Day 4: Dilations

Warm-Up: Given the line segment with points A(-1, 4) and B(2, 5) graph the image after the following transformations, identify the coordinates of the image, and write the Algebraic Rule for #1 & 2.

1) Reflect over the line y = x. A'(4, -1) B'(5, 2)

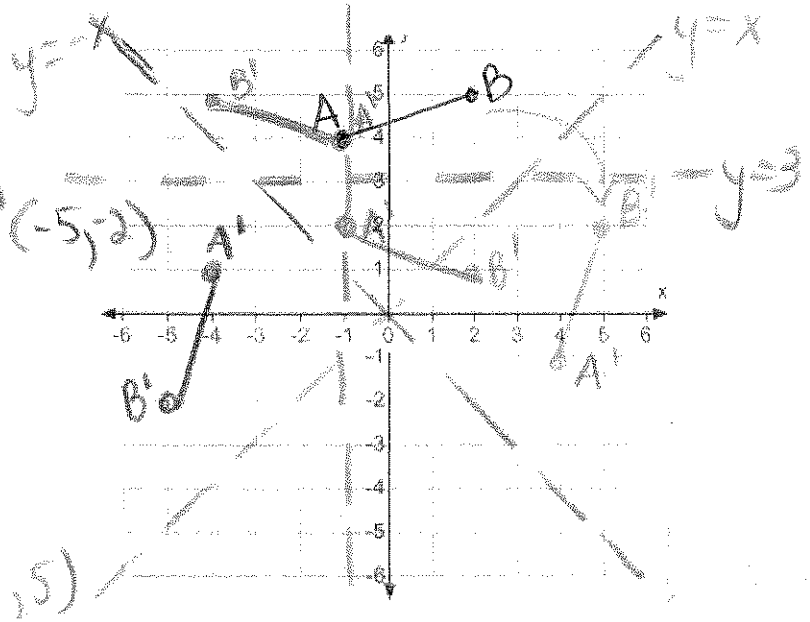
   Algebraic Rule: $(x, y) \rightarrow (y, x)$

2) Reflect over the line y = -x A'(-4, 1) B'(-5, 2)

   Algebraic Rule: $(x, y) \rightarrow (-y, -x)$

3) Reflect over the line y = 3. A'(-1, 2) B'(2, 1)

4) Reflect over the line x = -1. A'(-1, 4) unchanged because on the line B'(-4, 5)



Dilations – Discovery Activity

Alice in Wonderland

In the story, Alice's Adventures in Wonderland, Alice changes size many times during her adventures. The changes occur when she drinks a potion or eats a cake. Problems occur throughout her adventures because Alice does not know when she will grow larger or smaller.

Part 1

As Alice goes through her adventure, she encounters the following potions and cakes:

Red potion – shrink by $\frac{1}{9}$	Chocolate cake – grow by 12 times
Blue potion – shrink by $\frac{1}{36}$	Red velvet cake – grow by 18 times
Green potion – shrink by $\frac{1}{15}$	Carrot cake – grow by 9 times
Yellow potion – shrink by $\frac{1}{4}$	Lemon cake – grow by 10 times

Find Alice's height after she drinks each potion or eats each bite of cake. If everything goes correctly, Alice will return to her normal height by the end.

Starting Height	Alice Eats or Drinks	Scale factor from above	New Height
54 inches	Red potion	$\frac{1}{9}$	$54 \cdot \frac{1}{9} = 6$ inches
6 inches	Chocolate cake	12	$6'' \cdot 12 = 72''$
$72''$	Yellow potion	$\frac{1}{4}$	$72'' \cdot \frac{1}{4} = 18''$
$18''$	Carrot cake	9	$18'' \cdot 9 = 162''$
$162''$	Blue potion	$\frac{1}{36}$	$162'' \cdot \frac{1}{36} = 4.5''$
$4.5''$	Lemon cake	10	$4.5'' \cdot 10 = 45''$
$45''$	Green potion	$\frac{1}{15}$	$45'' \cdot \frac{1}{15} = 3''$
$3''$	Red velvet cake	18	$3'' \cdot 18 = 54''$

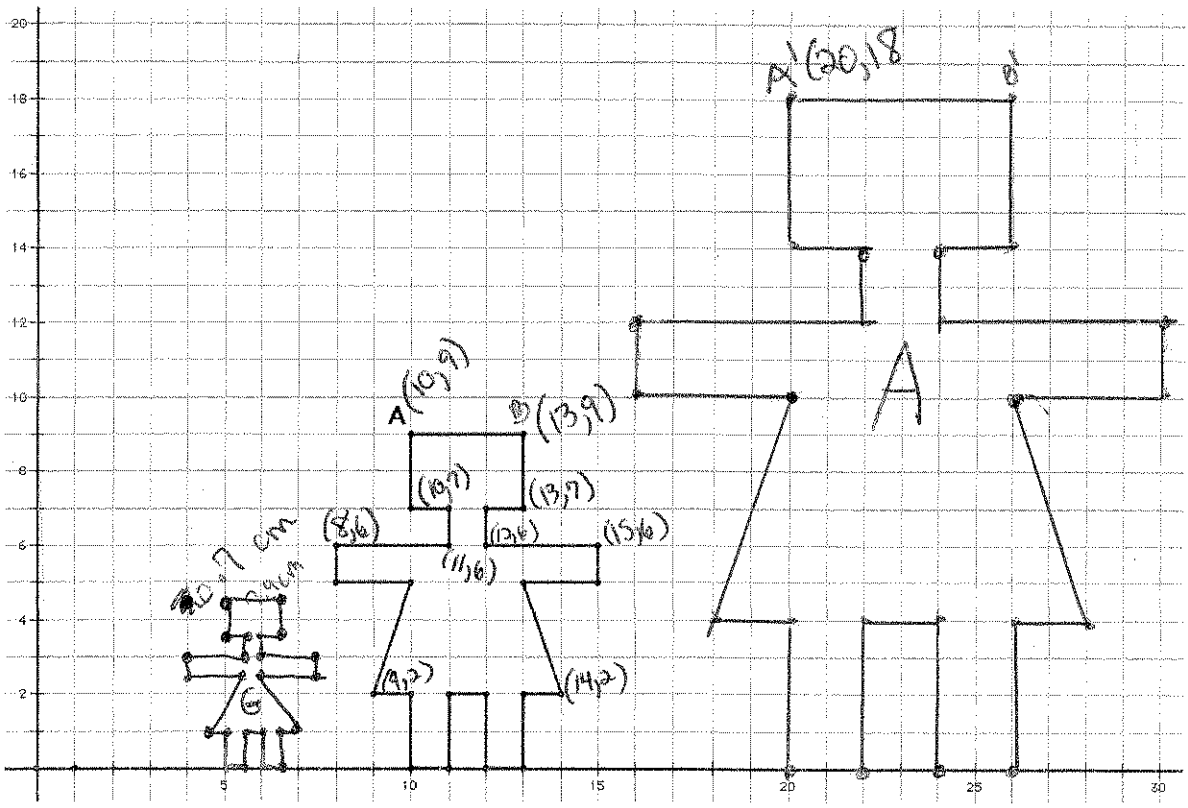
Part 2

A) The graph on the next page shows Alice at her normal height.

B) Place a ruler so that it goes through the origin and point A. Plot point A' such that it is twice as far from the origin as point A. Do the same with all of the other points. Connect the points to show Alice after she has grown.

1. How many times larger is the new Alice? twice as large (2x)
2. How much farther away from the origin is the new Alice? twice as far (2x)
3. What are the coordinates for point A? (10,9) Point A'? (20,18)
4. What arithmetic operation do you think happened to the coordinates of A? multiply x-coordinate by 2, multiply y-coordinate by 2
5. Write your conjecture by completing the Algebraic Rule $(x, y) \rightarrow (2x, 2y)$

C) Test your conjecture by looking at some of the other points and determining if their coordinates follow the same pattern.



D) What arithmetic operation on the coordinates do you think would shrink Alice in half?

half the x-coordinate, half the y-coordinate

E) Write your conjecture as an Algebraic rule.

$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$ OR $(x, y) \rightarrow (\frac{x}{2}, \frac{y}{2})$

F) If Alice shrinks in half, how far away from the origin will her image be from her preimage?

the image will be half as far from the origin as the preimage

G) Draw the image of Alice if she is shrunk by a scale factor of ½ from her original height.

H) What would the Algebra Rule be if Alice is shrunk by a factor of ½ from her original height?

$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$ *Should match your conjecture from earlier

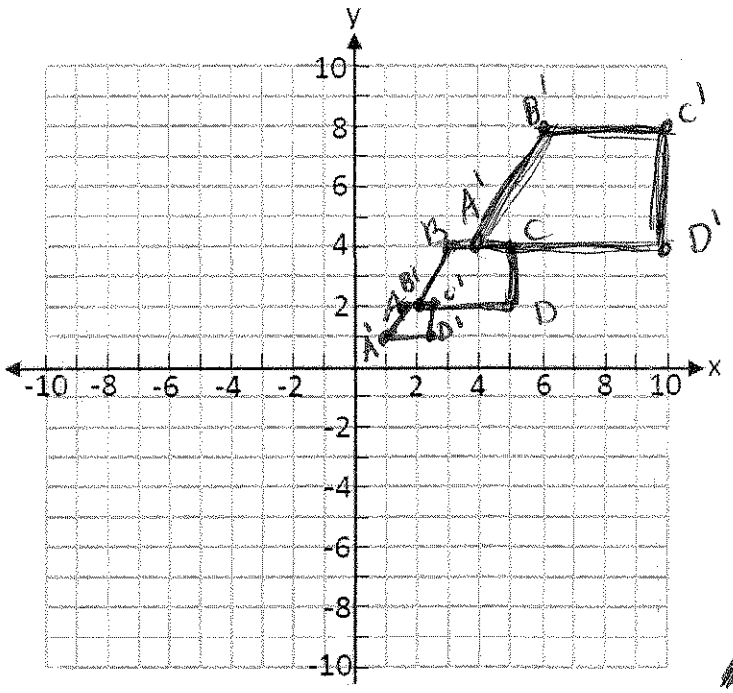
Summary: A dilation is

- an enlargement of the preimage if the scale factor is greater than 1 ($SF > 1$)
- a reduction of the preimage if the scale factor is between 0 and 1 ($0 < SF < 1$)
- If the scale factor is 1, then the preimage and image are congruent.

A dilation is NOT an isometry. (Remember from Day 1, an isometry is a transformation in which the preimage and image are congruent figures)

Practice: Day 4 Dilations Activity

1. Graph and connect these points: (2, 2) (3, 4) (5, 2) (5, 4).



A'(4,4) B'(6,8) D'(10,4) C'(10,8)

2. Graph a new figure on the same coordinate plane by applying a scale factor of 2.

What is the Algebraic Rule for this transformation? $(x,y) \rightarrow (2x,2y)$

How do the preimage and image compare? Describe the figure and the coordinate pairs.

The image is enlarged by a factor of 2. the coordinates are doubled.

3. Graph a new figure on the same coordinate plane by applying a scale factor of 1/2.

A'(1,1) B'(3/2 or 1.5, 2) D'(5/2 or 2.5, 1) C'(5/2, 2)

What is the Algebraic Rule for this transformation? $(x,y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

Compare the preimage to the dilated figure. Describe the figure and the coordinate pairs.

The image is reduced by a factor of 1/2. the coordinates are halved.

4. What happens when you apply a scale factor greater than 1 to a set of coordinates?

an enlargement of the preimage occurs

5. What happens when you apply a scale factor less than 1 to a set of coordinates?

a reduction of the preimage occurs

6. What happens when you apply a scale factor of 1 to a set of coordinates?

an isometry occurs ⇒ the preimage and image are congruent

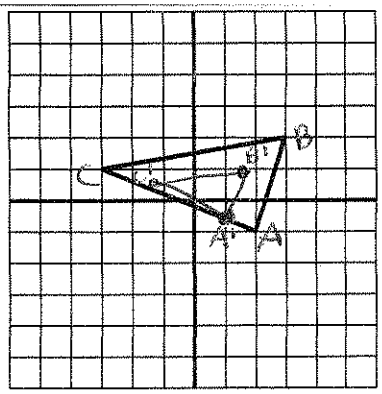
Practice: Dilations with Coordinates

For each problem, graph the image points, and describe the transformation that occurred. Specify if the transformation is an enlargement or reduction and by what scale factor. Then, examine the coordinates to create an Algebraic Rule.

1) The coordinates of △ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of △A'B'C' are A'(1, -1/2), B'(3/2, 1), and C'(-3/2, 1/2).

A reduction by 1/2

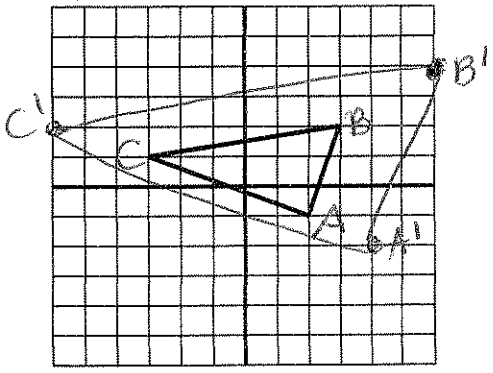
$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$



2) The coordinates of △ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of △A'B'C' are A'(4, -2), B'(6, 4), and C'(-6, 2).

An enlargement by 2

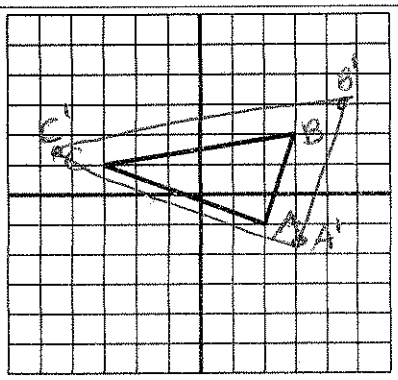
$(x, y) \rightarrow (2x, 2y)$



3) The coordinates of △ABC are A(2, -1), B(3, 2) and C(-3, 1). The coordinates of △A'B'C' are A'(3, -3/2), B'(9/2, 3), and C'(-9/2, 3/2).

A reduction by 1.5 or 3/2

$(x, y) \rightarrow (\frac{3}{2}x, \frac{3}{2}y)$



Summarize with Algebraic Rules:

What type of transformation does the following algebraic rule produce?

$(x, y) \rightarrow (ax, ay)$

if $a > 1$ then enlargement by SF = a

if $0 < a < 1$ then reduction by SF = a