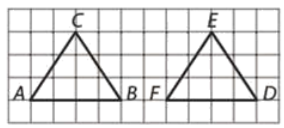
Day 1: Introduction to Transformations and Translations

Warm-Up:

Transformations: Translations

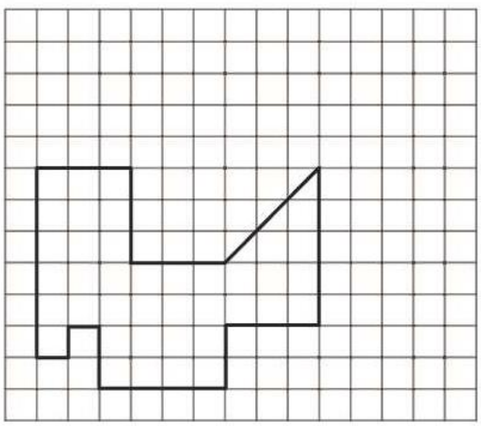
A **translation**, or a slide, is the movement of a figure from one position to another without turning. To the right are examples of a horizontal slide and a vertical slide.

horizontal slide

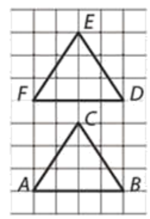


6 units to the right

Look at the figure below. Slide the figure 4 units to the right and 4 units up. Draw the image on the graph.



vertical slide

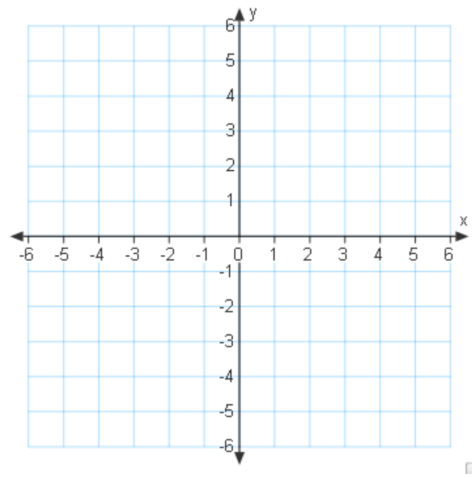


4 units up

Prerequisite Skill: Graphing Lines

Graph the following lines.

1) x = 2
2) y = 4
3) y = x (Hint: this is y = 1x + 0)
4) y = -x (Hint: this is y = -1x + 0)



Introduction to Transformations and Translations

Congruent figures ____ .

When two figures are congruent, you can move one so that ______________________________

____.

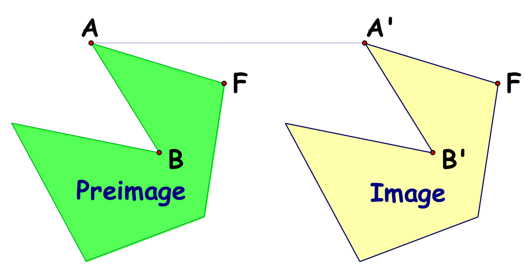
Transformation of a geometric figure: change in its _______________, _____________, or _________.

Preimage – __________________ figure

Notation: ___________

Image – ________ or _________________ figure

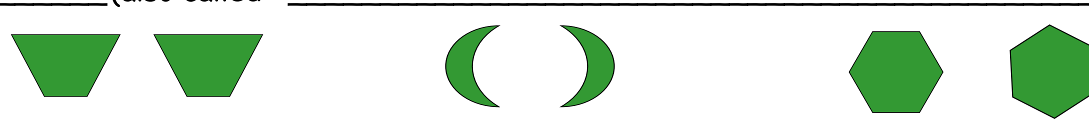
Notation: ___________



Isometry – transformation in which preimage and image are the _____________ ______________ and

_________________(also called: ____)

Examples:



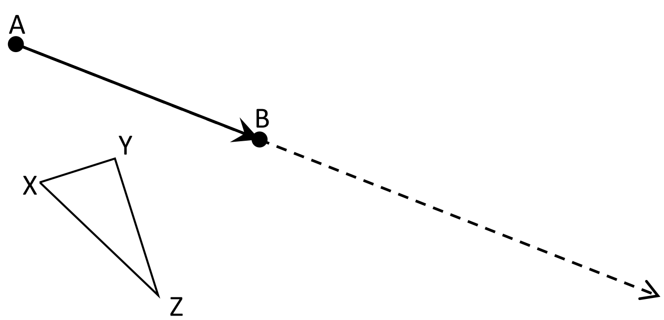
_____________________ , _______________________ , and ____________________

Translation – an isometry that maps all points the ___________ ______________________ and the ___________ ____________________.

Activity 1: Patty Paper Translation

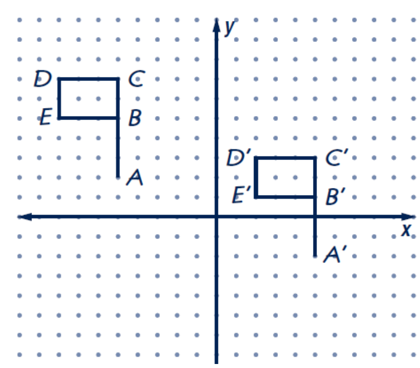
The translation T is defined by T(A) = B … meaning that it slides the figure the distance AB in the direction that $\overrightarrow{AB}$ goes.

1) Place the patty paper over this page. Trace the triangle and points A and B.
2) Slide the patty paper along $\overrightarrow{AB}$ so that the A on the patty paper is on top of B on this sheet and B on the patty paper is still on $\overrightarrow{AB}$ on this sheet.
3) The position of the triangle on your patty paper now corresponds to the image of $\triangle XYZ$ under the translation, T. If you press down hard with a sharp pencil, the image of the triangle can be seen on this page when you remove the patty paper.



Translation Vector – an arrow that indicates the **distance** and **direction** to translate a figure in a plane. $\overrightarrow{AB}$ in the activity above is an example of a translation vector.

The notation for a vector is: $\langle -a,\ b \rangle$ for a translation *a* units to the left and *b* units up.

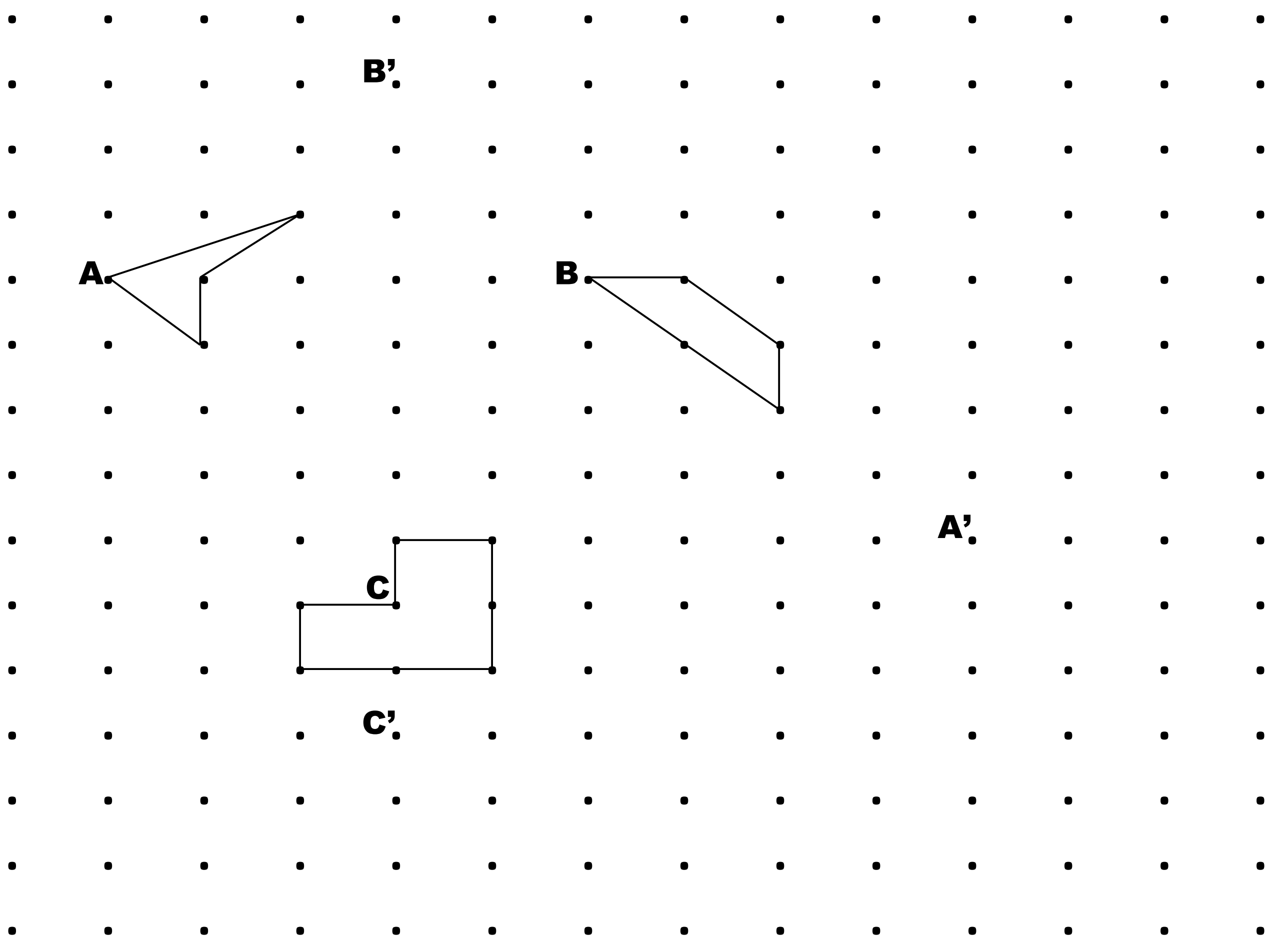


Three ways to describe a transformation (using example shown right):
Always **be specific when completing **any** type of description!!

1) **Words:** Translation to the right 10 units and down 4 units.
2) **Algebraic rule** (motion rule): T: $(x, y) \rightarrow (x + 10, y - 4)$
3) **Vector:** $\langle 10, -4 \rangle$

Activity 2: Dot Paper Translations

1) Use the dots to help you draw the image of the first figure so that A maps to A′.
2) Use the dots to help you draw the image of the second figure so that B maps to B′.
3) Use the dots to help you draw the image of the third figure so that C maps to C′.
4) Complete each of the following translation rules using your mappings from 1 – 3 above.
   a) For A, the translation rule is: T:(x, y) → (________, ________) or <______, ______>
   b) For B, the translation rule is: T:(x, y) → (________, ________) or <______, ______>
   c) For C, the translation rule is: T:(x, y) → (________, ________) or <______, ______>



Checkpoint: △GEO has coordinates G(-2, 5), E(-4, 1) O(0, -2). A translation maps G to G′ (3, 1).

1. Find the coordinates of:
   a) E′ (______, ______) b) O′ (______, ______)
2. The translation rule is: (x, y) → (________, ________) or <______, ______>
3. Specifically describe the transformation: ____

Day 2: Reflections

Warm-Up:
Using the points A(3, -4), B(1, 3), C(-2,1), D(-3, -5), perform each rule and give the resulting image points and the requested information.

1) translate right 2, down 5

Algebraic Rule: ___________________

2) translate left 6, up 4

Algebraic Rule: ___________________

3) translate using the rule (x, y) -> (x, y - 6)

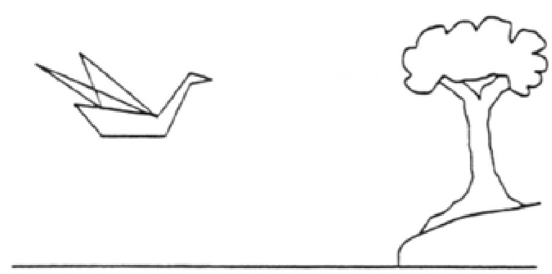
Description: ___________________

4) translate using the vector < -1, 2 >

Description: ___________________

Reflections

Reflections Introduction
Bill is sitting on a boat on a smooth, placid mountain lake. In the distance he sees the scene picture below, in which a swan is flying over the lake toward a distant tree. He also sees an image of the swan in the lake. Draw a picture of what Bill sees in the lake. Answer the questions.



1. How did you draw your picture?

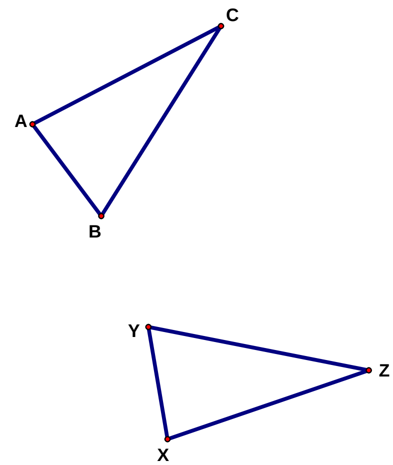
2. What type of transformation would **you** call this?

3. What **transformations term** would be used to describe the swan?

4. What transformations term would you use to describe the swan's reflection?

Reflection Exploration

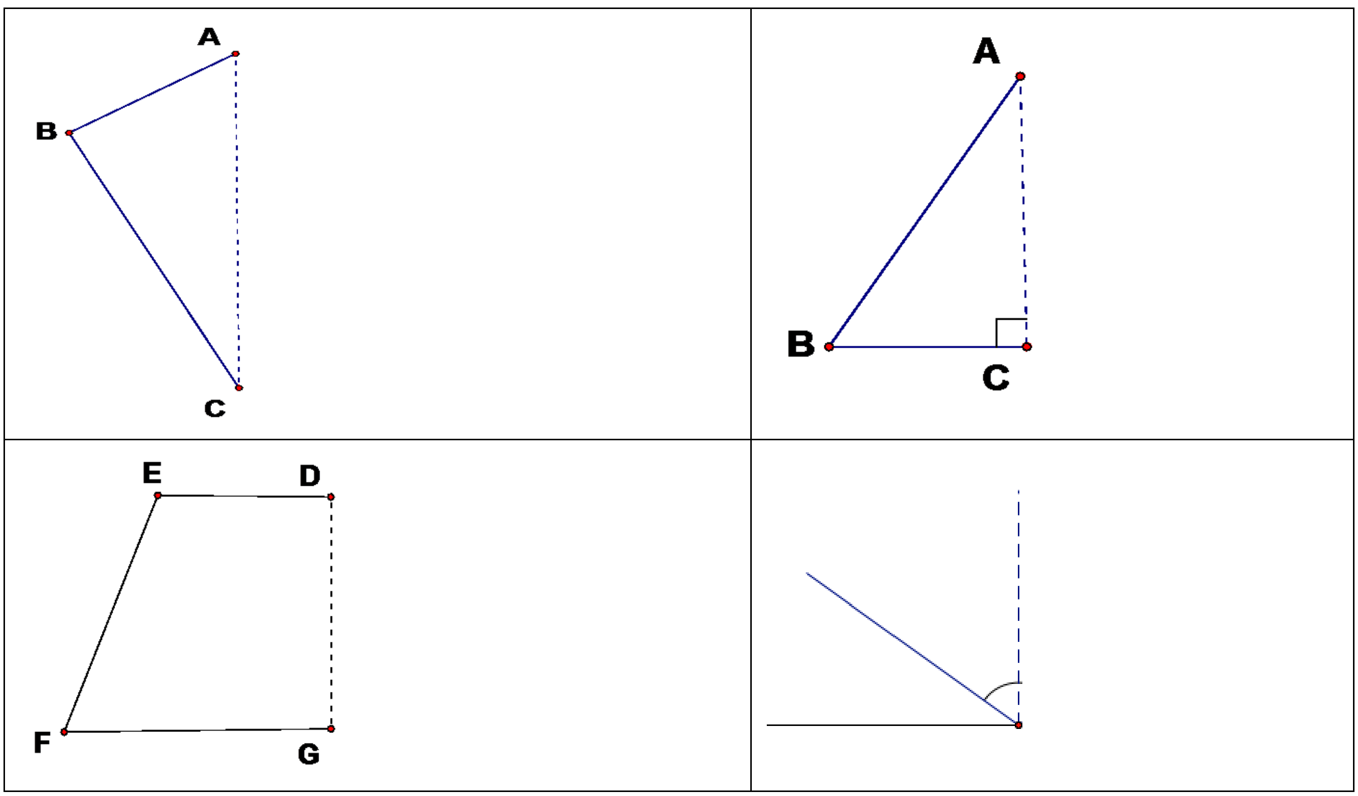
1) $\triangle ABC$ and $\triangle XYZ$ are reflections of each other. While holding the paper towards the light, fold the paper so that the triangles coincide (line up on top of each other). Crease the fold. Then open your paper back up and trace over this fold line using a straightedge to keep it neat.



2) Using a straightedge, draw $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$. Look at each segment in relationship to the reflection line. What appears to be true about the reflection line?

Patty Paper Reflections

Use patty paper to reflect each figure across the dashed line. Label the image points with proper notation.



Checkpoint: **Reflections:**

- A reflection is a transformation in which the image is a mirror image of the preimage.
- A point on the line of reflection maps to ________________________ .
- Other points map to the ___________________________ side of the reflection line so that the

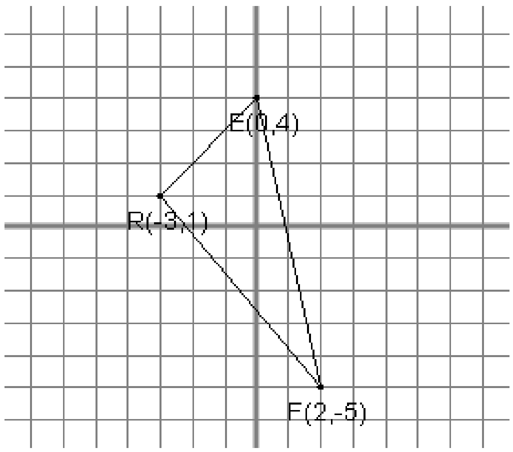
reflection line is the _________________________ of the segment joining the preimage and the image.

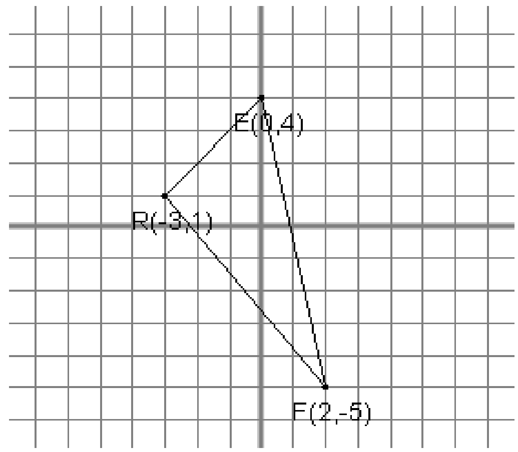
- Preimage and image points are equidistant from the ____________________ line.
- Notation for reflections is $R_{\text{line of reflection}}$. Example: $R_{x\text{-axis}}$ means reflection across the x-axis.

Activity: Reflections in the coordinate plane. Given ΔREF: R(-3, 1), E(0, 4), F(2, -5)

1) On the first grid, draw the reflection of ΔREF in the x-axis. Notation: $R_{x\text{-axis}}$
Record the new coordinates: R'(_____ , _____), E'(_____ , _____), F'(_____ , _____)

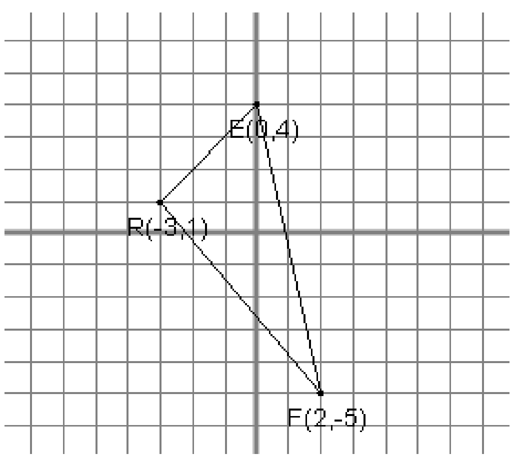
2) On the second grid, draw the reflection of ΔREF in the y-axis. Notation: __________
Record the new coordinates: R'(_____ , _____), E'(_____ , _____), F'(_____ , _____)

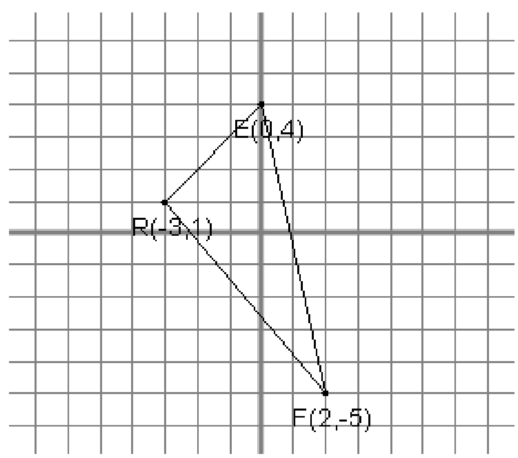




3) Graph the line y = x on the third coordinate grid. Trace ΔREF and the line y = x on patty paper. Then flip the patty paper over and line it up again to see where the triangle's image would be if you reflected it in the line y = x. Record the new coordinates: R'(_____ , _____), E'(_____ , _____), F'(_____ , _____)

4) Graph the line y = -x on the fourth coordinate grid. Trace ΔREF and the line y = -x on patty paper. Then flip the patty paper over and line it up again to see where the triangle's image would be if you reflected it in the line y = -x. Record the new coordinates: R'(_____ , _____), E'(_____ , _____), F'(_____ , _____)





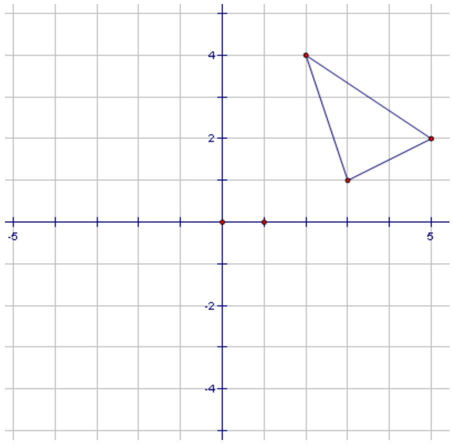
Checkpoint: Look at the patterns and complete the rule. Then write the rule using proper notation.

1. Reflection in the x-axis maps (x, y) → (_______, _______) ____________
2. Reflection in the y-axis maps (x, y) → (_______, _______) ____________
3. Reflection in the line y = x maps (x, y) → (_______, _______) ____________
4. Reflection in the line y = -x maps (x, y) → (_______, _______) ____________

Practice: Find the image of the following transformations and give the requested information.

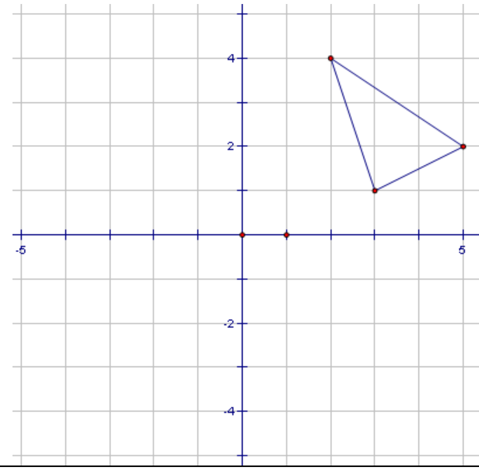
Hint: If you get stuck, review the Checkpoints after today's activities. ☺

The points (2,4), (3,1), (5,2) are reflected with the rule $(x, y) \rightarrow (x, -y)$



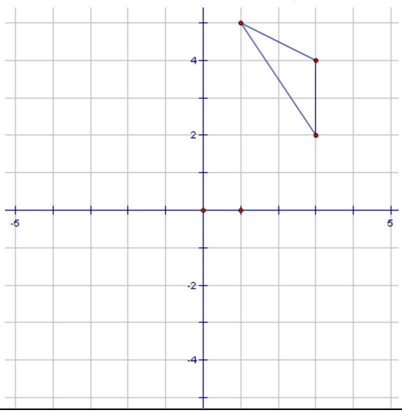
Description:

The points (2,4), (3,1), (5,2) are reflected with the rule $(x, y) \rightarrow (-x, y)$



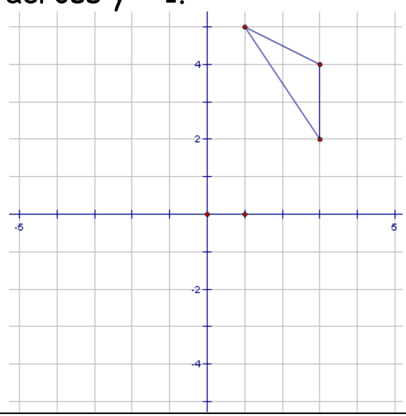
Description:

The points (3,2), (1,5), (3,4) are reflected across the x - axis.



Algebraic Rule:

The points (3,2), (1,5), (3,4) are reflected across y = 1.



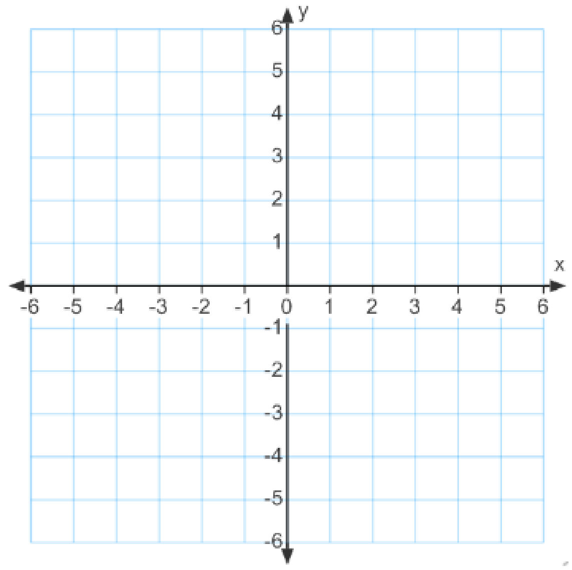
Summarize with Algebraic Rules:

What type of transformation does each of the following algebraic rules produce?

(x, y) → (x, -y)	(x, y) → (-x, y)
(x, y) → (-x, -y)	
(x, y) → (y, x)	(x, y) → (-y, -x)
Can you figure out this one on your own? Describe the reflection the results from the following algebraic rule (x, y) → (x, y)	

Day 3: Rotations

Warm-Up: Given triangle ABC with A(-1, 4), B(4, 3) and C(1, -5), graph the image points after the following transformations, identify the coordinates of the image, and write the Algebraic Rule for each.



1) Translate left 3, up 2

Points: Algebraic Rule:

2) Translate right 2, down 1

Points: Algebraic Rule:

3) Reflect over the x-axis

Points: Algebraic Rule:

4) Reflect over the y-axis

Points: Algebraic Rule:

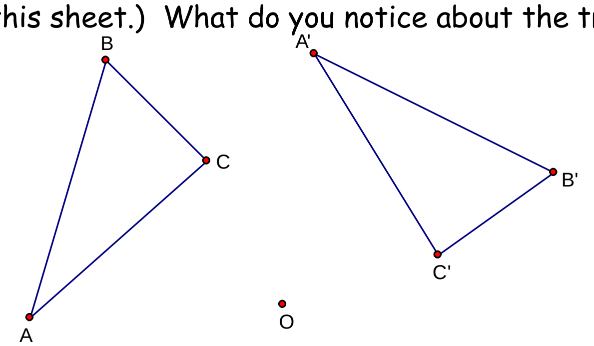
5) Solve the following system $4m + 18n = 80$

$12m + 34n = 160$

Rotations – Discovery Activity

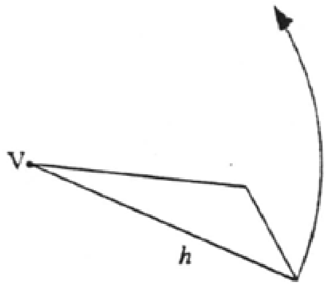
1.Exploration. Triangle A'B'C' is a rotation of Triangle ABC about the center O.

1) Using a compass, draw the circle that has center O and goes through point A.
2) Using a compass, draw the circle that has center O and goes through point B.
3) Using a compass, draw the circle that has center O and goes through point C.
4) What do you know notice about points A', B', and C'?
5) Trace Triangle ABC and point O on patty paper. Put your pencil point on top of the patty paper on point O and turn the patty paper around and around in both directions (keeping the O on your patty paper on top of the O on this sheet.) What do you notice about the triangle as it rotates around in either direction?



2. Bill rotates figure h using center V as shown by the arrow. Draw and label the image of figure h. Explain how you made your drawing.

- What method did you use?
- What does the arrow tell you?
- What is point V? What happens to point V after the motion is performed?

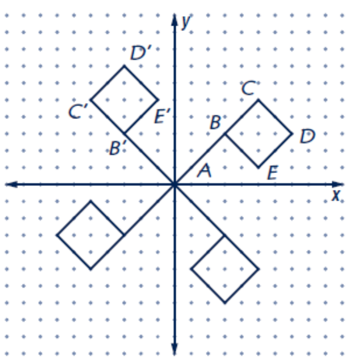


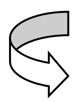
3. Summary

This type of transformation is called a __________________. To rotate an object, you must specify the __________________ of rotation, the _______________ around which the rotation is to occur, and the direction.

4. Visualizing Rotations Centered About the Origin

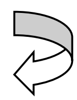
The flag shown below is rotated about the origin 90°, 180°, and 270°. Flag ABCDE is the preimage. Flag A'B'C'D'E' is a 90° counterclockwise rotation of ABCDE.





__________-____________

____________Degrees!



____

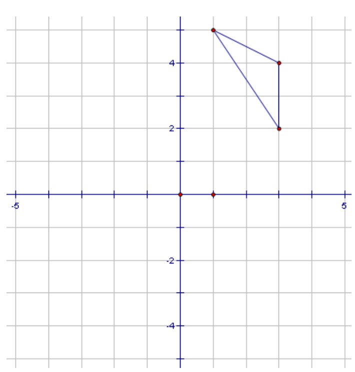
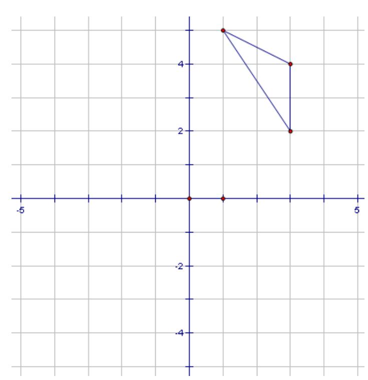
___________ Degrees!

NOTE: Unless otherwise specified, the standard for rotations is **counterclockwise!**

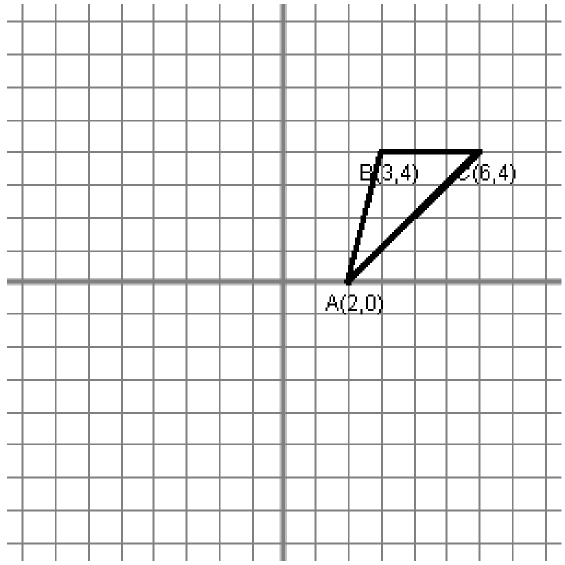
Notation for Rotations

R ________________________ , __________________________ **Example:** $R_{O,\ 90}$

5. Practice: Use patty paper to complete the following. Hint: trace the axes and triangle, then after the rotation be sure that the axes line up

Perform a 90°, counterclockwise rotation.	Perform a 180°, counterclockwise rotation.
	

5. Rotations on the Coordinate Plane Exploration



1) Triangle ABC has coordinates A(2, 0), B(3, 4), C(6, 4). Trace the triangle and the x- and y-axes on patty paper.
2) Rotate Triangle ABC 90°, using the axes you traced to help you line it back up. Record the new coordinates. A'(______ , ______), B'(______ , ______), C'(______ , ______)

3) Rotate Triangle ABC 270°, using the axes you traced to help you line it up. Record the new coordinates. A'(______ , ______), B'(______ , ______), C'(______ , ______)

4) Rotate Triangle ABC 180°, using the axes you traced to help you line it back up correctly. Record the new coordinates. A'(______ , ______), B'(______ , ______), C'(______ , ______)

Checkpoint: Look at the patterns and complete the rule. Then write the rule using <u>proper notation</u> for 1 – 3.

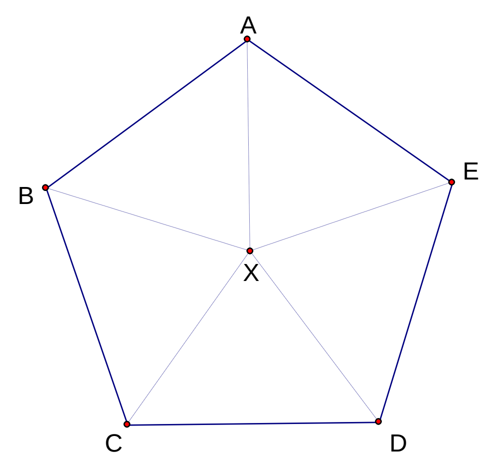
1. A 90° counter-clockwise rotation maps (x, y) → (_______, _______). ____________
2. A 270° counter-clockwise rotation maps (x, y) → (_______, _______). ____________
3. A 180° rotation maps (x, y) → (_______, _______). ____________
4. A rotation of 270° clockwise is equivalent to a rotation of ____.
5. A rotation of 270° counterclockwise is equivalent to a rotation of __________________________________.

Summarize with Algebraic Rules:

What type of transformation does each of the following algebraic rules produce?

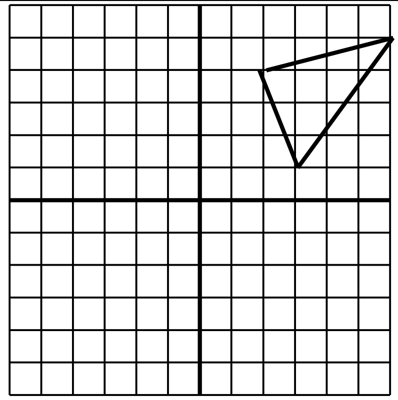
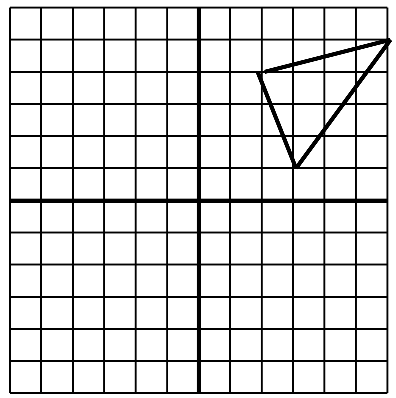
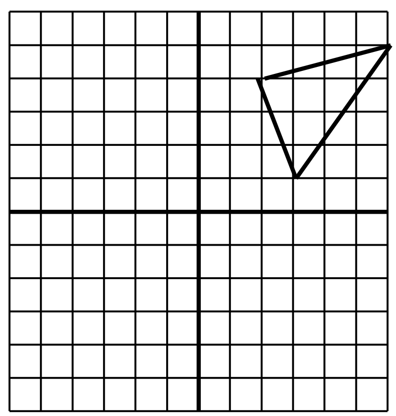
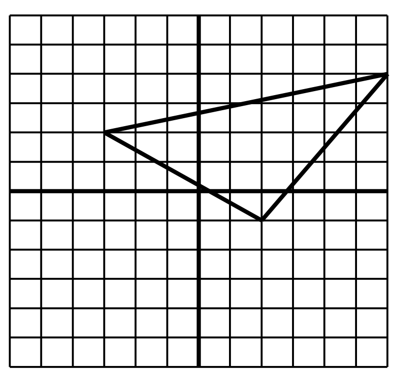
(x, y) → (-y, x)	(x, y) → (-x, -y)
(x, y) → (y, -x)	
Can you figure out this one on your own? Describe the rotation the results from the following algebraic rule (x, y) → (x, y)	
The rotation from the algebraic rule (x, y) → (x, y) is the same as another transformation. Describe that transformation.	

Practice

ABCDE is a regular pentagon. A regular polygon has all congruent angles and all congruent side lengths. (Hint: How many degrees are in a circle?) Name the image of point E for a counterclockwise 72° rotation about X. Name the image of point A for a clockwise 216° rotation about X. Describe 2 transformations with a preimage of point D and image of B.	

Practice: Rotations with Coordinates

For each problem graph the image points. Specifically describe in words the rotation that occurred. Then, write the Algebraic Rule for the rotation.

1) The coordinates of ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of A'B'C' are A'(-1, 3), B'(-5, 6), and C'(-4, 2). Description: Algebraic Rule:	
2) The coordinates of ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of A'B'C' are A'(1, -3), B'(5, -6), and C'(4, -2). Description: Algebraic Rule:	
3) The coordinates of ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of A'B'C' are A'(-3, -1), B'(-6, -5), and C'(-2, -4). Description: Algebraic Rule:	
4) The coordinates of ABC are A(2, -1), B(6, 4) and C(-3, 2). The coordinates of A'B'C' are A'(-1, -2), B'(4, -6), and C'(2, 3). Description: Algebraic Rule:	

Day 4: Dilations

Warm-Up: Given the line segment with points A(-1, 4) and B(2, 5) graph the image after the following transformations, write the coordinates of the image, and write the Algebraic Rule for #1 & 2.

1) Reflect over the line y = x.

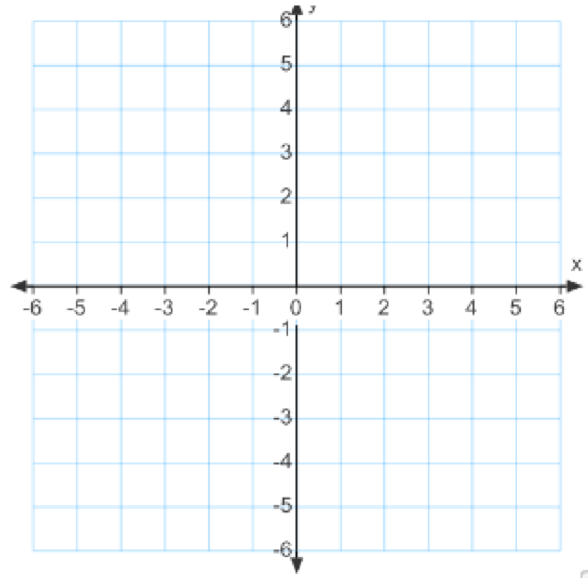
   Algebraic Rule:

2) Reflect over the line y = -x

   Algebraic Rule:

3) Reflect over the line y = 3.

4) Reflect over the line x = -1.



Dilations – Discovery Activity

Alice in Wonderland

In the story, Alice's Adventures in Wonderland, Alice changes size many times during her adventures. The changes occur when she drinks a potion or eats a cake. Problems occur throughout her adventures because Alice does not know when she will grow larger or smaller.

Part 1

As Alice goes through her adventure, she encounters the following potions and cakes:

Red potion – shrink by $\frac{1}{9}$

Blue potion – shrink by $\frac{1}{36}$

Green potion – shrink by $\frac{1}{15}$

Yellow potion – shrink by $\frac{1}{4}$

Chocolate cake – grow by 12 times

Red velvet cake – grow by 18 times

Carrot cake – grow by 9 times

Lemon cake – grow by 10 times

Find Alice's height after she drinks each potion or eats each bite of cake. If everything goes correctly, Alice will return to her normal height by the end.

Starting Height	Alice Eats or Drinks	Scale factor from above	New Height
54 inches	Red potion	$\frac{1}{9}$	6 inches
6 inches	Chocolate cake		
	Yellow potion		
	Carrot cake		
	Blue potion		
	Lemon cake		
	Green potion		
	Red velvet cake		

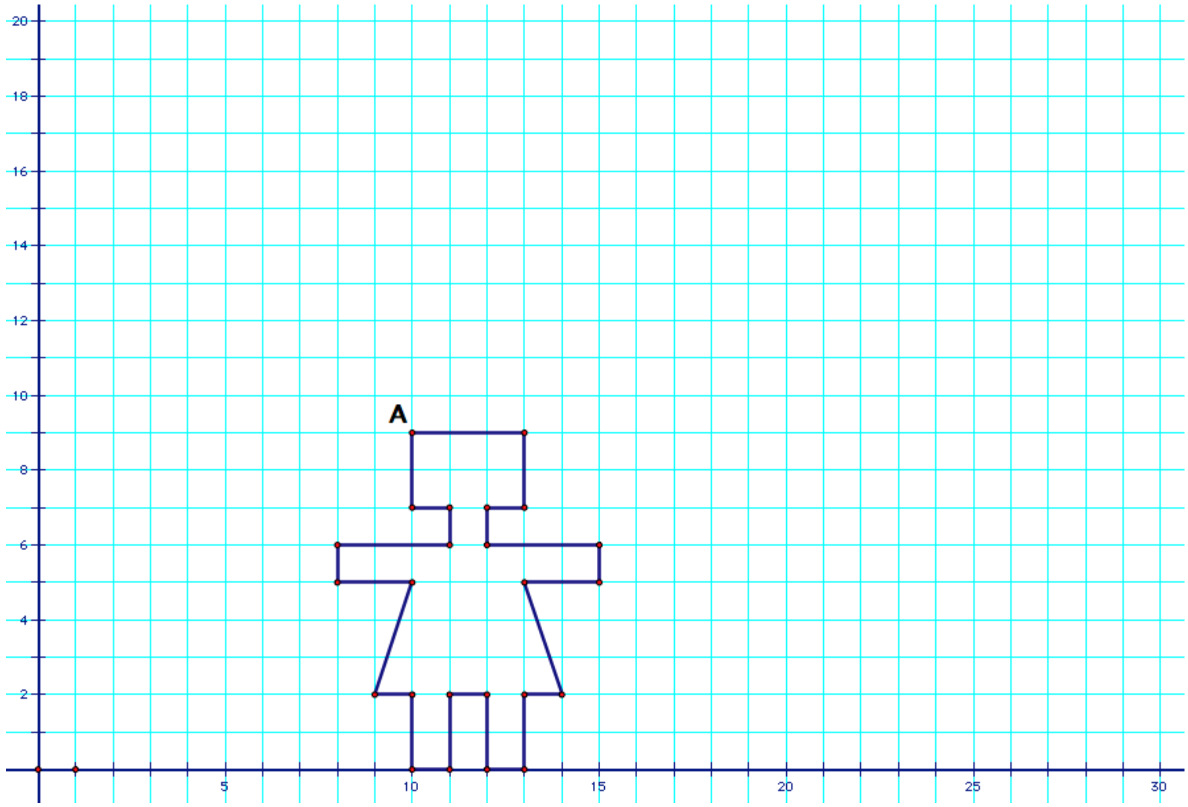
Part 2

A) The graph on the next page shows Alice at her normal height.

B) Place a ruler so that it goes through the origin and point A. Plot point A' such that it is twice as far from the origin as point A. Do the same with all of the other points. Connect the points to show Alice after she has grown.

1. How many times larger is the new Alice? ________________________
2. How much farther away from the origin is the new Alice? _______________________
3. What are the coordinates for point A? ____________ Point A'? ____________
4. What arithmetic operation do you think happened to the coordinates of A?
5. Write your conclusion as an Algebraic Rule $(x, y) \rightarrow (\quad , \quad)$

C) Test your conclusion by looking at some of the other points and determining if their coordinates follow the same pattern.



D) What arithmetic operation on the coordinates do you think would shrink Alice in half?

E) Write your conclusion as an Algebraic rule.

F) If Alice shrinks in half, how far away from the origin will her image be from her preimage?

G) Draw the image of Alice if she is shrunk by a scale factor of ½ from her original height.

H) What would the Algebra Rule be if Alice is shrunk by a factor of ½ from her original height?

Summary: A <u>dilation</u> is

- An enlargement of the pre-image if the ____________ is __________________________________.
- A reduction of the pre-image if the ____________ is __________________________________.
- If the scale factor is 1, then the pre-image and image are __________________________.

The ________________ of dilation is a fixed point in the plane about which all points reference too.

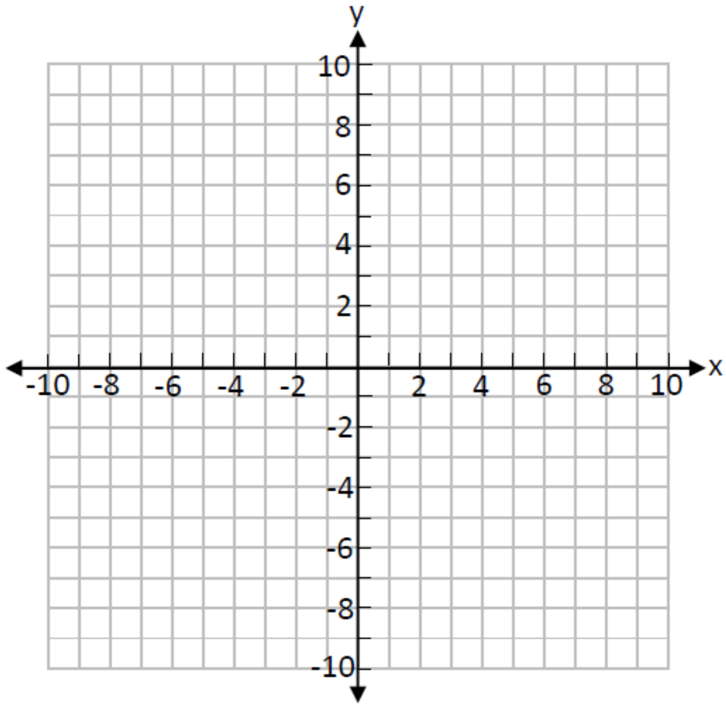
Circle the appropriate choice for the following characteristic/property ~

A dilation is SOMETIMES / ALWAYS / NEVER an 'Isometry'.

The amount by which the image grows or shrinks is called the "_______________ _________________".

Practice: Day 4 Dilations Activity

1. Graph and connect these points: (2, 2) (3, 4) (5, 2) (5, 4).



2. Graph a new figure on the same coordinate plane by applying a scale factor of 2.

   What is the Algebraic Rule for this transformation? ______________________________
   How do the preimage and image compare? Describe the figure and the coordinate pairs.

3. Graph a new figure on the same coordinate plane by applying a scale factor of 1/2.

   What is the Algebraic Rule for this transformation? ______________________________
   Compare the preimage to the dilated figure. Describe the figure and the coordinate pairs.

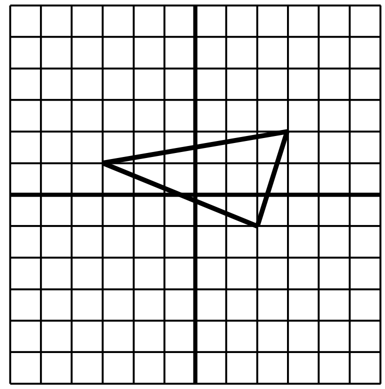
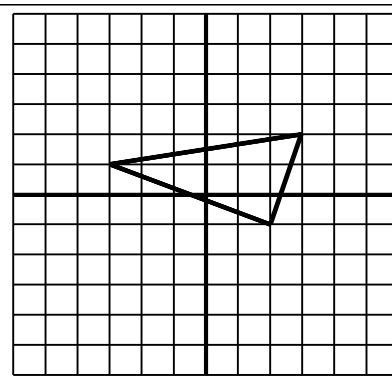
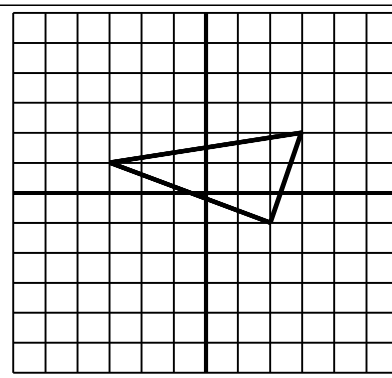
4. What happens when you apply a scale factor greater than 1 to a set of coordinates?

5. What happens when you apply a scale factor less than 1 to a set of coordinates?

6. What happens when you apply a scale factor of 1 to a set of coordinates?

Practice: Dilations with Coordinates

For each problem, graph the image points, and describe the transformation that occurred. Specify if the transformation is an enlargement or reduction and by what scale factor. Then, examine the coordinates to create an Algebraic Rule.

1) The coordinates of ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of A'B'C' are A'(1, -1/2), B'(3/2, 1), and C'(-3/2, 1/2). Transformation: Algebraic Rule:	
2) The coordinates of ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of A'B'C' are A'(4, -2), B'(6, 4), and C'(-6, 2). Transformation: Algebraic Rule:	
3) The coordinates of ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of A'B'C' are A'(3, -3/2), B'(9/2, 3), and C'(-9/2, 3/2). Transformation: Algebraic Rule:	

Summarize with Algebraic Rules:

What type of transformation does the following algebraic rule produce?

$(x, y) \rightarrow (ax, ay)$

if $a > 1$ then________________________

if $0 < a < 1$ then________________________