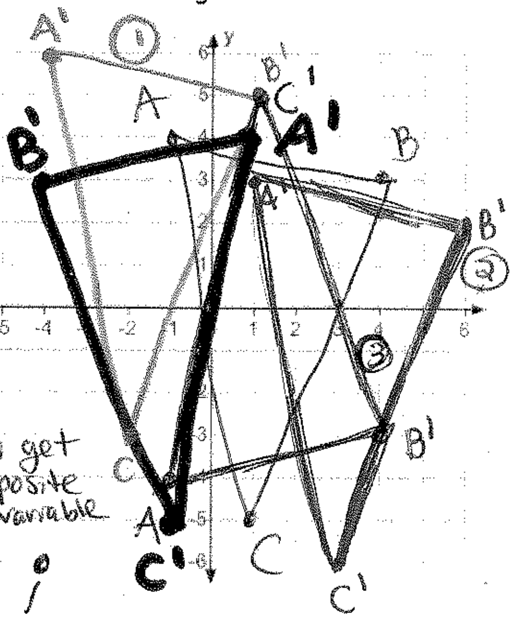
Day 3: Rotations

Warm-Up: Given triangle ABC with A(-1, 4), B(4, 3) and C(1, -5), graph the image points after the following transformations, identify the coordinates of the image, and write the Algebraic Rule for each.

1) Translate left 3, up 2 $A'(-4,6)\ B'(1,5)\ C'(-2,-3)$
   $(x,y) \to (x-3, y+2)$
2) Translate right 2, down 1 $A'(1,3)\ B'(6,2)\ C'(3,-6)$
   $(x,y) \to (x+2, y-1)$
3) Reflect over the x-axis $A'(-1,-4)\ B'(4,-3)\ C'(1,5)$
   $(x,y) \to (x,-y)$
4) Reflect over the y-axis $A'(1,4)\ B'(-4,3)\ C'(-1,-5)$
   $(x,y) \to (-x,y)$
5) Solve the following system $4m + 18n = 80$ $\cdot -3$ ① need to multiply to get same # & opposite signs on 1 variable
   $12m + 34n = 160$
   ② add equations to solve for variable
   $+ \ -12m - 54n = -240$
   $-20n = -80$
   $n = 4$
   $\boxed{(2,4)}$

③ substitute in 1st variable in 1 equation to get 2nd variable
$4m + 18n = 80$
$4m + 18(4) = 80$
$4m + 72 = 80$
$4m = 8$
$m = 2$

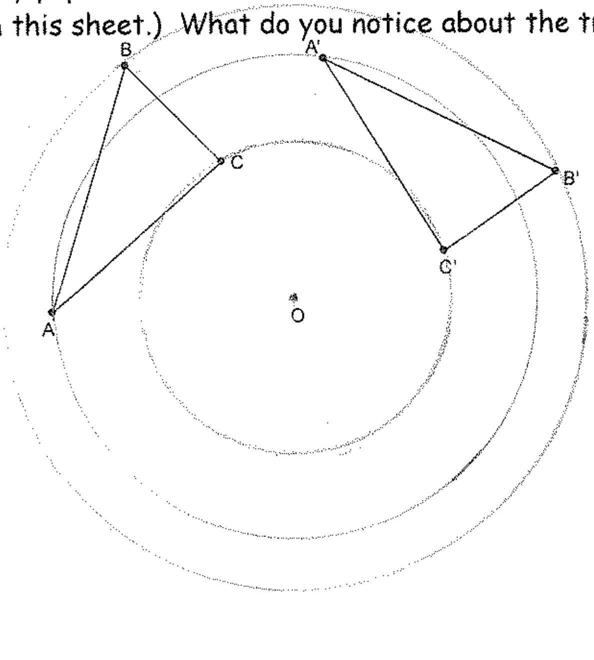


Rotations – Discovery Activity

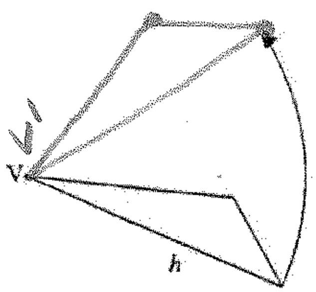
1. Exploration. Triangle *A'B'C'* is a rotation of Triangle *ABC* about the center *O*.

1) Using a compass, draw the circle that has center O and goes through point A.
2) Using a compass, draw the circle that has center O and goes through point B.
3) Using a compass, draw the circle that has center O and goes through point C.
4) What do you know notice about points A', B', and C'? Each point is on the same circle as it's preimage point is.
5) Trace Triangle ABC and point O on patty paper. Put your pencil point on top of the patty paper on point O and turn the patty paper around and around in both directions (keeping the O on your patty paper on top of the O on this sheet.) What do you notice about the triangle as it rotates around in either direction?

The points of the triangle lie on the circles as you rotate the triangle.



2. Bill rotates figure h using center V as shown by the arrow. Draw and label the image of figure h. Explain how you made your drawing.



- What method did you use?
  patty paper rotation with pencil on center V
- What does the arrow tell you?
  the angle of rotation + direction of rotation
- What is point V? What happens to point V after the motion is performed?
  V is the center of rotation. V stays fixed even after the motion is performed.

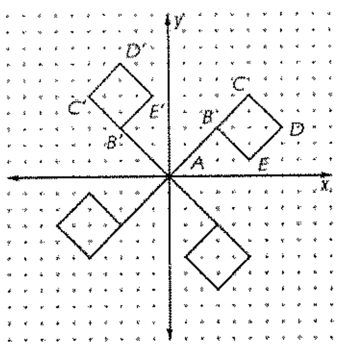
3. Summary

This type of transformation is called a rotation. To rotate an object, you must specify the angle of rotation, the point around which the rotation is to occur, and the direction.

* The standard for rotations, unless otherwise noted, is counterclockwise.

4. Visualizing Rotations Centered About the Origin

The flag shown below is rotated about the origin 90°, 180°, and 270°. Flag ABCDE is the preimage. Flag A'B'C'D'E' is a 90° counterclockwise rotation of ABCDE.

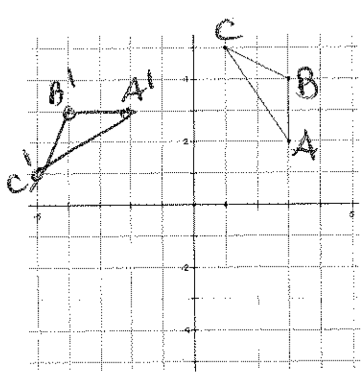
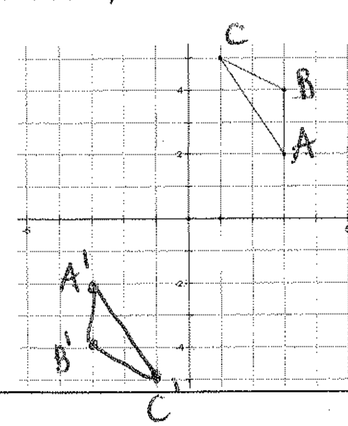


Counter-Clockwise
Positive Degrees!

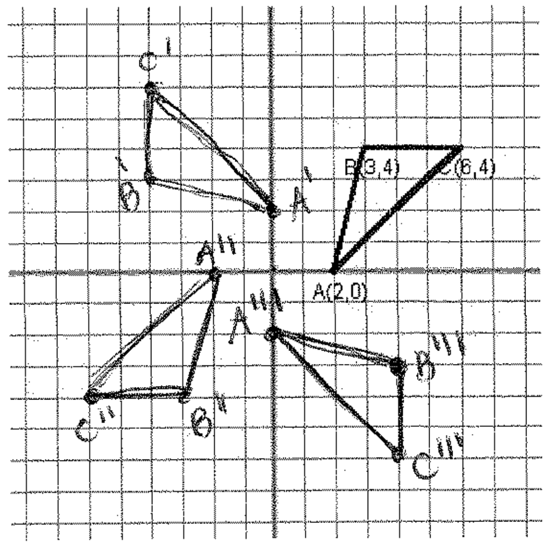
Clockwise
Negative Degrees!

NOTE: Unless otherwise specified, the standard for rotations is **counterclockwise!**

5. Practice: Use patty paper to complete the following.

Perform a 90°, counterclockwise rotation. 	Perform a 180°, counterclockwise rotation. 

5. Rotations on the Coordinate Plane Exploration



1) Triangle *ABC* has coordinates *A*(2, 0), B(3, 4), *C*(6, 4). Trace the triangle and the x- and y-axes on patty paper.

2) Rotate Triangle *ABC* 90°, using the axes you traced to help you line it back up. Record the new coordinates. A'(0 , 2), B'(−4 , 3), C'(−4 , 6)

3) Rotate Triangle *ABC* 270°, using the axes you traced to help you line it up. Record the new coordinates. A'(0 , −2), B'(4 , −3), C'(4 , −6)
A''' on my picture B''' C'''

4) Rotate Triangle *ABC* 180°, using the axes you traced to help you line it back up correctly. Record the new coordinates. A'(−2 , 0), B'(−3 , −4), C'(−6 , −4)

Checkpoint: Look at the patterns and complete the rule. Then write the rule using proper notation for 1 – 3.

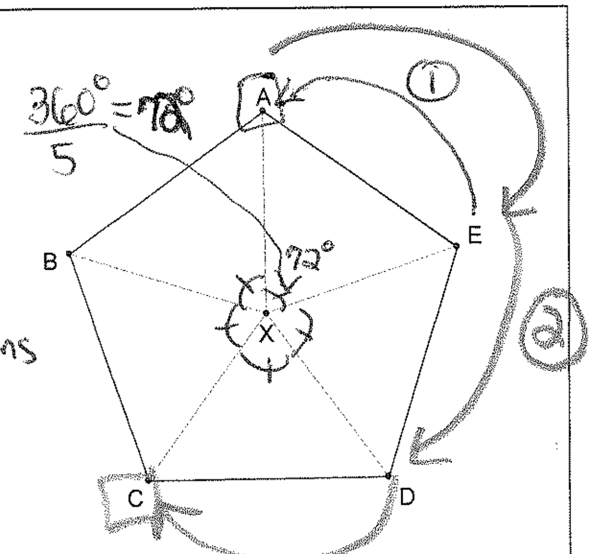
1. A 90° counter-clockwise rotation maps (x, y) → (−y , x). $R_{O,90}$
2. A 270° counter-clockwise rotation maps (x, y) → (y , −x). $R_{O,270}$
3. A 180° rotation maps (x, y) → (−x , −y). $R_{O,180}$
4. A rotation of 270° clockwise is equivalent to a rotation of 90° counterclockwise.
5. A rotation of 270° counterclockwise is equivalent to a rotation of 90° clockwise.

Summarize with Algebraic Rules: *Remember, unless told other wise the two standards are (1) counterclockwise direction (2) origin is the center

What type of transformation does each of the following algebraic rules produce?

$(x, y) \to (-y, x)$ Rotate 90° Counterclockwise	$(x, y) \to (-x, -y)$ Rotate 180° (counterclockwise OR clockwise)
$(x, y) \to (y, -x)$ Rotate 270° counterclockwise	
Can you figure out this one on your own? Describe the rotation the results from the following algebraic rule $(x, y) \to (x, y)$ Rotate 360° or 0°	
The rotation from the algebraic rule $(x, y) \to (x, y)$ is the same as another transformation. Describe that transformation.	

Practice

ABCDE is a regular pentagon. A regular polygon has all congruent angles and all congruent side lengths. (1) Name the image of point E for a counterclockwise 72° rotation about X. $\boxed{A}$ (2) Name the image of point A for a clockwise 216° rotation about X. $\frac{216°}{72} = 3$ turns $\boxed{C}$ Describe 2 transformations with a preimage of point D and image of B. • 144° clockwise rotation from D with a center of X.	

• 216° rotation from D with a center of X.
(Remember: counterclockwise is the standard & is understood to be the direction if not told otherwise)

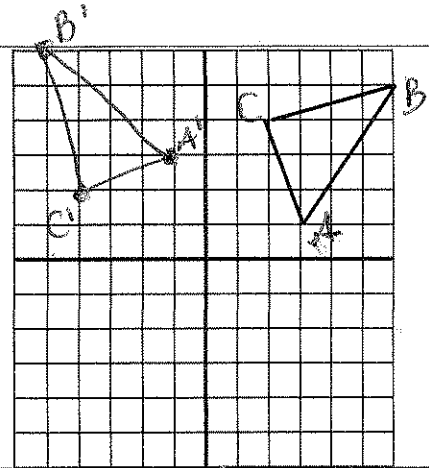
Practice: Rotations with Coordinates

For each problem graph the image points. Specifically describe in words the rotation that occurred. Then, write the Algebraic Rule for the rotation.

1) The coordinates of △ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of △A'B'C' are A'(-1, 3), B'(-5, 6), and C'(-4, 2).

90° rotation

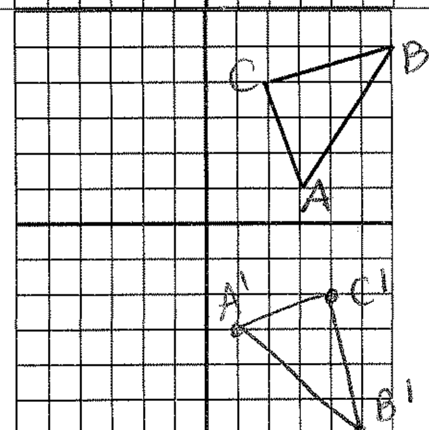
$(x, y) \to (-y, x)$



2) The coordinates of △ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of △A'B'C' are A'(1, -3), B'(5, -6), and C'(4, -2).

270° rotation
(or 90° clockwise rotation)

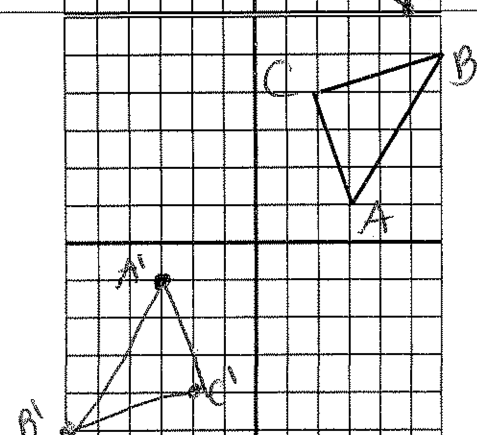
$(x, y) \to (y, -x)$



3) The coordinates of △ABC are A(3, 1), B(6, 5) and C(2, 4). The coordinates of △A'B'C' are A'(-3, -1), B'(-6, -5), and C'(-2, -4).

180° rotation
(or double reflection over x-axis + y-axis)

$(x, y) \to (-x, -y)$



4) The coordinates of △ABC are A(2, -1), B(6, 4) and C(-3, 2). The coordinates of △A'B'C' are A'(-1, -2), B'(4, -6), and C'(2, 3).

270° rotation
(or 90° clockwise rotation)

$(x, y) \to (y, -x)$