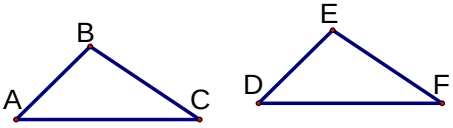
If _____ and the included side of one triangle are congruent to _____ and the included side of another triangle, then the 2 triangles are _____.



If $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$, and $\angle C \cong \angle F$, then $\Delta ACB \cong$ _____

Angle – Angle - Side Theorem (or AAS Theorem):

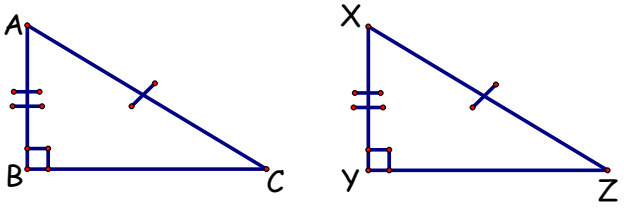
If _____ and a non- included side of one triangle are congruent to _____ and a _____ non-included side of another triangle, then the 2 triangles are _____.



If $\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$, and $\angle C \cong \angle F$, then $\Delta CAB \cong$ _____

Hypotenuse-Leg Theorem (or HL Theorem):

If the _____ and _____ of one right triangle are congruent to the _____ and _____ of another right triangle, then the triangles are congruent.



If $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, and $\angle B$ and $\angle Y$ are right angles, then $\Delta ABC \cong \Delta$_____.

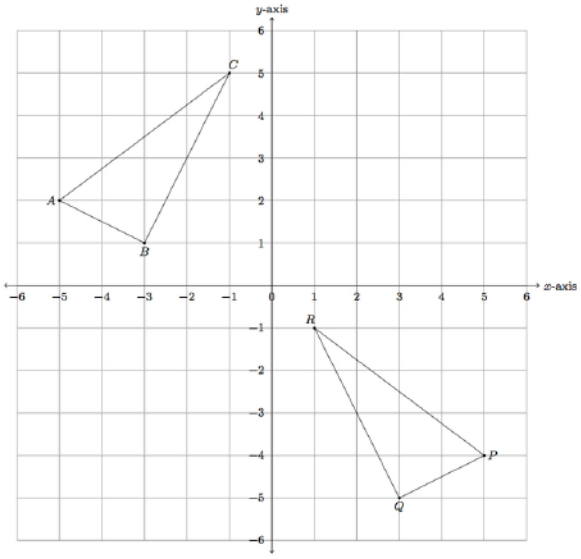
Day 9: Triangle Congruence and Similarity

Warm-Up: Triangles ABC and PQR are shown below in the coordinate plane:

a. Show that ABC is congruent to PQR with a reflection followed by a translation.

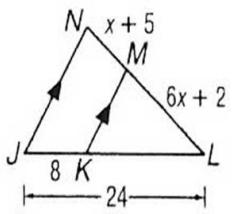
b. If you reverse the order of your reflection and translation in part (a) does it still map ABC to PQR?

c. Find a second way, different from your work in part (a), to map ABC to PQR using translations, rotations, and/or reflections.



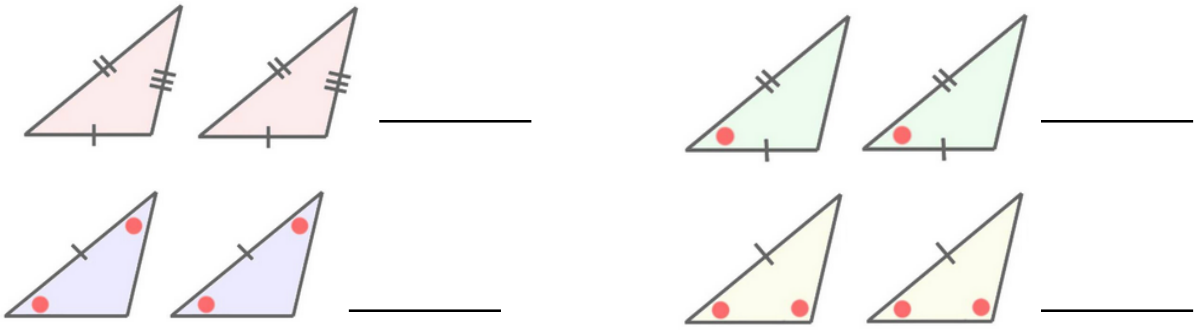
d. Explain why the triangles are similar and write a similarity statement. Then, find the value of x and the lengths of the segments requested.

$\overline{NL}$ and $\overline{ML}$



Congruence and Similarity Lesson

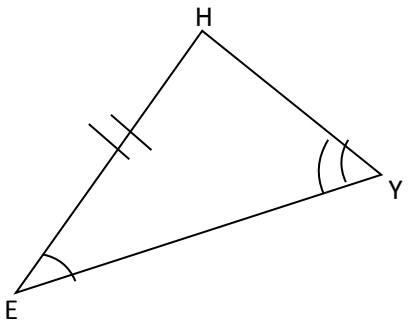
Review: What are the 4 shortcuts to knowing that two triangles are congruent?



Once we use one of the shortcuts to show that triangles are congruent, we know that the other 3 parts have congruent matches. In Geometry, we state that "corresponding parts of congruent triangles are congruent, or CPCTC."

#1: ΔHEY is congruent to ΔMAN by _____.

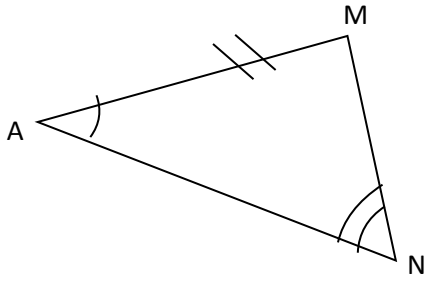
What **other** parts of the triangles are congruent **by CPCTC?**



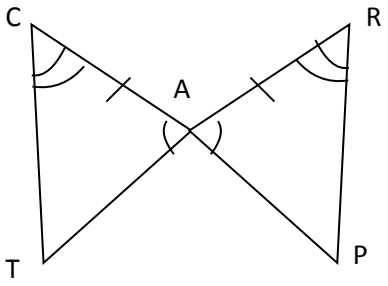
_____ ≅ _____

_____ ≅ _____

_____ ≅ _____



#2:



ΔCAT ≅ _____, by _____

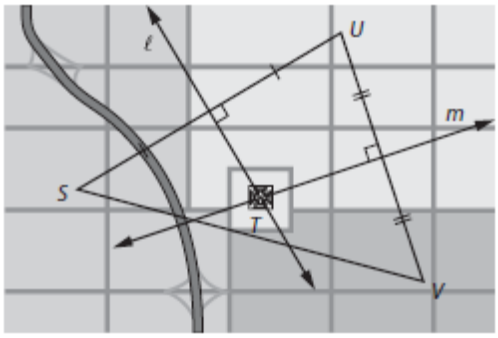
THEREFORE:

_____ ≅ _____, by CPCTC

_____ ≅ _____, by CPCTC

_____ ≅ _____, by CPCTC

Example: Plans for the location of a telecommunications tower that is to serve three northern suburbs of Milwaukee are shown below. Design specifications indicate the tower should be located so that it is equidistant from the center *S*, *U*, and *V* of each of the suburbs. In the diagram, line ℓ is the *perpendicular bisector* of *SU*. Line *m* is the perpendicular bisector of *UV*.

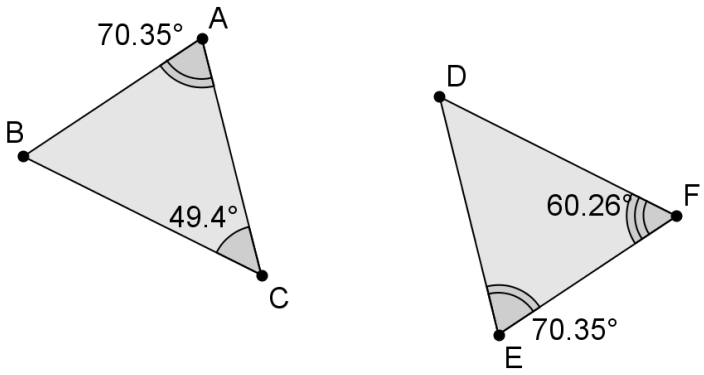


a. Draw line *TS* and line *TU*. How can you show that *TS* = *TU*.

b. Draw *line TV* on your diagram. Prove that *TU* = *TV*.

c. Explain why the tower should be located at point *T*.

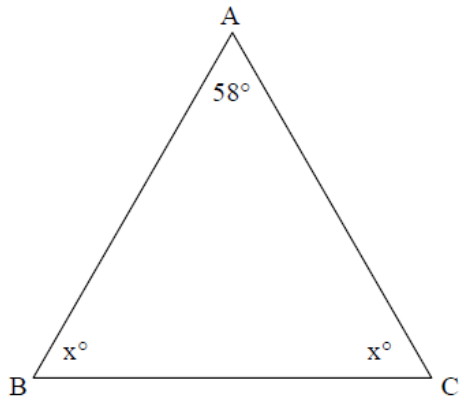
Are the following triangles congruent? Explain.



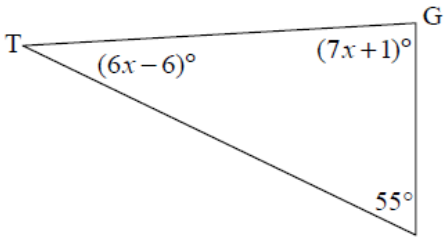
In the previous example, we needed to use the idea that the three angles of a triangle add to _____.

Let's play with this theorem for a bit…
Solve for the missing variables:

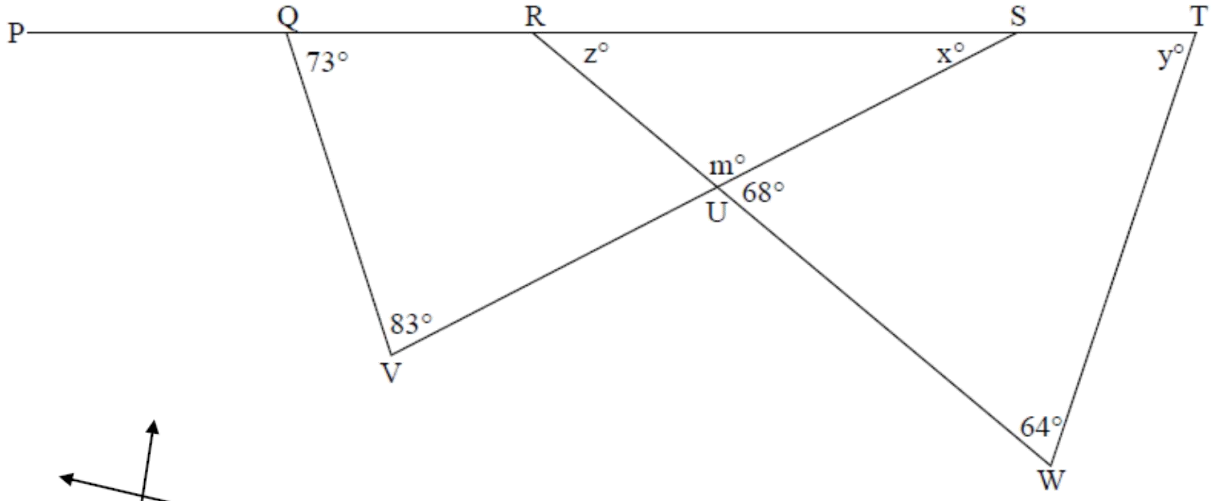
1.



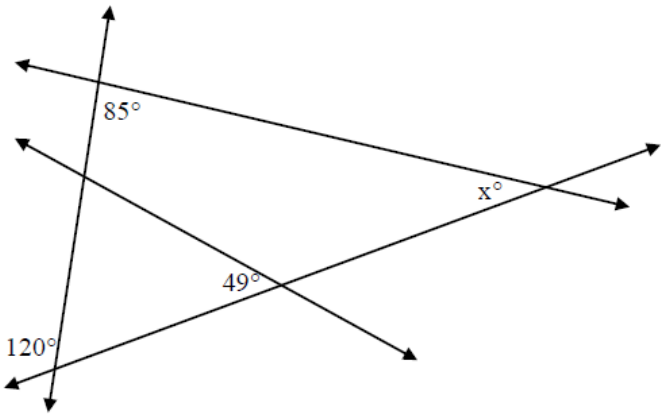
2.



3.



4.



Day 10: Midsegments, Isosceles Triangles

Warm-up:

1. Simplify

(x-7)(x+4)

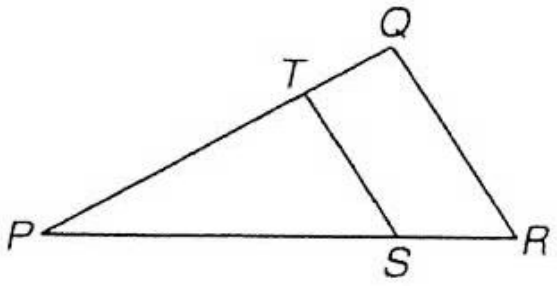
2. Solve for x.

$$\frac{x}{x+5} = \frac{x+3}{x+13}$$

$d^2 = (\text{change in x})^2 + (\text{change in y})^2$

3. Given a triangle with vertices A (2, 5), B(3, 6) and C(1, 6), use the distance formula to decide whether triangle ABC is scalene, isosceles, or equilateral. Show ALL work!!

4. Given $\overline{TS} \| \overline{QR}$, explain why the triangles are similar and write a similarity statement. Then use TS = 6, PS = x + 7, QR = 8, and SR = x – 1, to find PS and PR.

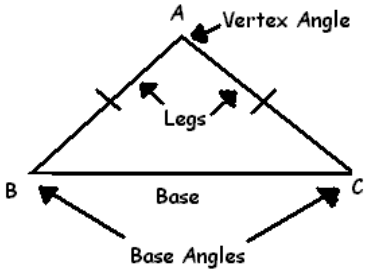


Midsegments, Isosceles Triangles Notes/Practice

Isosceles and Equilateral Triangles

Isosceles triangles are commonly found in the real world in buildings and bridges.

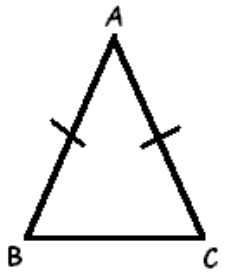
- The congruent sides of an isosceles triangle are its _____.
- The third side is the _____.
- The two congruent sides form the _____ _____.
- The other two angles are the _____ _____.



Isosceles Triangle Theorem:

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

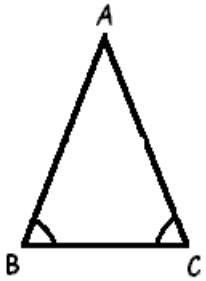
Ex. If _____ $\cong$ _____ , then _____ $\cong$ _____ .



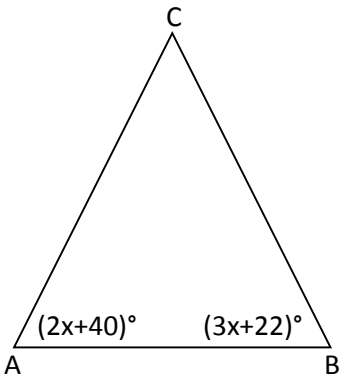
Converse of the Isosceles Triangle Theorem:

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

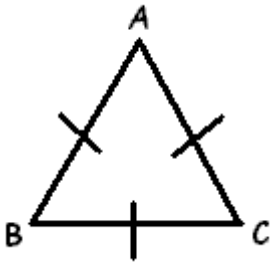
Ex. If _____ $\cong$ _____ , then _____ $\cong$ _____ .



Example: Triangle ABC is isosceles with vertex C. What is the value of x? What is the measure of each angle?



Corollary to Isosceles Triangle Theorem:
If a triangle is equilateral, then the triangle is equiangular.



Ex: If _____ ≅ _____ ≅ _____ ,

then _____ ≅ _____ ≅ _____ .

Example: Given triangle ABC, what is the measure of angle A?

How would you define the **midpoint** of a segment?

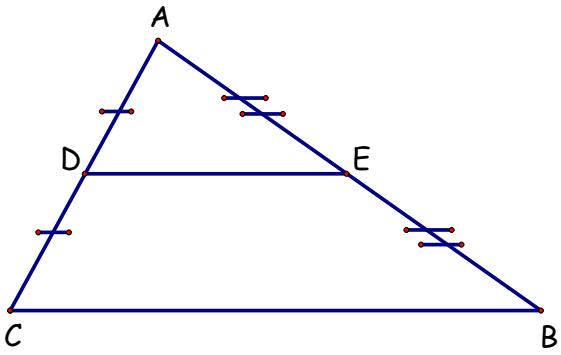
Midsegment of a triangle (the triangle doesn't have to be isosceles or equilateral)

A midsegment of a triangle is a segment connecting the midpoints of 2 sides. It measures _____ the length of the other side and is _____ to the other side

$\overline{DE}$ is a midsegment of $\triangle ABC$

DE = _____

and _____ // _____

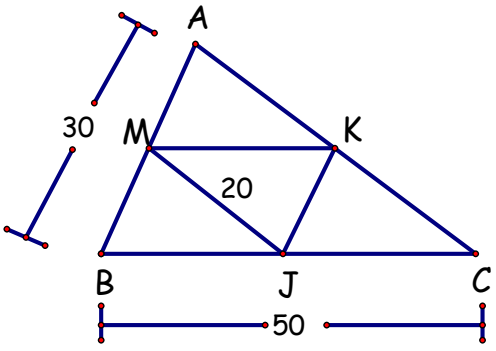


Example 1: In triangle ABC, **M**, **J**, and **K** are midpoints

AB = ___ KJ = ___

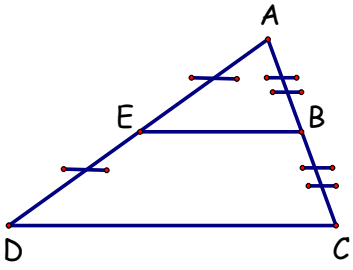
BC = ___ MK = ___

AC = ___ MJ = ___



Example 2: AB = 10, CD = 18

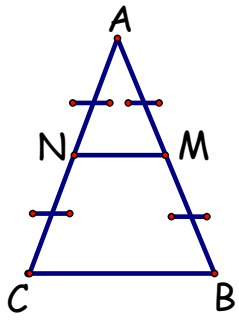
BC = ___ EB = ___



Example 3: Given $m\angle A = 42°$,

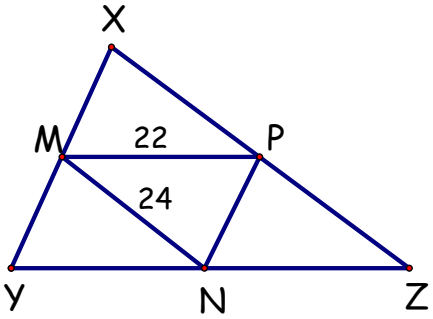
Find $m\angle AMN$ = _____

$m\angle ANM$ = _____



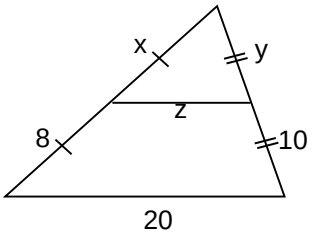
Example 4: In ΔXYZ, M, N, and P are midpoints.

The perimeter of ΔMNP is 60. Find XY and YZ.

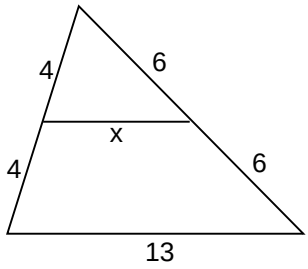


Practice…Directions: Find the values of the variables. You must show all work to receive full credit. Figures are not drawn to scale.

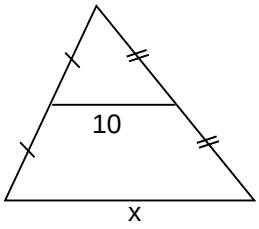
1. x = _____ y = _____ z = _____



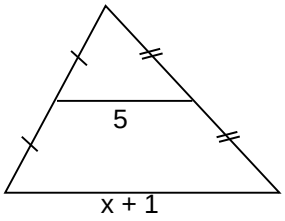
2. x = _____



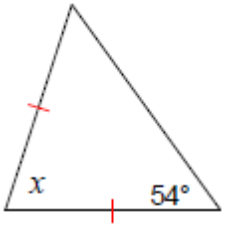
3. x = _____



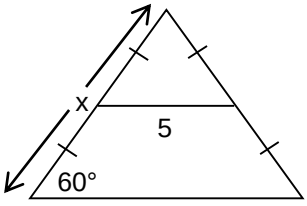
4. x = _____



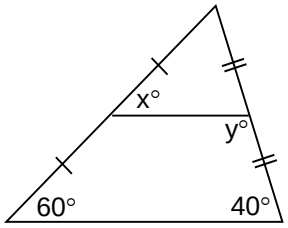
5. x = _____



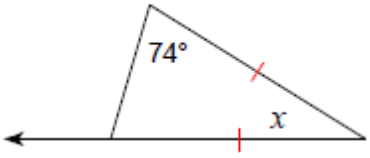
6. x = _____



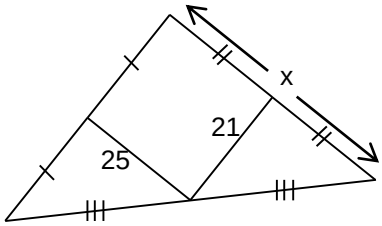
7. x = _____ y = _____



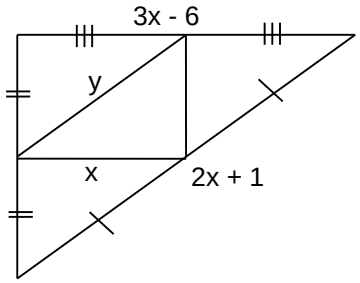
8. x = _____



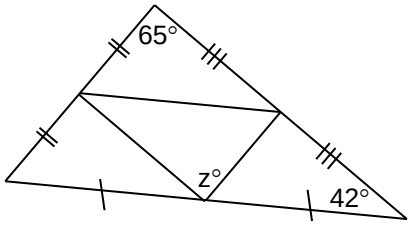
9. x = _____



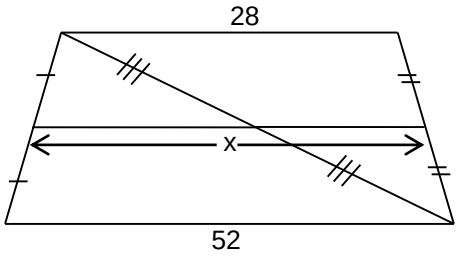
10. x = _____ y = _____



11. z = _____

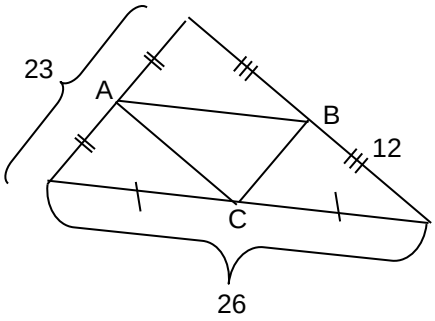


12. x = _____

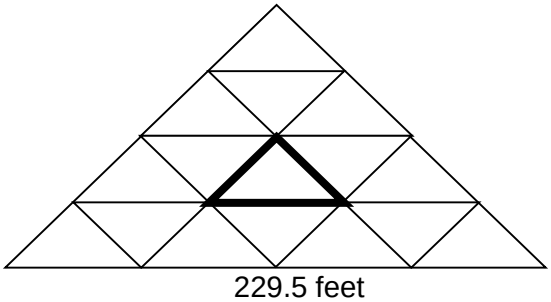


13. Sadie is designing a kite. The diagonals measure 28 in and 48 in. She wants to decorate the mid-segments of the triangles formed by the diagonals with purple ribbon. How much ribbon must she purchase? Draw a picture! (Hint: Do you remember which sides are congruent on a kite?)

14. Find the perimeter of ΔABC.



15. One side of the Rock and Roll Hall of Fame is an isosceles triangle made up of smaller triangles based on mid-segments. The length of the base of the building is 229.5 feet. What would the base of the bold triangle be?



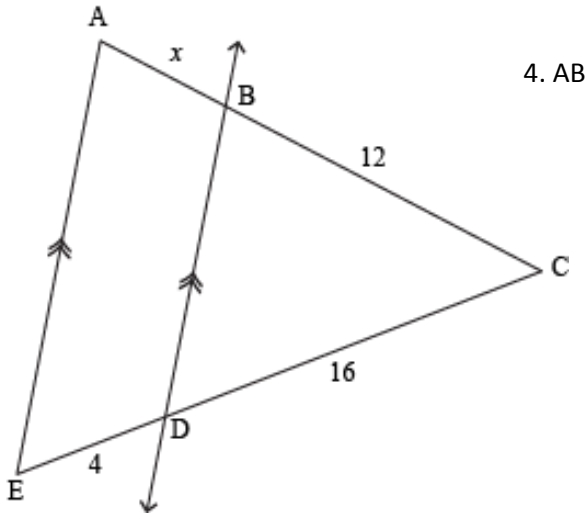
Day 11: Segment addition, Angle addition, and Quiz 2

Warm-Up:

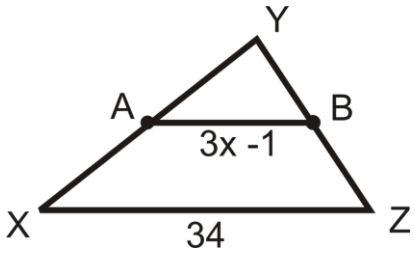
1. Multiply. (x+5)(x+7)

2. Solve for x.
$$\frac{x}{x+6}=\frac{x+1}{x-2}$$

3. Solve for x.



4. AB is a midsegment. Solve for x.

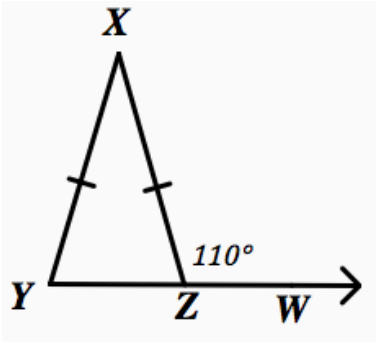


5. **Prove or Disprove**: Is the triangle with vertices R (-2, -2), S (1, 4), and T (4 ,-5) an equilateral triangle. How do you know?

6. **Prove or Disprove:** Are two triangles are congruent? ΔABC has the vertices A (-4, 1), B (-3, 4), and C (-1, 1). ΔDEF has the vertices D (2, -3), E (5, -2), and F (2, 0).

7. **Error Analysis:** Two students are asked to find the angle measures of ΔXYZ, given that ΔXYZ is isosceles. Their work is shown below. Is either answer correct? Explain your reasoning.

Esteban's Answer	Dashan's Answer
$m\angle Z = 70°$. Since an isosceles triangle has two congruent angles, $m\angle X = m\angle Y = 55°$	$m\angle Z = 70°$. Since base angles are congruent, $m\angle Y = 70°$ also. This leaves 40° for $m\angle X$.

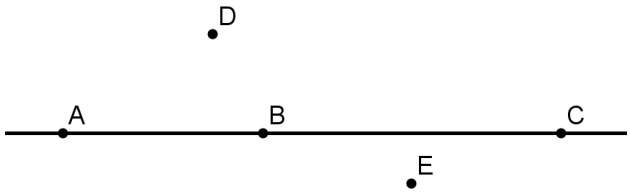


Lesson – Angle Addition, Segment Addition, and Quiz

The word **between** in Geometry has a special meaning: a point is *between* two others if all three points are collinear (on the same line) and it is "between" the other two.

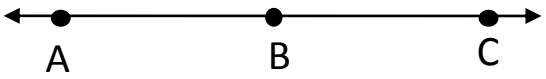
Example:

1. Is B between A and C?_____
2. Is D between A and C?_____
3. Is E between A and C?_____



Segment addition postulate: If three points A, B, and C are collinear and B is between A and C,

then _____.



Example: If AB = 5, and BC = 6, then AC = _____

Angle Addition Postulate:

If C is in the interior of ∠DAB then

_____

Example: Find m∠DAB, given m∠DAC = 30°and m∠CAB = 35°.

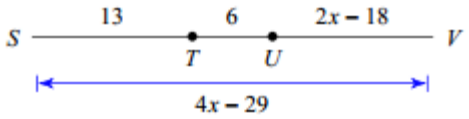
Practice: Segment Addition Postulate

Points A, B and C are collinear. Point B is between A and C. Solve for the requested values.

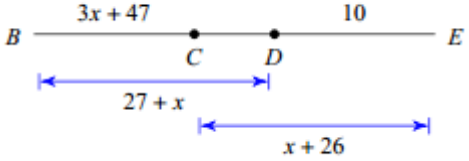
1. $AC = 3x + 3$, $AB = -1 + 2x$, and $BC = 11$. Find x.

2. $AC = 22$, $BC = x + 14$, and $AB = x + 10$. Find x.

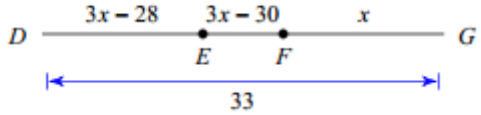
3. Solve for x.



Find CE



5. Find DE



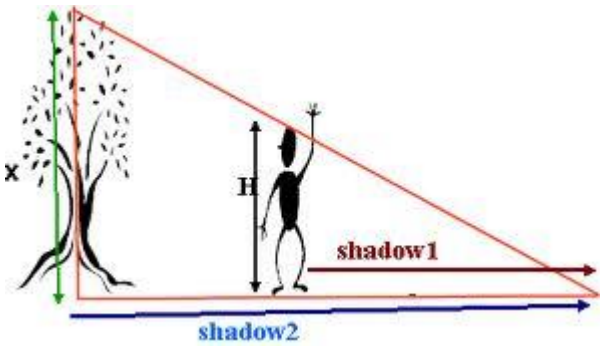
6. Points A, B, C, D, and E are collinear and in that order. Find AC if $AE = x + 50$ and $CE = x + 32$.

Day 12: Line segments and points, Cross sections, and rotations

Warm-up:

1. Height of person (H) = 64 in.
   Length of shadow 1 = 80 in.
   Length of shadow 2 = 120 in.

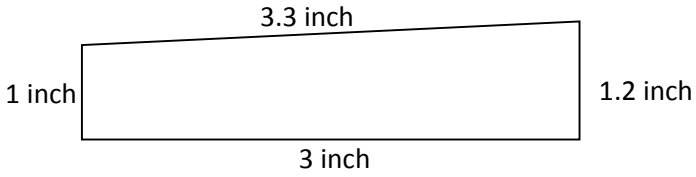
The two triangles in the figure are similar. Explain why this is true.



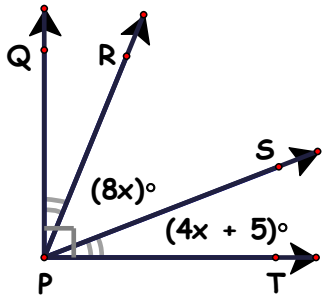
How tall is the tree? Justify your answer.

2. A contractor is installing a new counter top in a kitchen. The figure shown here is a model for a counter top. The measurements on the model diagram are given. If the actual countertop will be similar to the model, explain one thing that must be true about the actual countertop.

If the ratio of sides of the model to the actual countertop is 1:30, what are the dimensions of the actual countertop?



3. Find the m∠RPS.



4. H is in the interior of IGF. You are given that m∠IGH = 2x + 5, m∠FGH = 47, m∠IGF = 18x – 12. Find m∠IGH and m∠FGI. (Hint: Draw a diagram!)

5. If T is in the interior of ∠ABC, and m∠ABC = 3x, m∠ABT = x + 3, and m∠TBC = 13, find m∠ABC.

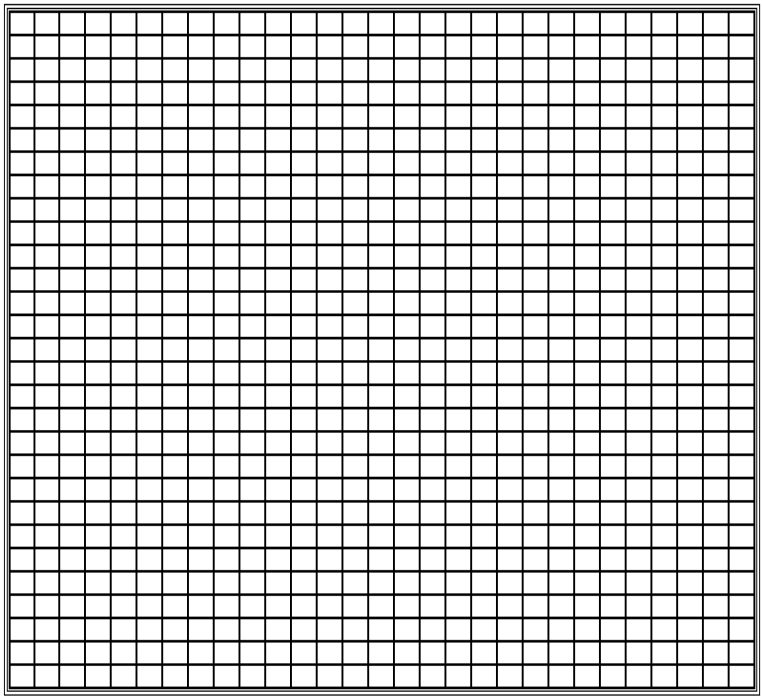
The Lesson: Line segments and points, Cross sections, and rotations

A _____ of a segment is **a point** that divides a segment into two congruent segments.

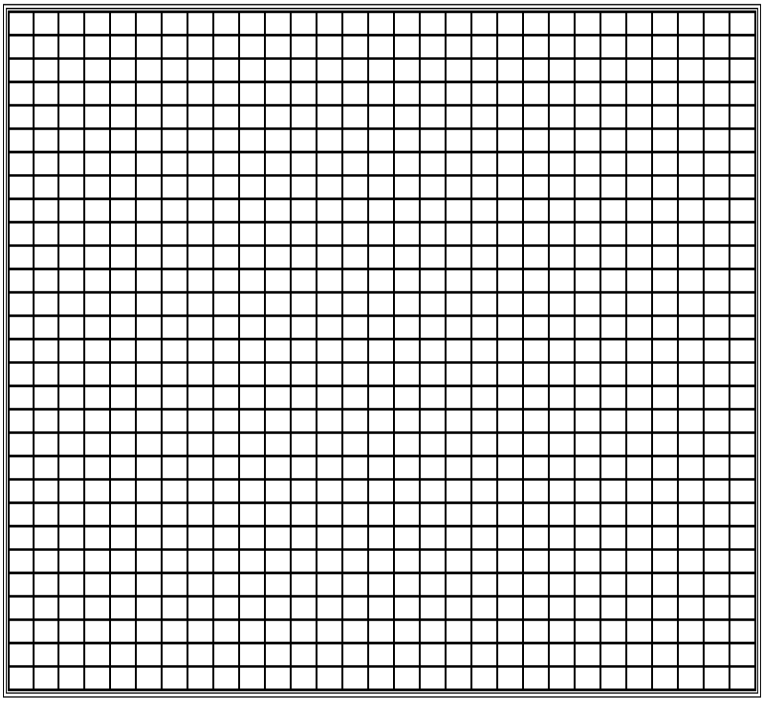
Midpoint Formula:

Examples:

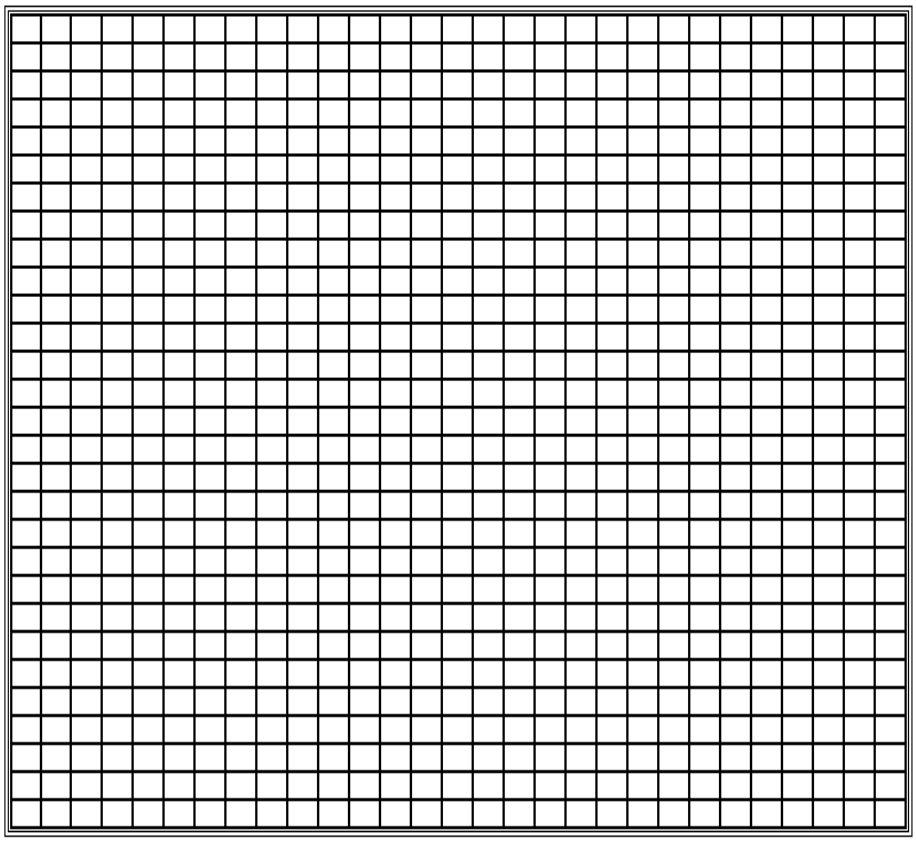
1. Billy and Evan are standing on a coordinate grid. Billy is currently on point (-1, 3) and Evan is currently on point (2, -7). They decide to meet in the middle of the segment connecting their locations. What point should they walk to?



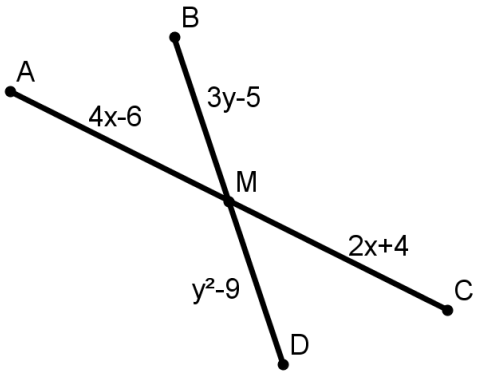
2. Sarah is also standing on a coordinate grid at point (-5, 0). Her friend is somewhere on the grid. Sarah walks in a straight line to point (4, 8) and realizes that she's gone exactly half of the distance to her friend's location. On what point is her friend (assuming that her friend did not move).



3. Caitlyn and Jack are standing on a coordinate grid. Caitlyn is at point (4, 12) and Jack is at point (-2, -3). Cailtyn can walk much faster than Jack, so they agree to met at the point 1/3 the distance from Jack (so Caitlyn would walk 2/3 the distance and Jack would walk 1/3 the distance). On what point do they meet?



4. In the following diagram, $\overline{AC}$ and $\overline{BD}$ bisect each other.



a. What is the official name for point M?

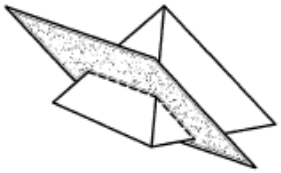
b. Find the value(s) of x and y.

c. Find AM, MC, BM, and BD. (Two points together with no line or segment above them mean "the distance between." AM translates to "the distance between A and M.)

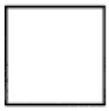
Just as we can cut segments into pieces, we can cut 3-dimensional objects:

Cross Section: _____

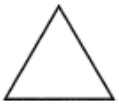
1 A square pyramid is cut along the shaded plane shown below.

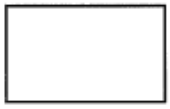


Which of the following is the cross-section of this solid?

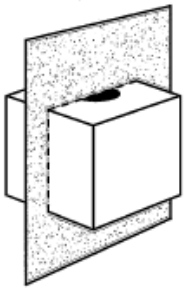
Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

2 A cube with a cylinder cut from its center is cut along the plane shown below.

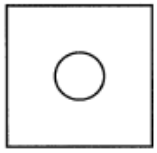


Which of the following is the cross-section of this solid?

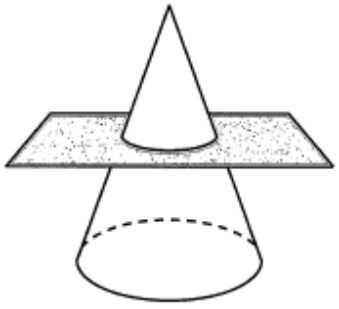
Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

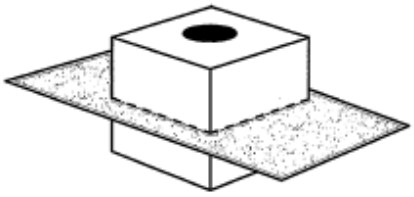
3 A cross-section is cut from the circular cone below.



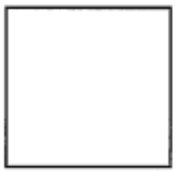
What is the shape of the cross-section?

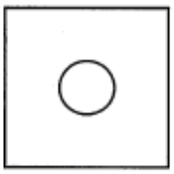
Ⓐ Square
Ⓑ Semicircle
Ⓒ Triangle
Ⓓ Circle

4 A cube with a cylinder cut from its center is cut along the plane shown below.



Which of the following is the cross-section of this solid?

Ⓕ 

Ⓗ 

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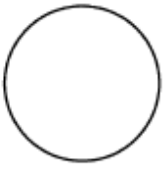
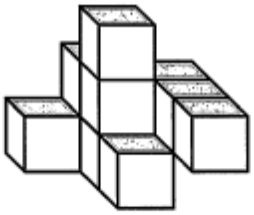
Ⓙ 

5 Which drawing represents the top view of this solid?



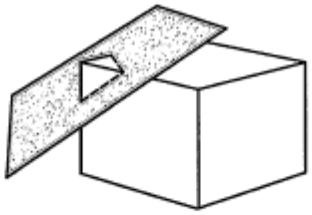
Ⓐ 

Ⓒ 

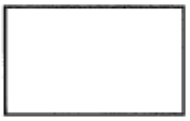
Ⓑ 

Ⓓ 

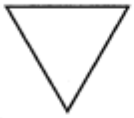
6 A rectangular prism is cut along the shaded plane shown below.



Which of the following is the cross-section of this solid?

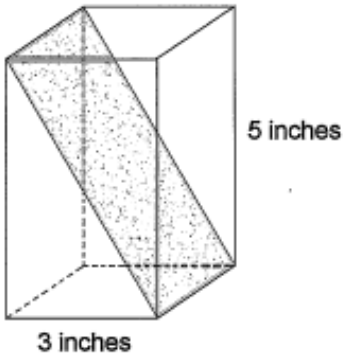
Ⓕ 

Ⓖ 

Ⓗ 

Ⓙ 

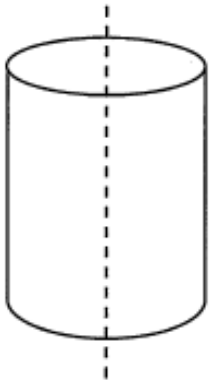
9 Andrew had a piece of foam in the shape of a rectangular prism as shown below. The base is a square with sides 3 inches long, and the piece is 5 inches tall. He cut the foam along the diagonal plane shown by the shaded area.



Which of the following is closest to the area of the shaded diagonal plane?

Ⓐ 19.3 square inches
Ⓑ 12 square inches
Ⓒ 15.8 square inches
Ⓓ 17.5 square inches

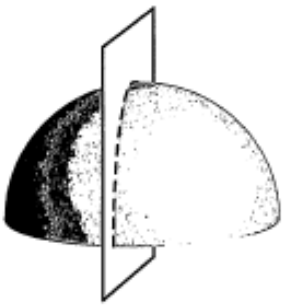
11 A cross-section is cut from the cylinder below.



What is the shape of the cross-section?

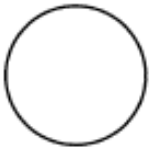
Ⓐ Rectangle
Ⓑ Circle
Ⓒ Semicircle
Ⓓ Oval

18 A hemisphere is cut along the plane shown below.



Which of the following is the cross-section of this solid?

Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

Use the original solid to **describe** the shape of each **cross section**.

7. **Cylinder:** Cross section parallel to the base_____

Cross section perpendicular to the base_____

Diagonal - from corner to corner _____

8. **Square Pyramid**: Cross section parallel to the base_____

Cross section perpendicular to the base_____

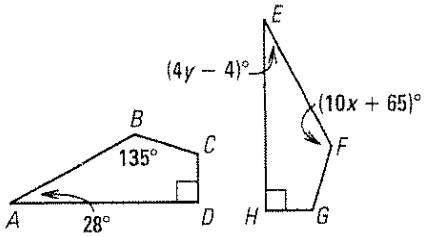
Diagonal - from corner to corner _____

Day 13: Test Review

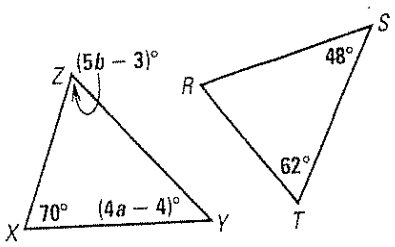
Warm-Up

Use the given information to find the indicated values.

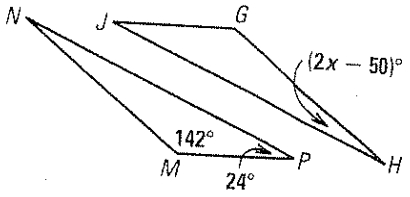
1. Given ABCD ≅ EFGH, find the values of **x** and **y**.



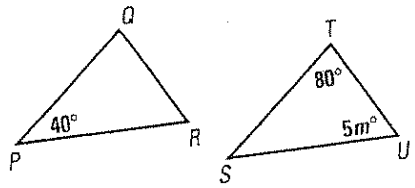
2. Given ΔXYZ ≅ ΔRST, find the values of **a** and **b**.



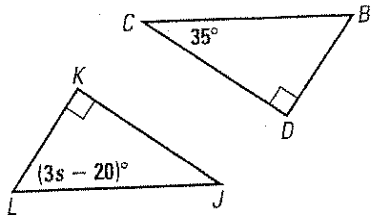
3. Given ∠M ≅ ∠G & ∠N ≅ ∠H find the value of **x**.



4. Given ∠P ≅ ∠S & ∠Q ≅ ∠T find the value of **m**.



5. Given ∠K ≅ ∠D & ∠J ≅ ∠C find the value of **s**.



6. Given ∠A ≅ ∠X & ∠C ≅ ∠Z find the value of r.