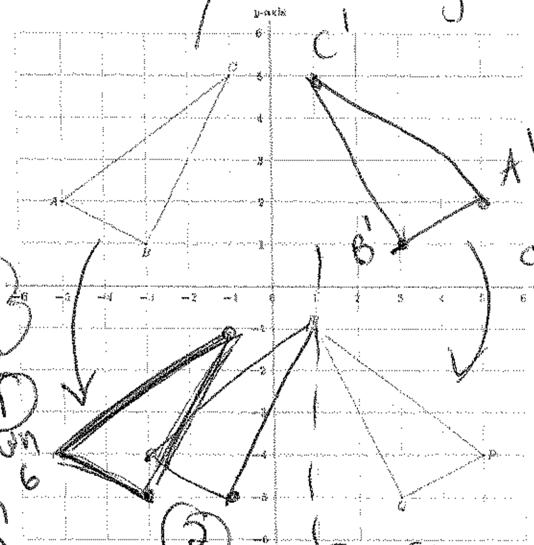
Day 9: Triangle Congruence and Similarity

part a

Warm-Up: Triangles ABC and PQR are shown below in the coordinate plane:

1 refl over y-axis



a. Show that ABC is congruent to PQR with a reflection followed by a translation.

answers vary • Sample: refl over y-axis then down 6

b. If you reverse the order of your reflection and translation in part (a) does it still map ABC to PQR?

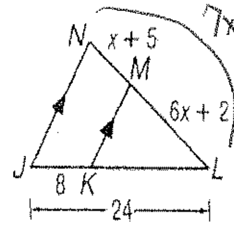
yes

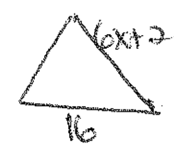
c. Find a second way, different from your work in part (a), to map ABC to PQR using translations, rotations, and/or reflections.

answers vary • Sample: translate down 6 + right 2 THEN refl. over x=1

see bold at left for start

d. **Explain why the triangles are similar and write a similarity statement. Then, find the value of x and the lengths of the segments requested.** $\overline{NL}$ and $\overline{ML}$





3rd Do $\frac{\text{full side}}{\text{full side}} = \frac{\text{full side}}{\text{full side}}$

4th solve $\frac{7x+7}{6x+2} = \frac{24}{16}$

$16(7x+7) = 24(6x+2)$

$112x + 112 = 144x + 48$

$\boxed{x = 2}$

1st • ∠MK ≅ ∠LNJ and ∠LKM ≅ ∠LJN because // lines make corresponding ∠s ≅

• $\boxed{\triangle JNL \sim \triangle KML \text{ by AA} \sim}$

2nd and draw small △ separately + mark in it

Congruence and Similarity Lesson

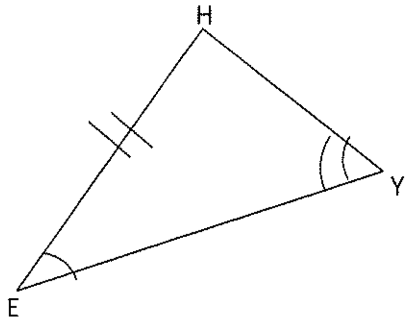
Review: What are the 4 shortcuts to knowing that two triangles are congruent?

SSS ≅, SAS ≅, ASA ≅, and AAS ≅

Once we use one of the shortcuts to show that triangles are congruent, we know that the other 3 parts have congruent matches. In Geometry, we state that "corresponding parts of congruent triangles are congruent, or CPCTC."

#1: ΔHEY is congruent to ΔMAN by AAS. (AAS not ASA because side is not included)

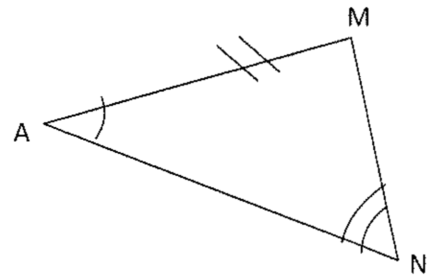
What **other** parts of the triangles are congruent **by CPCTC**?



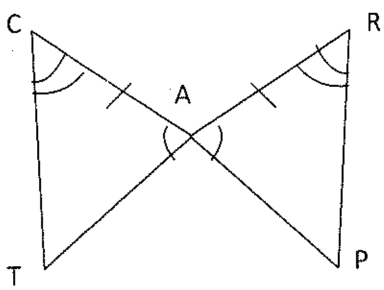
$\overline{HY} \cong \overline{MN}$

$\angle H \cong \angle M$

$\overline{EY} \cong \overline{AN}$



#2:



$\Delta CAT \cong \Delta RAP$ by ASA (ASA not AAS because side is included)

THEREFORE:

$\overline{AT} \cong \overline{AP}$, by CPCTC

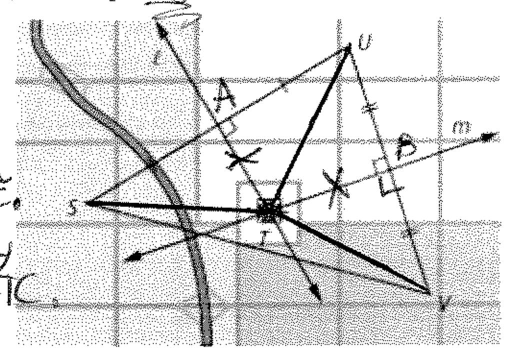
$\angle T \cong \angle P$, by CPCTC

$\overline{CT} \cong \overline{RP}$, by CPCTC

Note: If writing name for angles like the one at vertex A with multiple angles at a vertex, you must use 3 letters $\angle CAT \cong \angle RAP$

Example: Plans for the location of a telecommunications tower that is to serve three northern suburbs of Milwaukee ~~are shown below.~~ Design specifications indicate the tower should be located so that it is equidistant from the center S, U, and V of each of the suburbs. In the diagram, line ℓ is the *perpendicular bisector* of SU. Line m is the perpendicular bisector of UV. = GOAL

1st) Check that all info from problem is marked in the diagram ✓



a. Draw line TS and line TU. How can you show that $TS = TU$.

$\overline{AT} \cong \overline{AT}$ by reflexive property, $\angle SAT$ and $\angle UAT$ are both rt $\angle$s so they're $\cong$. $\Delta SAT \cong \Delta UAT$ by SAS so then $\overline{TS} \cong \overline{TU}$ by CPCTC.

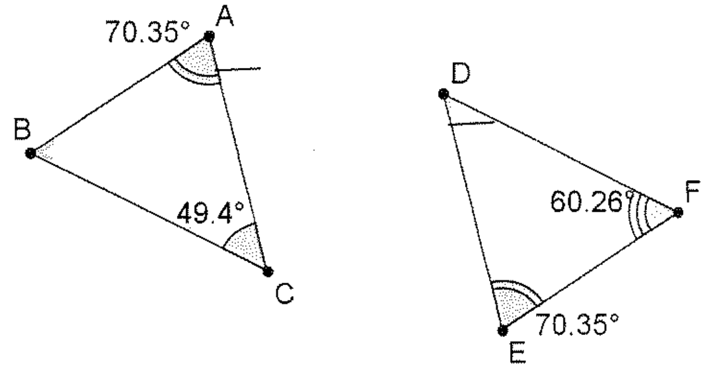
b. Draw ~~line~~ TV on your diagram. Prove that $TU = TV$.

$\overline{TB} \cong \overline{TB}$ by reflexive property. $\angle VBT$ and VBT are both rt $\angle$s so they're $\cong$. $\Delta UBT \cong \Delta VBT$ by SAS so $\overline{TU} \cong \overline{TV}$ by CPCTC.

c. Explain why the tower should be located at point T.

If $\overline{TS} \cong \overline{TU}$ and $\overline{TU} \cong \overline{TV}$ then $\overline{TS} \cong \overline{TV}$ and $\overline{TS} \cong \overline{TU} \cong \overline{TV}$ so T is equidistant from S, U, and V. A tower at T would be in the center.

Are the following triangles congruent? Explain.



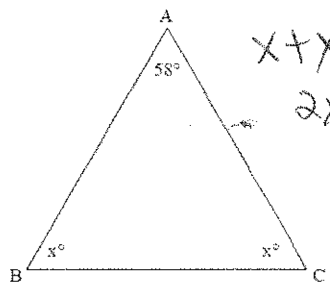
$180 - 70.35 - 49.4 =$

$180 - 70.35 - 60.26 =$

In the previous example, we needed to use the idea that the three angles of a triangle add to $\underline{180°}$.

Let's play with this theorem for a bit…
Solve for the missing variables:

1.



$x + x + 58 = 180$

$2x + 58 = 180$

$2x = 122$

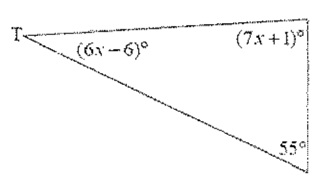
$\boxed{x = 61}$

2. $55 + 6x - 6 + 7x + 1 = 180$

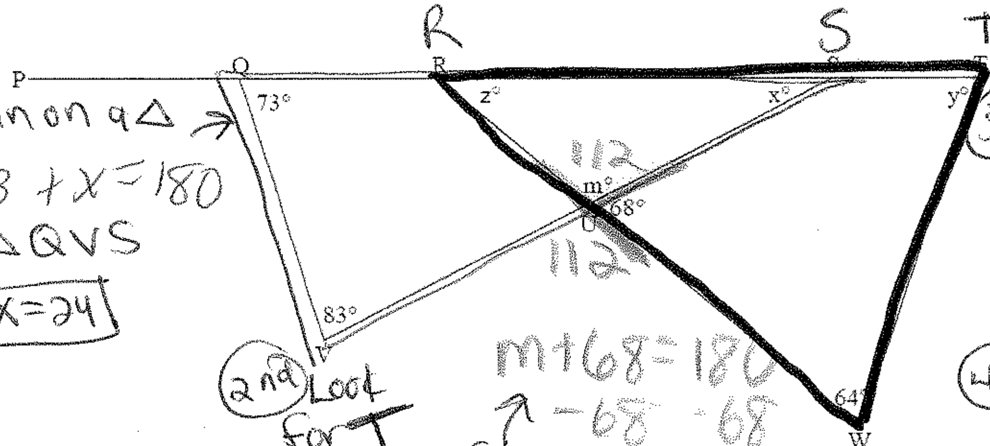
$13x + 50 = 180$

$13x = 130$

$\boxed{x = 10}$



3.



1st zoom in on a △

$73 + 83 + x = 180$

using △QVS

$\boxed{x = 24}$

2nd Look for ⊥

→ linear pair angles sum to 180°

$m + 68 = 180$

$-68 \quad -68$

$\boxed{m = 112}$

3rd look at small △RSU

$z + x + 112 = 180$

$z + 24 + 112 = 180$

$\boxed{z = 44}$

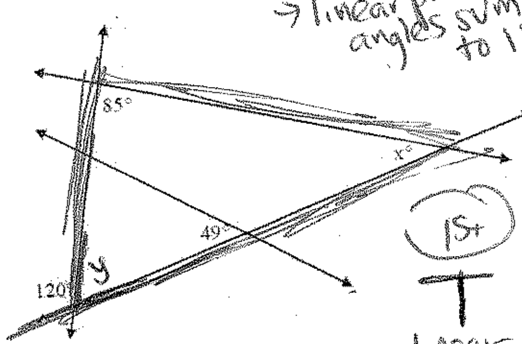
4th look at right side △RTW

$z + y + 64 = 180$

$44 + y + 64 = 180$

$\boxed{y = 72}$

4.



1st

⊥

linear pair angles

$180 - 120$

$\boxed{y = 60}$

2nd

Red △

$85 + x + y = 180$

$85 + x + 60 = 180$

$\boxed{x = 35}$