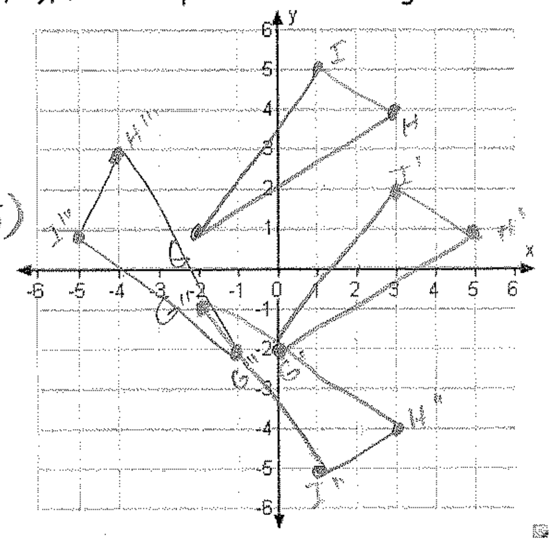
Day 5: Compositions

Warm-Up: Given triangle GHI with G(-2, 1), H(3, 4), and I(1, 5), find the points of the image under the following transformations and write the Algebraic Rule.



1) Translate right 2, down 3

G'(0,-2), H'(5,1), I'(3,2)

$(x, y) \rightarrow (x+2), (y-3)$

2) Reflect over the x-axis

G'(-2,-1), H'(3,-4), I'(1,-5)

$(x, y) \rightarrow (x, -y)$

3) Rotate 90 degrees, counter-clockwise

G'(-1,-2), H'(-4

$(x, y) \rightarrow (-y, x)$

4) Dilate with a scale factor of 3

G'(6,3), H'(9,12), I'(3,15)

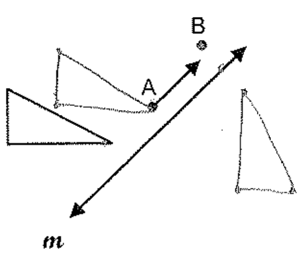
$(x, y) \rightarrow (3x, 3y)$

Compositions

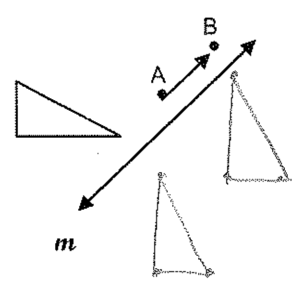
A **glide reflection** is the composition of a translation and a reflection where the translation motion is parallel to the reflection line.

Discovery Activity: Use patty paper to complete the transformations below:

1. Translate A→B, then reflect over line m



2. Reflect over line m, then translate A→B.

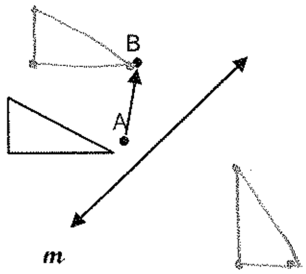


3. Does it matter which transformation is done first in a glide reflection?

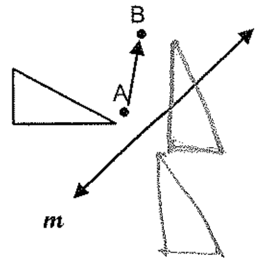
no

Use patty paper to complete the transformations below:

4. Translate A→B, then reflect over line m



5. Reflect over line m, then translate A→B.



6. Is this a glide reflection? Why or why not?

no - The line of reflection and translation motion are not parallel

7. The translation $T(x, y) \rightarrow (x + 4, y + 2)$ is followed by the translation $T(x, y) \rightarrow (x - 8, y - 5)$

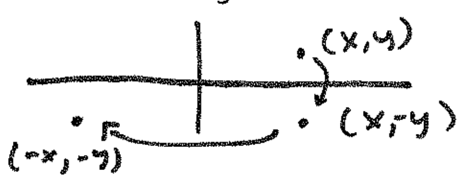
Write this as a single translation. $(x, y) \rightarrow (x-4, y-3)$

8. A 90° rotation followed by a 90° rotation clockwise . Write this as a single rotation.

$R_{0,180}$ → Rotate 180°

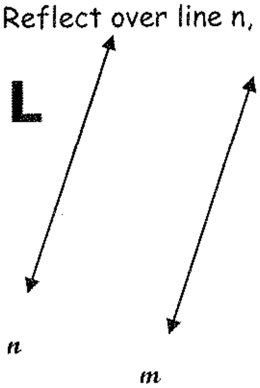
9. The reflection over the x-axis is followed by the reflection over the y-axis. Can this be written as a single reflection? yes $R_{0,180}$

Can this be written as a single transformation? Draw a sketch to support your answer.



Use patty paper to complete the transformations below:

10. Reflect over line n, then reflect over line m.



Measure the distance from the preimage to the final image.

4 cm

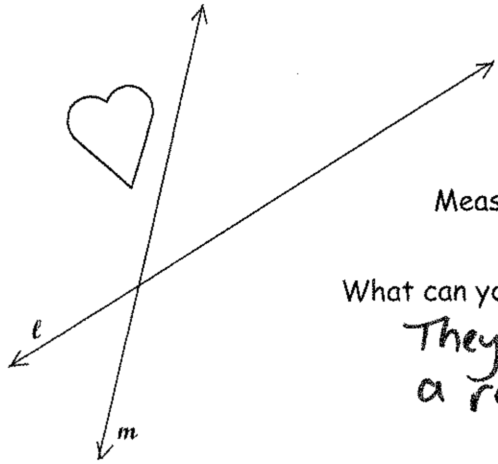
Measure the distance from line n to line m.

2 cm

What can you conclude about consecutive reflections over parallel lines?

They produce a translation such that the distance between the preimage + image is double the distance between the parallel lines.

11. Reflect over line m, then reflect over line l.



Measure the acute angle formed by line l and line m.

45°

Measure the angle of rotation (from the preimage to the final image)

90°

What can you conclude about consecutive reflections over intersecting lines?

They produce the image the same as a rotation twice the measure of the angle between the lines.

12. Two lines intersect at a 50° angle. Write the composition of two reflections over the lines as a single transformation. rotation of 100°

13. Two parallel lines are 3 cm apart. Describe the composition of two reflections over the lines as a single transformation. translation 6cm in direction ⊥ to ‖ reflection lines

14. A figure is reflected over the y-axis and then reflected over the line y = x. Write as a single transformation. $(x, y) \rightarrow (-x, y) \rightarrow (y, -x)$

Summary

A composition is a sequence of transformations.

Two reflections across parallel lines is the same as a translation.

A rotation is the same as a double reflection around nonparallel (intersecting) lines.

The point of rotation is the intersection of the non parallel lines

Same Orientation: Facing the same direction.

TIP to check: If vertices are labeled alphabetically with ABC and A'B'C', read them in alphabetical order. They should read both clockwise or both counterclockwise.

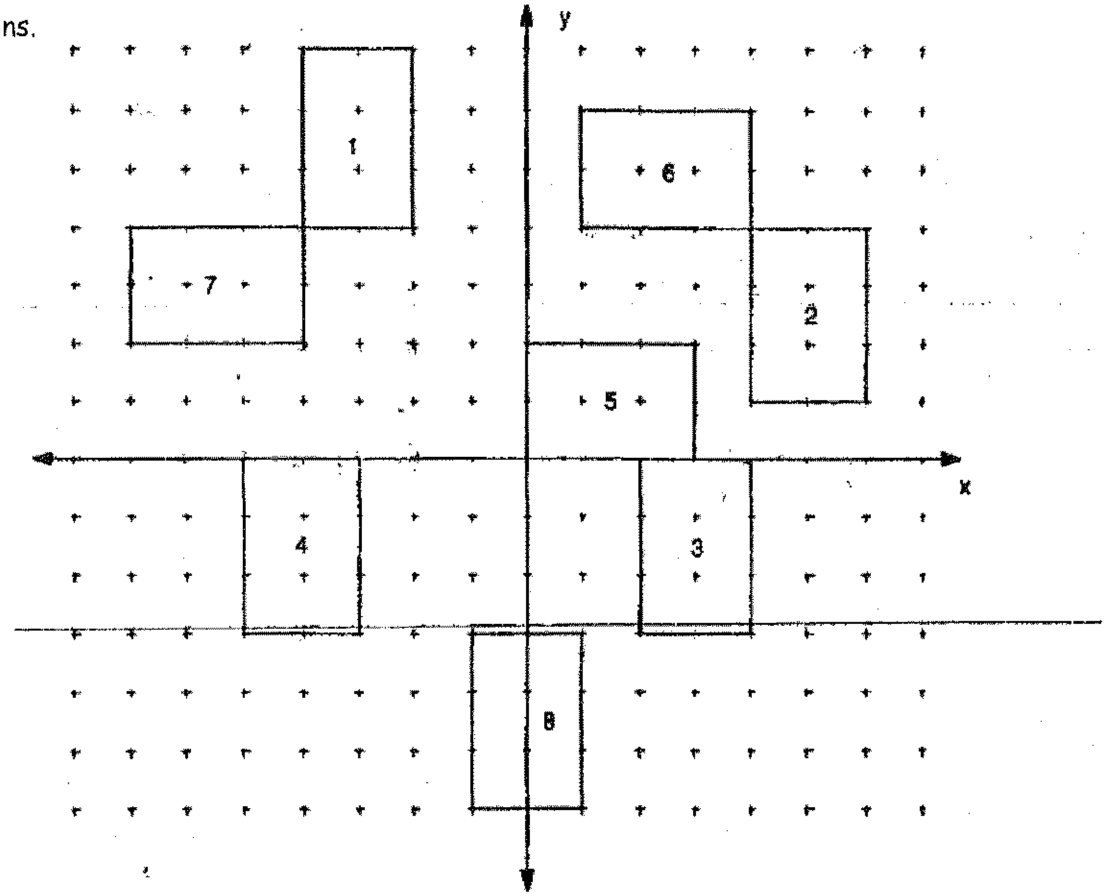
Opposite Orientation: Facing the opposite direction.

TIP to check: If vertices are labeled alphabetically with ABC and A'B'C', read them in alphabetical order. They should read one clockwise and one counterclockwise.

Orientation can be helpful in describing and identifying transformations.

Practice 1: Compositions of Transformations with Coordinates

All of the rectangles are congruent. For each problem, start with the rectangle indicated. Then perform compositions of transformations specified. Perform the transformations in the order specified, one after the other. Determine which rectangle you land on after performing the transformations.



1. Reflect figure 1 over the y-axis. Translate it three units down then rotate it 90° counter-clockwise around (3,1). Which figure does figure 1 now match? **Answer: figure 5**
2. Translate figure 2 one unit down. Reflect it over the x-axis then reflect it over the line x = 4. Which figure does figure 2 now match? 3
3. Reflect figure 3 over the y-axis. Rotate 90° clockwise around (-2, 0) then glide 5 units to the right. Which figure does figure 3 now match? 5
4. Rotate figure 4 90° clockwise around (-3,0). Then reflect over the line y =2 then translate one unit to the left. Which figure does figure 4 now match? 7
5. Translate figure 5 five units to the left. Then rotate 90° clockwise around (-2,2). Then translate up two spaces. Which figure does figure 5 now match? 1
6. Rotate figure 6 90° clockwise around (4,4) then translate three units down. Which figure does figure 6 now match? 2
7. Rotate figure 7 90° clockwise around (-4,4) then reflect over the line x = -4. Which figure does figure 7 now match? 1
8. Reflect figure 8 over the x-axis. Then translate four units to the left. Then reflect over the line y = 1.5 Which figure does figure 8 now match? 4

Practice 2: Composition of Motions with Algebraic Rules

For each problem, there is a composition of motions listed. Write algebraic rules for each of the transformations. Then, determine a single algebraic rule that would accomplish the same motion with a single transformation.

1) Translate the triangle 4 units right and 2 units up, and then reflect the triangle over the line y=x.

$(x,y) \to (x+4, y+2)$ $(x,y) \to (y,x)$

$\boxed{(x,y) \to (y+2, x+4)}$

2) Rotate the triangle 90 degrees counter clockwise, and then dilate the figure by a scale factor of 3.

$(x,y) \to (-y,x)$ $(x,y) \to (3x, 3y)$

$\boxed{(x,y) \to (-3y, 3x)}$

3) Translate the triangle 4 units left and 2 units down, and then reflect the triangle over the y-axis.

$(x,y) \to (x-4, y-2)$ $(x,y) \to (-x,y)$

$\boxed{(x,y) \to (-(x-4), y-2)}$

4) Rotate the triangle 90 degrees clockwise, and then dilate the figure by a scale factor of 1/3.

$(x,y) \to (y,-x)$ $(x,y) \to (\frac{1}{3}x, \frac{1}{3}y)$

$\boxed{(x,y) \to (\frac{1}{3}y, -\frac{1}{3}x)}$

5) Translate the triangle 4 units right and 2 units down, and then reflect the triangle over the x-axis.

$(x,y) \to (x+4, y-2)$ $(x,y) \to (x,-y)$

$\boxed{(x,y) \to (x+4, -(y-2))}$

6) Rotate the triangle 180 degrees counter clockwise, and then dilate the figure by a scale factor of 2.

$(x,y) \to (-x,-y)$ $(x,y) \to (2x, 2y)$

$\boxed{(x,y) \to (-2x, -2y)}$

7) Translate the triangle 4 units left and 2 units up, and then reflect the triangle over the line y=x.

$(x,y) \to (x-4, y+2)$ $(x,y) \to (y,x)$

$\boxed{(x,y) \to (y+2, x-4)}$

8) Rotate the triangle 180 degrees clockwise, and then dilate the figure by a scale factor of 1/2.

$(x,y) \to (-x,-y)$ $(x,y) \to (\frac{1}{2}x, \frac{1}{2}y)$

$\boxed{(x,y) \to (-\frac{1}{2}x, -\frac{1}{2}y)}$