Day 10: Midsegments, Isosceles Triangles

Warm-up:

$d^2 = (\text{change in } x)^2 + (\text{change in } y)^2$

1. Given a triangle with vertices A (2, 5), B(3, 6) and C(1, 6), use the distance formula to decide whether triangle ABC is scalene, isosceles, or equilateral. Show ALL work!!

$\sqrt{(AB)^2} = \sqrt{(2-3)^2 + (5-6)^2}$

$AB = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\sqrt{(BC)^2} = \sqrt{(3-1)^2 + (6-6)^2}$

$BC = \sqrt{(2)^2 + 0^2} = \sqrt{4} = 2$

$\sqrt{(AC)^2} = \sqrt{(2-1)^2 + (5-6)^2}$

$AC = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$AB = \sqrt{2}$, $AC = \sqrt{2}$, $BC = 2$

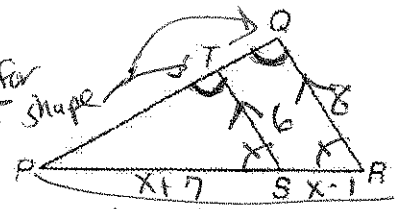
$\triangle ABC$ is isosceles because 2 sides $\cong$

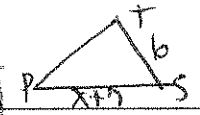
2. Given $\overline{TS} \| \overline{QR}$, explain why the triangles are similar and write a similarity statement. Then use TS = 6, PS = x + 7, QR = 8, and SR = x – 1, to find PS and PR.

- $\angle PTS \cong \angle PQR$ and $\angle PST \cong \angle PRQ$ (corresponding angles $\cong$ when lines are $\|$) → look for F shape
- $\triangle PTS \sim \triangle PQR$ by AA $\sim$
- use $\frac{\text{full side}}{\text{full side}} = \frac{\text{full side}}{\text{full side}}$ → $\frac{6}{8} = \frac{x+7}{2x+6}$ → $x = 5$

PS = 12

PR = 16





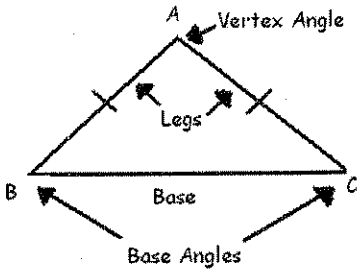
* be careful ... x is not always the end of the problem!

Midsegments, Isosceles Triangles Notes/Practice

Isosceles and Equilateral Triangles

Isosceles triangles are commonly found in the real world in buildings and bridges.

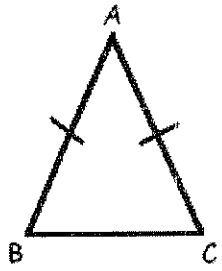
- The congruent sides of an isosceles triangle are its legs.
- The third side is the base.
- The two congruent sides form the vertex angle.
- The other two angles are the base angles.



Isosceles Triangle Theorem:

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

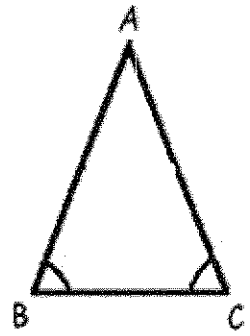
Ex. If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



Converse of the Isosceles Triangle Theorem:

If two angles of a triangle are congruent, then the sides opposite those ~~sides~~ angles are congruent.

Ex. If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



Example: Triangle ABC is isosceles with vertex C. What is the value of x? What is the measure of each angle?

$2x+40 = 3x+22$

$18 = x$

$m\angle A = 2(18)+40 = 76$

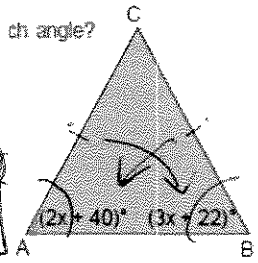
$m\angle B = 3(18)+22 = 76$

$m\angle A + m\angle B + m\angle C = 180$

$76 + 76 + m\angle C = 180$

$152 + m\angle C = 180$

$m\angle C = 28$

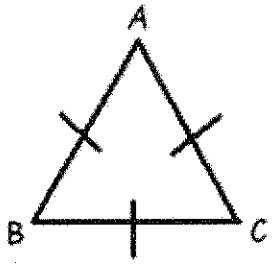


Corollary to Isosceles Triangle Theorem:

If a triangle is equilateral, then the triangle is equiangular.

Ex. If $\overline{AB} \cong \overline{AC} \cong \overline{BC}$, then

$\angle A \cong \angle B \cong \angle C$.

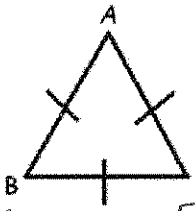


Example: Given triangle ABC, what is the measure of angle A?

If $m\angle A = x$

then $x + x + x = 180$

so $3x = 180$ and $x = 60$ so $m\angle A = 60$



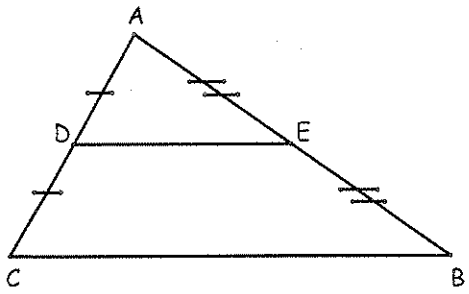
How would you define the **midpoint** of a segment?

The midpoint of a segment divides a segment into 2 congruent segments.

Ex/ M is the mdpt of $\overline{DE}$

Midsegment of a triangle (doesn't have to be isosceles or equilateral)

A midsegment of a triangle is a segment connecting the midpoints of 2 sides. It measures half the length of the opposite side

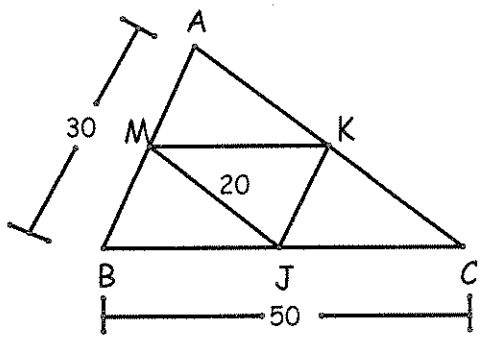


$\overline{DE}$ is a midsegment of $\triangle ABC$

$DE = \frac{1}{2} BC$

and $\overline{DE} \parallel \overline{CB}$

Example 1: In triangle ABC, **M**, **J**, and **K** are midpoints



AB = 30 KJ = 15 $\frac{1}{2}(30)$

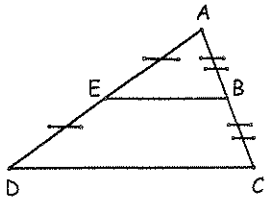
BC = 50 MK = 25 $\frac{1}{2}(50)$

AC = 40 MJ = 20

$= 2 \cdot MJ$

$= 2 \cdot 20$

Example 2: AB = 10, CD = 18



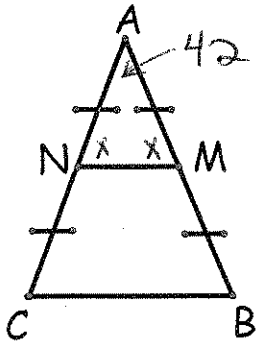
BC = 10 EB = 9

$= \frac{1}{2}(18)$

Example 3: Given m<A = 42°,

Find $m\angle AMN$ = 69°

$m\angle ANM$ = 69°



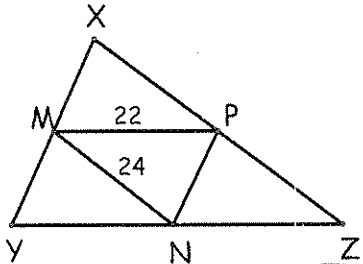
$42 + x + x = 180$

$2x = 138$

$x = 69$

Example 4: In $\triangle XYZ$, M, N, and P are midpoints.

The perimeter of $\triangle MNP$ is 60. Find XY and YZ.



$pe\triangle MNP = MP + MN + NP = 60$

$22 + 24 + NP = 60$

$46 + NP = 60$

$NP = 14$

$XY = 2(NP)$

$= 2(14)$

$XY = 28$

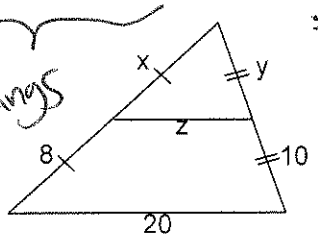
$YZ = 2MP = 2(22) = 44 = YZ$

Practice…Directions: Find the values of the variables. You must show all work to receive full credit. Figures are not drawn to scale.

1. x = 8 y = 10 z = 10

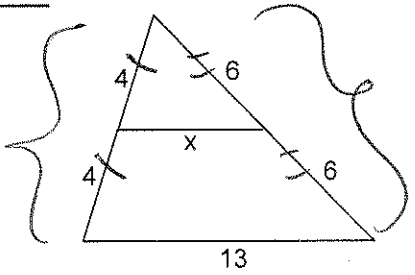
use markings

$= \frac{1}{2}(20)$



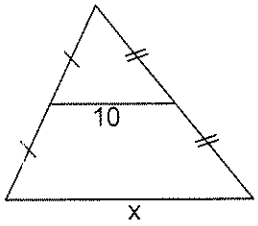
2. x = 6.5 $= \frac{1}{2}(13)$ → midsegment is half of base

numbers marked show ≅ sides



3. x = 20

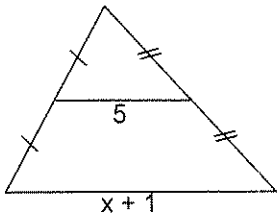
$= 10 \cdot 2$



4. x = 9

$\frac{1}{2}(x+1) = 5$

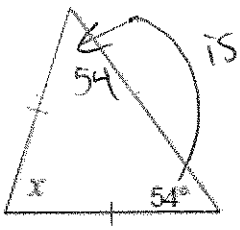
$x + 1 = 10$



5. x = 72

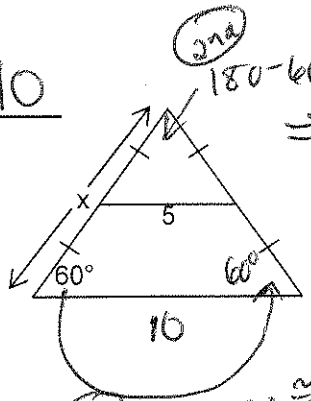
$54 + 54 + x = 180$

$108 + x = 180$



ISOS → ≅ sides

6. x = 10



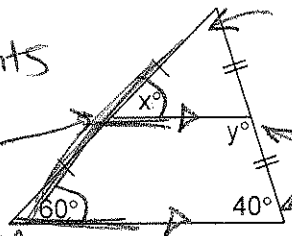
2nd $180 - 60 - 60 = 60$

⇒ 3rd △ is equiangular so it is equilateral

$x = 10$

1st ISOS △ so ≅ ∠s

7. x = 60 y = 140



1st midsegments are // to base

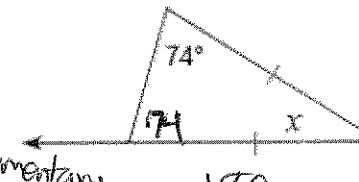
2nd corresponding angles are ≅ … look for F (or rotated F)

$x = 60$

3rd same side interior angles are supplementary … look for ⊾ or ⊿

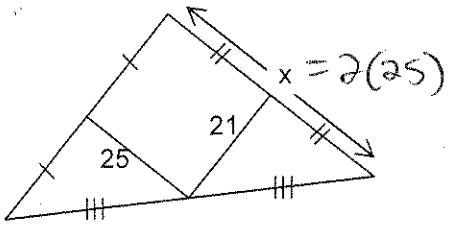
$180 - 40 = 140 = y$

8. x = 32

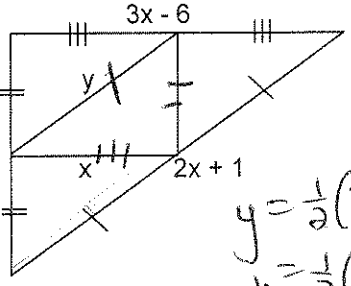


$180 - 74 - 74 = 32$

9. x = 50



10. x = 6 y = 13/2 or 6.5



$x = \frac{1}{2}(3x - 6)$

$2x = 3x - 6$

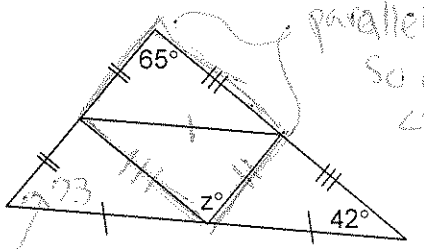
$6 = x$

$y = \frac{1}{2}(2x + 1)$

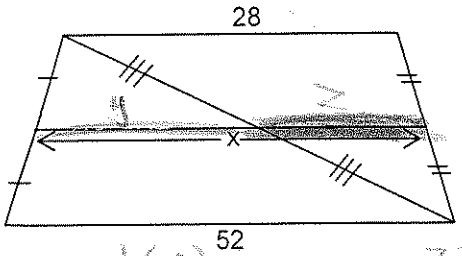
$y = \frac{1}{2}(2 \cdot 6 + 1)$

$y = \frac{1}{2}(13)$

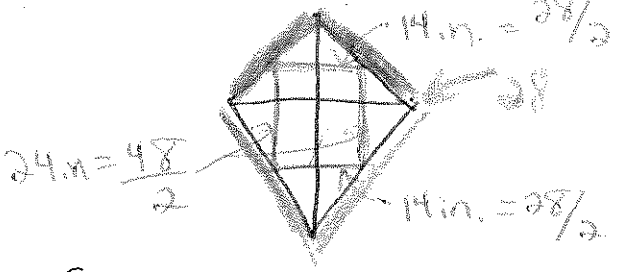
11. z = 65°



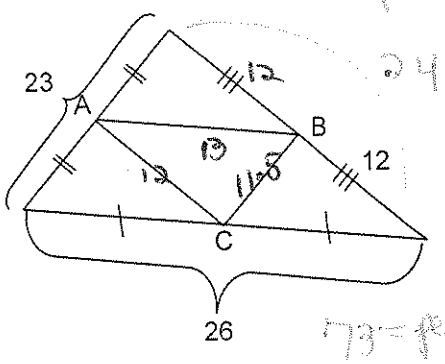
12. x = 40



13. Sadie is designing a kite. The diagonals measure 28 in and 48 in. She wants to decorate the mid-segments with purple ribbon. How much ribbon must she purchase? Draw a picture!



14. Find the perimeter of ΔABC.



15. One side of the Rock and Roll Hall of Fame is an isosceles triangle made up of smaller triangles based on mid-segments. The length of the base of the building is 229.5 feet. What would the base of the bold triangle be?